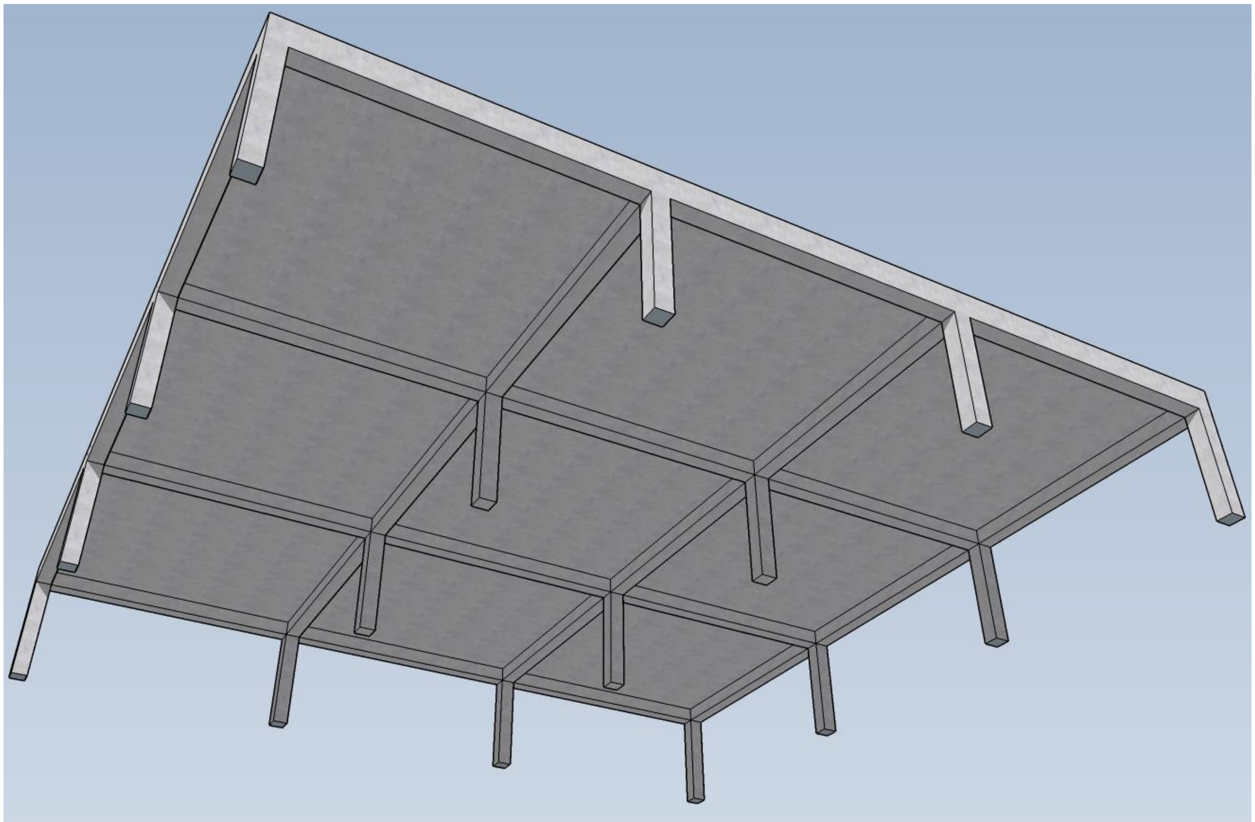
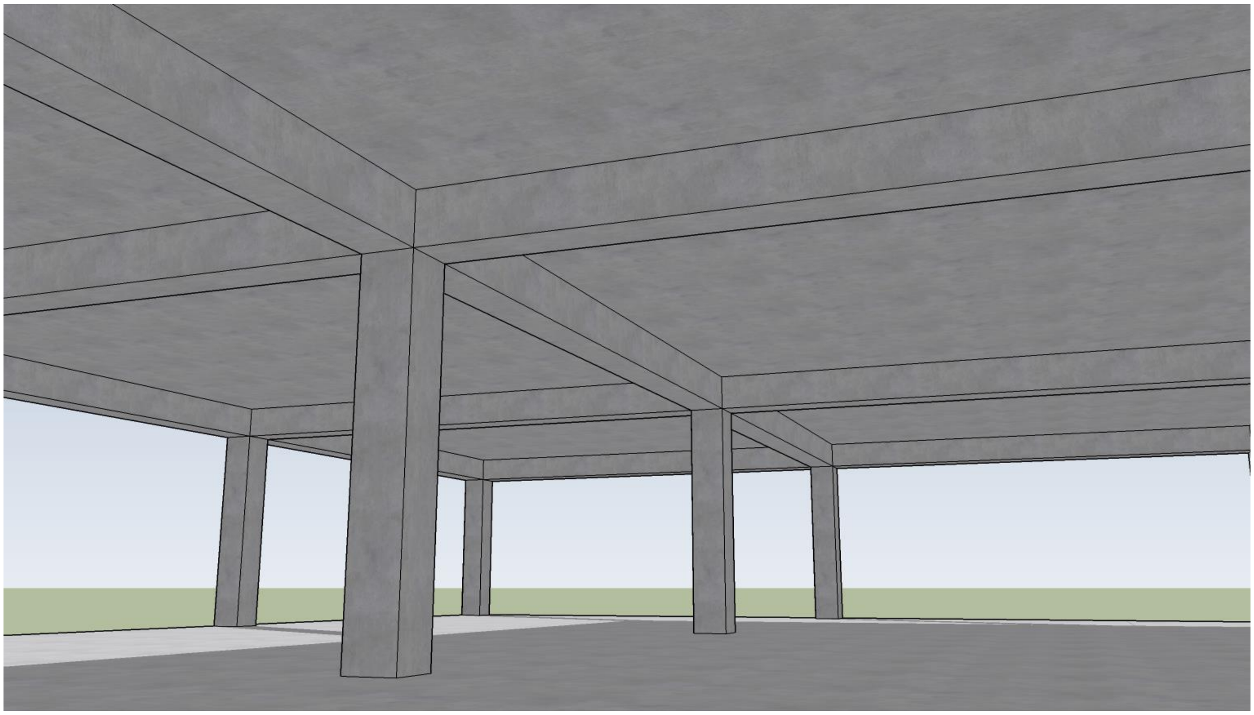
Two-Way Concrete Floor Slab with Beams Design and Detailing





Two-Way Concrete Floor Slab with Beams Design and Detailing

Design the slab system shown in Figure 1 for an intermediate floor where the story height = 12 ft, column cross-sectional dimensions = 18 in. x 18 in., edge beam dimensions = 14 in. x 27 in., interior beam dimensions = 14 in. x 20 in., and unfactored live load = 100 psf. The lateral loads are resisted by shear walls. Normal weight concrete with ultimate strength (fc'= 4000 psi) is used for all members, respectively. And reinforcement with Fy = 60,000 psi is used. Use the Equivalent Frame Method (EFM) and compare the results with spSlab model results.

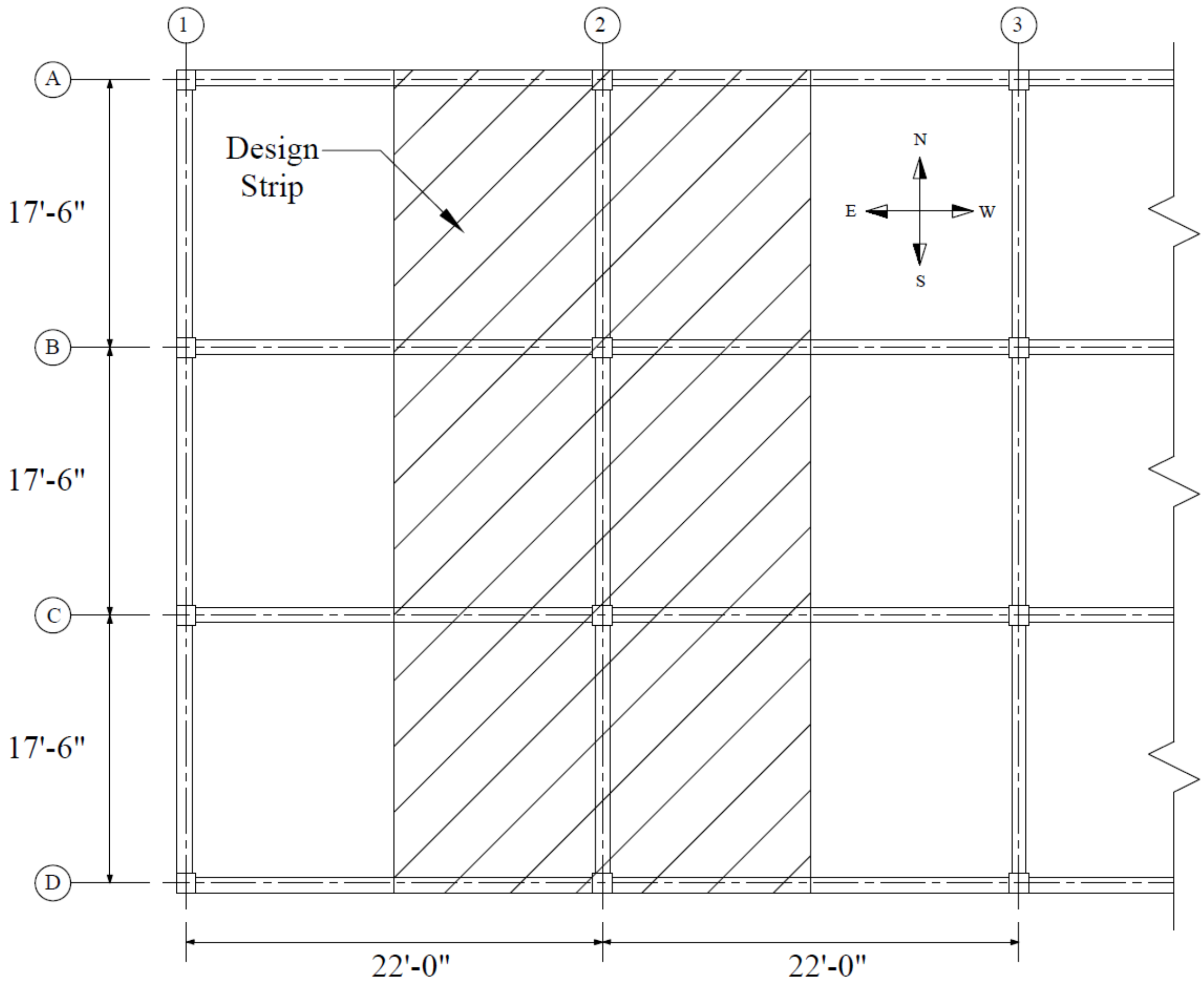


Figure 1 – Two-Way Slab with Beams Spanning between all Supports

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Minimum Design Loads for Buildings and Other Structures (ASCE/SEI 7-10)

International Code Council, 2012 International Building Code, Washington, D.C., 2012

References

Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association.

Concrete Floor Systems (Guide to Estimating and Economizing), Second Edition, 2002 David A. Fanella

Simplified Design of Reinforced Concrete Buildings, Fourth Edition, 2011 Mahmoud E. Kamara and Lawrence C. Novak

Design Data

Floor-to-Floor Height = 12 ft (provided by architectural drawings)

Columns = 18 x 18 in.

Interior beams = 14 x 20 in.

Edge beams = 14 x 27 in.

w_c = 150 pcf

f_c' = 4,000 psi

f_y = 60,000 psi

Live load, L_o = 100 psf (Office building) ***ASCE/SEI 7-10 (Table 4-1)***

Solution

1. Preliminary Slab Thickness Sizing

Control of deflections. ***ACI 318-14 (8.3.1.2)***

In lieu of detailed calculation for deflections, ACI 318 Code gives minimum thickness for two-way slab with beams spanning between supports on all sides in ***Table 8.3.1.2***.

Beam-to-slab flexural stiffness (relative stiffness) ratio (α_f) is computed as follows:

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{I_b}{I_s}$$ ***ACI 318-14 (8.10.2.7b)***

The moment of inertia for the effective beam and slab sections can be calculated as follows:

$$I_s = \frac{l_2 h^3}{12} \text{ and } I_b = \left(\frac{ba^3}{12}\right) \times f$$

Then,

$$\alpha_f = \left(\frac{b}{l_2}\right)\left(\frac{a}{h}\right)^3 f$$

<u>For Edge Beams:</u>

The effective beam and slab sections for the computation of stiffness ratio for edge beam is shown in Figure 2.

<u>For North-South Edge Beam:</u>

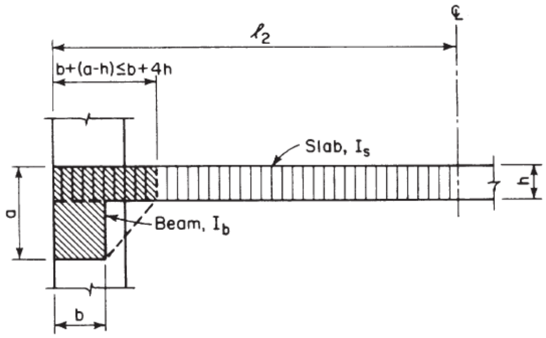
$$l_2 = \frac{22\times12}{2} + \frac{18}{2} = 141 \text{ in.}$$

$$\frac{a}{h} = \frac{27}{6} = 4.5$$

$$\frac{b}{h} = \frac{14}{6} = 2.33$$

$f = 1.47$ using Figure 3.

$$\alpha_f = \left(\frac{14}{141}\right)\left(\frac{27}{6}\right)^3 (1.47) = 13.30$$



<u>Figure 2 – Effective Beam and Slab Sections (Edge Beam)</u>

<u>For East-West Edge Beam:</u>

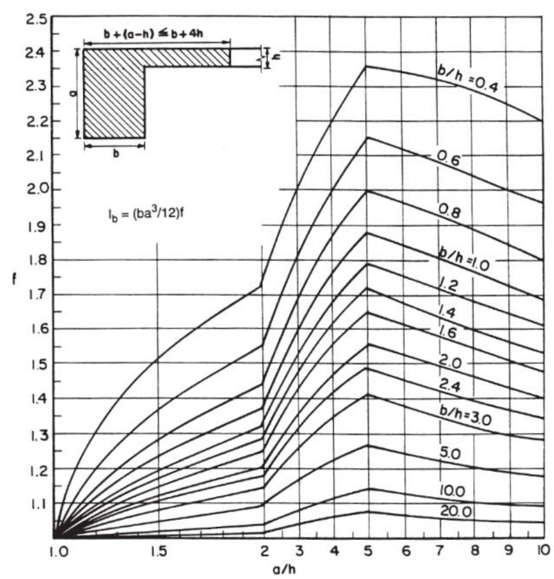
$$l_2 = \frac{17.5\times12}{2} + \frac{18}{2} = 114 \text{ in.}$$

$$\frac{a}{h} = \frac{27}{6} = 4.5$$

$$\frac{b}{h} = \frac{14}{6} = 2.33$$

$f = 1.47$ using Figure 3.

$$\alpha_f = \left(\frac{14}{114}\right)\left(\frac{27}{6}\right)^3 (1.47) = 16.45$$



<u>Figure 3 – Beam Stiffness (Edge Beam)</u>

For interior Beams:

The effective beam and slab sections for the computation of stiffness ratio for interior beam is shown in Figure 4.

For North-South Interior Beam:

$$l_2 = 22 \times 12 = 264 \text{ in.}$$

$$\frac{a}{h} = \frac{20}{6} = 3.33$$

$$\frac{b}{h} = \frac{14}{6} = 2.33$$

$f = 1.61$ using Figure 5.

$$\alpha_f = \left(\frac{14}{264}\right)\left(\frac{20}{6}\right)^3 (1.61) = 3.16$$

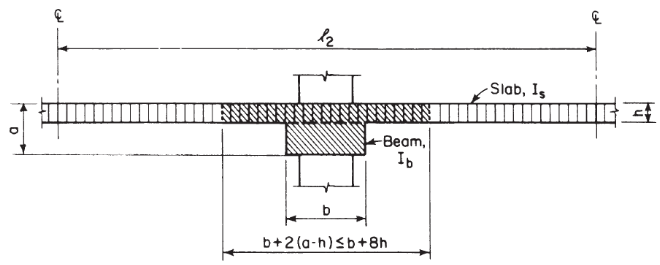


Figure 4 – Effective Beam and Slab Sections (Interior Beam)

For East-West Interior Beam:

$$l_2 = 17.5 \times 12 = 210 \text{ in.}$$

$$\frac{a}{h} = \frac{20}{6} = 3.33$$

$$\frac{b}{h} = \frac{14}{6} = 2.33$$

$f = 1.61$ using Figure 5.

$$\alpha_f = \left(\frac{14}{210}\right)\left(\frac{20}{6}\right)^3 (1.61) = 3.98$$

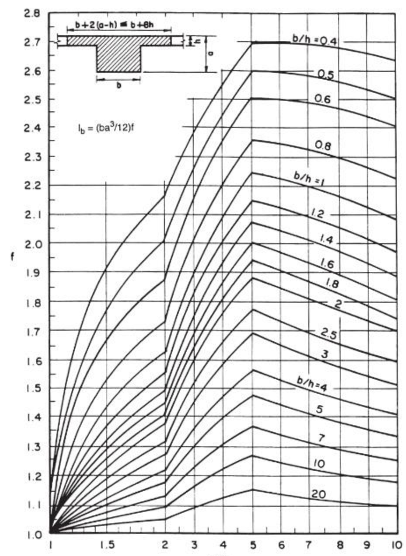


Figure 5 – Beam Stiffness (Interior Beam)

Since $\alpha_f > 2.0$ for all beams, the minimum slab thickness is given by:

$$h_{\min} = greater\ of \left\{ \begin{array}{c} \dfrac{l_n\left(0.8 + \dfrac{f_y}{200,000}\right)}{36 + 9\beta} \\ 5.0 \end{array} \right\}$$ **ACI 318-14 (8.3.1.2)**

Where:

l_n = clear span in the long direction measured face to face of columns = 20.5 ft = 246 in.

$$\beta = \frac{\text{clear span in the long direction}}{\text{clear span in the short direction}} = \frac{22 - 18/12}{17.2 - 18/12} = 1.28$$

$$h_{\min} = greater\ of \begin{Bmatrix} \dfrac{246\left(0.8+\dfrac{60,000}{200,000}\right)}{36+9(1.28)} = 5.7 \\ 5.0 \end{Bmatrix} = 5.7 \text{ in.}$$

Use 6 in. slab thickness.

2. Two-Way Slab Analysis and Design – Using Equivalent Frame Method (EFM)

ACI 318 states that a slab system shall be designed by any procedure satisfying equilibrium and geometric compatibility, provided that strength and serviceability criteria are satisfied. Distinction of two-systems from one-way systems is given by ***ACI 318-14 (R8.10.2.3 & R8.3.1.2)***.

ACI 318 permits the use of Direct Design Method (DDM) and Equivalent Frame Method (EFM) for the gravity load analysis of orthogonal frames and is applicable to flat plates, flat slabs, and slabs with beams. The following sections outline the solution per EFM and spSlab software. The solution per DDM can be found in the "Two-Way Plate Concrete Floor System Design" example.

EFM is the most comprehensive and detailed procedure provided by the ACI 318 for the analysis and design of two-way slab systems where the structure is modeled by a series of equivalent frames (interior and exterior) on column lines taken longitudinally and transversely through the building.

The equivalent frame consists of three parts:

1) Horizontal slab-beam strip, including any beams spanning in the direction of the frame. Different values of moment of inertia along the axis of slab-beams should be taken into account where the gross moment of inertia at any cross section outside of joints or column capitals shall be taken, and the moment of inertia of the slab-beam at the face of the column, bracket or capital divide by the quantity $(1-c_2/l_2)^2$ shall be assumed for the calculation of the moment of inertia of slab-beams from the center of the column to the face of the column, bracket or capital. ***ACI 318-14 (8.11.3)***
2) Columns or other vertical supporting members, extending above and below the slab. Different values of moment of inertia along the axis of columns should be taken into account where the moment of inertia of columns from top and bottom of the slab-beam at a joint shall be assumed to be infinite, and the gross cross section of the concrete is permitted to be used to determine the moment of inertia of columns at any cross section outside of joints or column capitals. ***ACI 318-14 (8.11.4)***
3) Elements of the structure (Torsional members) that provide moment transfer between the horizontal and vertical members. These elements shall be assumed to have a constant cross section throughout their length consisting of the greatest of the following: (1) portion of slab having a width equal to that of the column, bracket, or capital in the direction of the span for which moments are being determined, (2) portion of slab specified in (1) plus that part of the transverse beam above and below the slab for monolithic or fully composite construction, (3) the transverse beam includes that portion of slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness. ***ACI 318-14 (8.11.5)***

2.1. Equivalent frame method limitations

In EFM, live load shall be arranged in accordance with 6.4.3 which requires slab systems to be analyzed and designed for the most demanding set of forces established by investigating the effects of live load placed in various critical patterns. ***ACI 318-14 (8.11.1.2 & 6.4.3)***

Complete analysis must include representative interior and exterior equivalent frames in both the longitudinal and transverse directions of the floor. ***ACI 318-14 (8.11.2.1)***

Panels shall be rectangular, with a ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2. ***ACI 318-14 (8.10.2.3)***

2.2. Frame members of equivalent frame

Determine moment distribution factors and fixed-end moments for the equivalent frame members. The moment distribution procedure will be used to analyze the equivalent frame. Stiffness factors k, carry over factors COF, and fixed-end moment factors FEM for the slab-beams and column members are determined using the design aids tables at ***Appendix 20A of PCA Notes on ACI 318-11***. These calculations are shown below.

a. Flexural stiffness of slab-beams at both ends, K_{sb}.

$$\frac{c_{N1}}{\ell_1} = \frac{18}{(17.5\times12)} = 0.0857 \approx 0.1\ ,\ \frac{c_{N2}}{\ell_2} = \frac{18}{(22\times12)} = 0.0682$$

For $c_{F1} = c_{F2}$ stiffness factors, $k_{NF} = k_{FN} = 4.11$ ***PCA Notes on ACI 318-11 (Table A1)***

Thus, $K_{sb} = k_{NF}\dfrac{E_c I_{sb}}{\ell_1} = 4.11\dfrac{E_c I_{sb}}{\ell_1}$ ***PCA Notes on ACI 318-11 (Table A1)***

Where I_{sb} is the moment of inertia of slab-beam section shown in Figure 6 and can be computed with the aid of Figure 7 as follows:

$$I_{sb} = C_t\left(\frac{b_w h^3}{12}\right) = 2.72\left(\frac{14\times20^3}{12}\right) = 25387 \text{ in.}^4$$

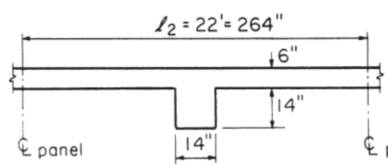


Figure 6 – Cross-Section of Slab-Beam

$$K_{sb} = 4.11\frac{E_c\times25{,}387}{17.5\times12} = 497\ E_c$$

Carry-over factor COF = 0.507 ***PCA Notes on ACI 318-11 (Table A1)***

Fixed-end moment FEM $= 0.0842 w_u \ell_2 \ell_1^2$ ***PCA Notes on ACI 318-11 (Table A1)***

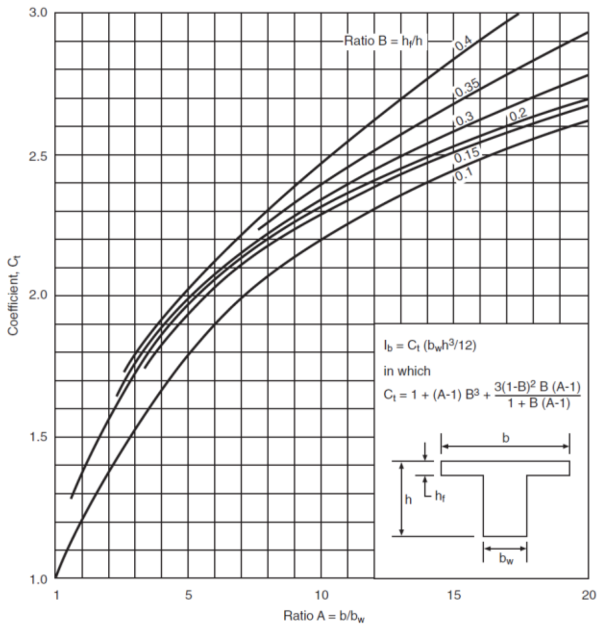


Figure 7 – Coefficient C_t for Gross Moment of Inertia of Flanged Sections

b. Flexural stiffness of column members at both ends, K_c.

Referring to ***Table A7, Appendix 20A***:

For Interior Columns:

$t_a = 20 - 6/2 = 17$ in. , $t_b = 3$ in.

$H = 12$ ft $= 144$ in., $H_c = 144 - 17 - 3 = 124$ in., $\dfrac{t_a}{t_b} = 5.67$, $\dfrac{H}{H_c} = 1.16$

Thus, $k_{c,\,top} = 6.82$ and $k_{c,\,bottom} = 4.99$ by interpolation.

$$I_c = \frac{c^4}{12} = \frac{(18)^4}{12} = 8{,}748 \text{ in.}^4$$

$$\ell_c = 12\,ft = 144 \text{ in.}$$

$$K_c = \frac{k_c E_{cc} I_c}{\ell_c}$$

PCA Notes on ACI 318-11 (Table A7)

$$K_{c,\,top} = \frac{6.82 \times 8748 \times E_c}{144} = 414 E_c$$

$$K_{c,\,bottom} = \frac{4.99 \times 8748 \times E_c}{144} = 303 E_c$$

For Exterior Columns:

$t_a = 27 - 6/2 = 24$ in. , $t_b = 3$ in.

$H = 12$ ft $= 144$ in., $H_c = 144 - 24 - 3 = 117$ in. , $\dfrac{t_a}{t_b} = 8.0$, $\dfrac{H}{H_c} = 1.23$

Thus, $k_{c,\ top} = 8.57$ and $k_{c,\ bottom} = 5.31$ by interpolation.

$$I_c = \frac{c^4}{12} = \frac{(18)^4}{12} = 8,748 \text{ in.}^4$$

$$\ell_c = 12\,ft = 144 \text{ in.}$$

$$K_c = \frac{k_c E_{cc} I_c}{\ell_c}$$

PCA Notes on ACI 318-11 (Table A7)

$$K_{c,\ top} = \frac{8.57 \times 8748 \times E_c}{144} = 521 E_c$$

$$K_{c,\ bottom} = \frac{5.31 \times 8748 \times E_c}{144} = 323 E_c$$

c. Torsional stiffness of torsional members, K_t.

$$K_t = \frac{9E_{cs}C}{[\ell_2(1-\frac{c_2}{\ell_2})^3]}$$

ACI 318-14 (R.8.11.5)

For Interior Columns:

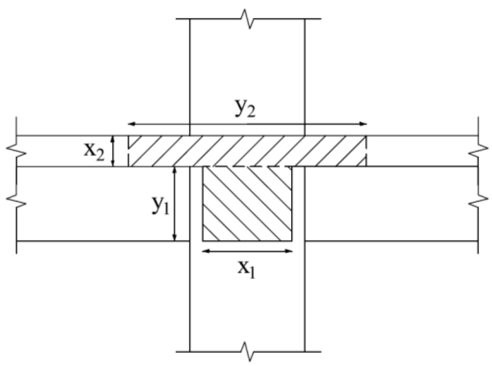
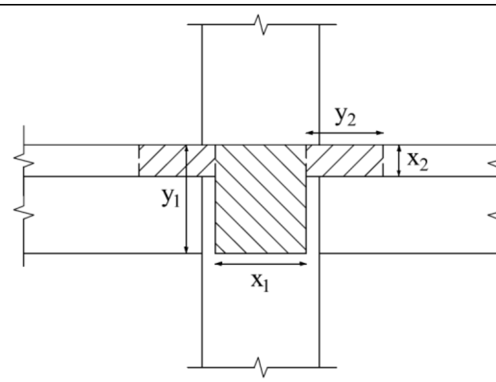
$$K_t = \frac{9E_c \times 11,698}{264(0.932)^3} = 493E_c$$

Where:

$$1-\frac{c_2}{\ell_2} = 1-\frac{18}{22\times 12} = 0.932$$

$$C = \sum(1-0.63\frac{x}{y})(\frac{x^3y}{3})$$

ACI 318-14 (Eq. 8.10.5.2b)

x_1 = 14 in y_1 = 14 in C_1 = 4738	x_2 = 6 in y_2 = 42 in C_2 = 2,752	x_1 = 14 in y_1 = 20 in C_1 = 10,226	x_2 = 6 in y_2 = 14 in C_2 = 736
$\sum$C = 4738 + 2,752 = 7,490 in^4		$\sum$C = 10,226 + 736 x 2 = 11,698 in^4	
			

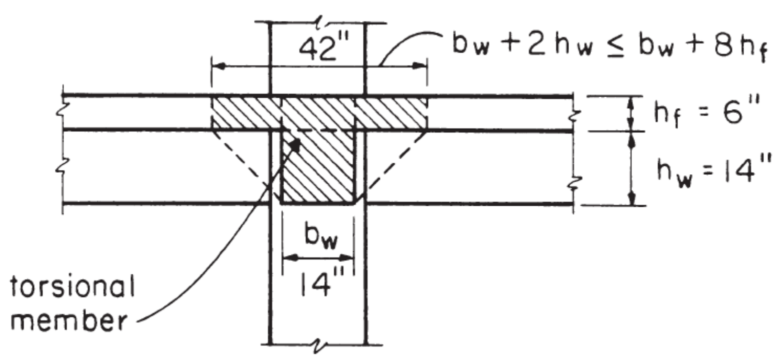


Figure 8 – Attached Torsional Member at Interior Column

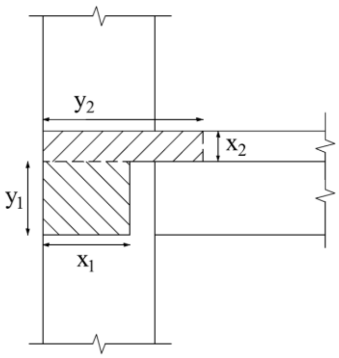
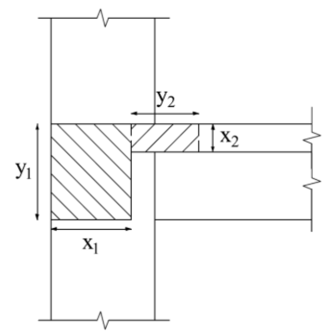
<u>For Exterior Columns:</u>

$$K_t = \frac{9E_c \times 17,868}{264(0.932)^3} = 752E_c$$

Where:

$$1 - \frac{c_2}{\ell_2} = 1 - \frac{18}{22 \times 12} = 0.932$$

$$C = \sum(1 - 0.63\frac{x}{y})(\frac{x^3 y}{3})$$ ***ACI 318-14 (Eq. 8.10.5.2b)***

<table>
<tr><td>x_1 = 14 in
y_1 = 21 in
C_1 = 11,141</td><td>x_2 = 6 in
y_2 = 35 in
C_2 = 2,248</td><td>x_1 = 14 in
y_1 = 27 in
C_1 = 16,628</td><td>x_2 = 6 in
y_2 = 21 in
C_2 = 1,240</td></tr>
<tr><td colspan="2">∑C = 11,141 + 2,248 = 13,389 in^4</td><td colspan="2">∑C = 16,628 + 1,240 = 17,868 in^4</td></tr>
<tr><td colspan="2">
</td><td colspan="2">
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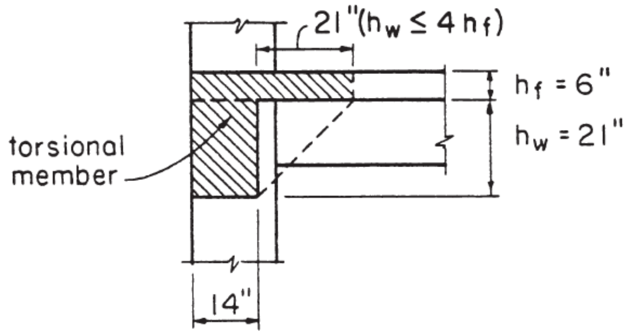


<u>Figure 9 – Attached Torsional Member at Exterior Column</u>

d. Increased torsional stiffness due to parallel beams, K_{ta}.

For Interior Columns:

$$K_{ta} = \frac{K_t I_{sb}}{I_s} = \frac{493E_c \times 25,387}{4752} = 2634E_c$$

Where:

$$I_{sb} = \frac{l_2 \times h^3}{12} = \frac{264 \times 6^3}{12} = 4752 \text{ in.}^4$$

For Exterior Columns:

$$K_{ta} = \frac{K_t I_{sb}}{I_s} = \frac{752E_c \times 25,387}{4752} = 4017E_c$$

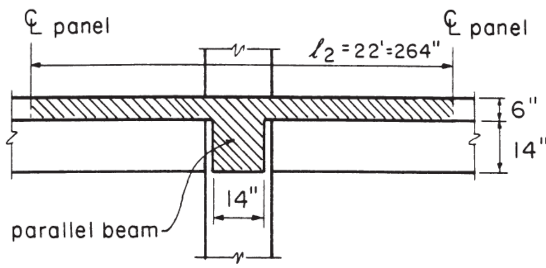


Figure 10 – Slab-Beam in the Direction of Analysis

e. Equivalent column stiffness K_{ec}.

$$K_{ec} = \frac{\sum K_c \times \sum K_{ta}}{\sum K_c + \sum K_{ta}}$$

Where $\sum K_{ta}$ is for two torsional members one on each side of the column, and $\sum K_c$ is for the upper and lower columns at the slab-beam joint of an intermediate floor.

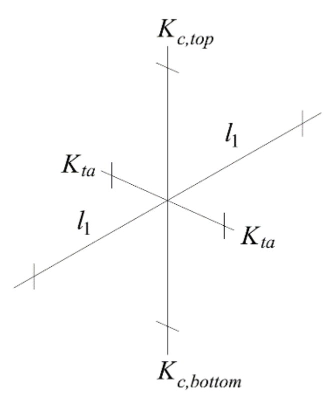


Figure 11 – Equivalent Column Stiffness

For Interior Columns:

$$K_{ec} = \frac{(303E_c + 414E_c)(2 \times 2634E_c)}{(303E_c + 414E_c) + (2 \times 2634E_c)} = 631E_c$$

For Exterior Columns:

$$K_{ec} = \frac{(323E_c + 521E_c)(2 \times 4017E_c)}{(323E_c + 521E_c) + (2 \times 4017E_c)} = 764E_c$$

f. Slab-beam joint distribution factors, DF.

At exterior joint,

$$DF = \frac{497E_C}{(497E_c + 764E_c)} = 0.394$$

At interior joint,

$$DF = \frac{497E_c}{(497E_c + 497E_c + 631E_c)} = 0.306$$

COF for slab-beam =0.507

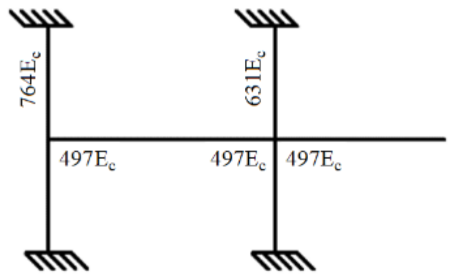


Figure 12 – Slab and Column Stiffness

2.3. Equivalent frame analysis

Determine negative and positive moments for the slab-beams using the moment distribution method.

With an unfactored live-to-dead load ratio:

$$\frac{L}{D} = \frac{100}{(150 \times 6/12)} = 1.33 > \frac{3}{4}$$

The frame will be analyzed for five loading conditions with pattern loading and partial live load as allowed by ***ACI 318-14 (6.4.3.3).***

a. Factored load and Fixed-End Moments (FEM's).

Factored dead load $q_{Du} = 1.2(75 + 9.3) = 101$ psf

Where (9.3 psf = (14 x 14) / 144 x 150 / 22 is the weight of beam stem per foot divided by l_2)

Factored live load $q_{Lu} = 1.6(100) = 160$ psf

Factored load $q_u = q_{Du} + q_{Lu} = 261$ psf

FEM's for slab-beam $= m_{NF} q_u \ell_2 \ell_1^2$ ***PCA Notes on ACI 318-11 (Table A1)***

FEM due to $q_{Du} + q_{Lu} = 0.0842 \times (0.261 \times 22) \times 17.5^2 = 148.1$ ft-kip

FEM due to $q_{Du} + \frac{3}{4} q_{Lu} = 0.0842 \times (0.221 \times 22) \times 17.5^2 = 125.4$ ft-kip

FEM due to $q_{Du} = 0.0842 \times (0.101 \times 22) \times 17.5^2 = 57.3$ ft-kip

b. Moment distribution.

Moment distribution for the five loading conditions is shown in Table 1. Counter-clockwise rotational moments acting on member ends are taken as positive. Positive span moments are determined from the following equation:

$$M_{u(midspan)} = M_o - \frac{(M_{uL} + M_{uR})}{2}$$

Where M_o is the moment at the midspan for a simple beam.

When the end moments are not equal, the maximum moment in the span does not occur at the midspan, but its value is close to that midspan for this example.

Positive moment in span 1-2 for loading (1):

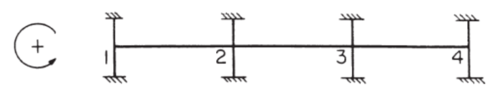
$$M_u^+ = (0.261\times 22)\frac{17.5^2}{8} - \frac{(93.1+167.7)}{2} = 89.4 \text{ ft-kip}$$

Positive moment span 2-3 for loading (1):

$$M_u^+ = (0.261\times 22)\frac{17.5^2}{8} - \frac{(153.6+153.6)}{2} = 66.2 \text{ ft-kip}$$

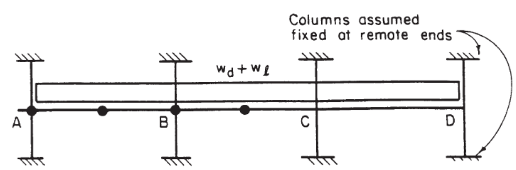
Table 1 – Moment Distribution for Partial Frame (Transverse Direction)

Joint	1	2		3		4
Member	1-2	2-1	2-3	3-2	3-4	4-3
DF	0.394	0.306	0.306	0.306	0.306	0.394
COF	0.507	0.507	0.507	0.507	0.507	0.507



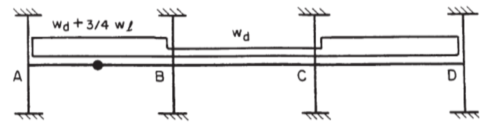
Loading (1) All spans loaded with full factored live load

FEM	148.1	-148.1	148.1	-148.1	148.1	-148.1
Dist	-58.4	0	0	0	0	58.4
CO	0	-29.6	0	0	29.6	0
Dist	0	9.1	9.1	-9.1	-9.1	0
CO	4.6	0	-4.6	4.6	0	-4.6
Dist	-1.8	1.4	1.4	-1.4	-1.4	1.8
CO	0.7	-0.9	-0.7	0.7	0.9	-0.7
Dist	-0.3	0.5	0.5	-0.5	-0.5	0.3
CO	0.3	-0.1	-0.3	0.3	0.1	-0.3
Dist	-0.1	0.1	0.1	-0.1	-0.1	0.1
M	93.1	-167.6	153.6	-153.6	167.6	-93.1
Midspan M	89.5		66.2		89.5	



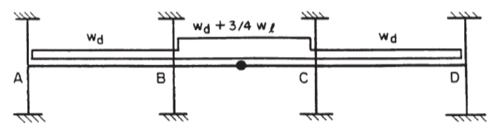
(1) Loading pattern for design moments in all spans with L ≤ 3/4 D

Loading (2) First and third spans loaded with 3/4 factored live load						
FEM	125.4	-125.4	57.3	-57.3	125.4	-125.4
Dist	-49.4	20.8	20.8	-20.8	-20.8	49.4
CO	10.6	-25.1	-10.6	10.6	25.1	-10.6
Dist	-4.2	10.9	10.9	-10.9	-10.9	4.2
CO	5.5	-2.1	-5.5	5.5	2.1	-5.5
Dist	-2.2	2.3	2.3	-2.3	-2.3	2.2
CO	1.2	-1.1	-1.2	1.2	1.1	-1.2
Dist	-0.5	0.7	0.7	-0.7	-0.7	0.5
CO	0.4	-0.2	-0.4	0.4	0.2	-0.4
Dist	-0.1	0.2	0.2	-0.2	-0.2	0.1
M	86.7	-119	74.5	-74.5	119	-86.7
Midspan M	83.3		10.6		83.3	



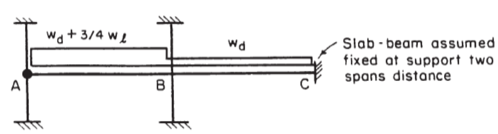
(2) Loading pattern for positive design moment in span AB*

Loading (3) Center span loaded with 3/4 factored live load						
FEM	57.3	-57.3	125.4	-125.4	57.3	-57.3
Dist	-22.6	-20.8	-20.8	20.8	20.8	22.6
CO	-10.6	-11.4	10.6	-10.6	11.4	10.6
Dist	4.2	0.3	0.3	-0.3	-0.3	-4.2
CO	0.1	2.1	-0.1	0.1	-2.1	-0.1
Dist	-0.1	-0.6	-0.6	0.6	0.6	0.1
CO	-0.3	0	0.3	-0.3	0	0.3
Dist	0.1	-0.1	-0.1	0.1	0.1	-0.1
CO	0	0.1	0	0	-0.1	0
Dist	0	0	0	0	0	0
M	28.1	-87.7	115	-115	87.7	-28.1
Midspan M	27.2		71.3		27.2	

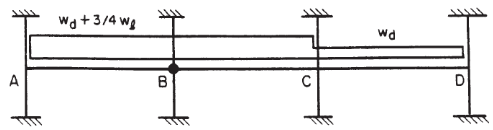


(3) Loading pattern for positive design moment in span BC*

Loading (4) First span loaded with 3/4 factored live load and beam-slab assumed fixed at support two spans away				
FEM	125.4	-125.4	57.3	-57.3
Dist	-49.4	20.8	20.8	0
CO	10.6	-25	0	10.6
Dist	-4.2	7.7	7.7	0
CO	3.9	-2.1	0	3.9
Dist	-1.5	0.6	0.6	0
CO	0.3	-0.8	0	0.3
Dist	-0.1	0.2	0.2	0
CO	0.1	-0.1	0	0.1
Dist	0	0	0	0
M	85.1	-124.1	86.6	-42.4
Midspan M	81.5		20.6	



(4) Loading pattern for negative design moment at support A*

Loading (5) First and second spans loaded with 3/4 factored live load							
FEM	125.4	-125.4	125.4	-125.4	57.3	-57.3	 (5) Loading pattern for negative design moment at support B*
Dist	-49.4	0.0	0.0	20.8	20.8	22.6	
CO	0.0	-25.1	10.6	0.0	11.4	10.6	
Dist	0.0	4.4	4.4	-3.5	-3.5	-4.2	
CO	2.2	0.0	-1.8	2.2	-2.1	-1.8	
Dist	-0.9	0.5	0.5	0.0	0.0	0.7	
CO	0.3	-0.4	0.0	0.3	0.4	0.0	
Dist	-0.1	0.1	0.1	-0.2	-0.2	0.0	
CO	0.1	-0.1	-0.1	0.1	0.0	-0.1	
Dist	0.0	0.0	0.0	0.0	0.0	0.0	
M	77.6	-146.0	139.1	-105.7	84.1	-29.5	
Midspan M	74.3		63.7		28.3		

						
Max M^-	93.1	-167.7	153.6	-153.6	167.7	-93.1
Max M^+	89.4		71.3		89.4	

2.4. Design moments

Positive and negative factored moments for the slab system in the direction of analysis are plotted in Figure 13. The negative design moments are taken at the faces of rectilinear supports but not at distances greater than $0.175\ell_1$ from the centers of supports. ***ACI 318-14 (8.11.6.1)***

$$\frac{18\text{in.}}{2} = 0.75 \text{ ft} < 0.175 \times 17.5 = 3.1 \text{ ft (use face of support location)}$$

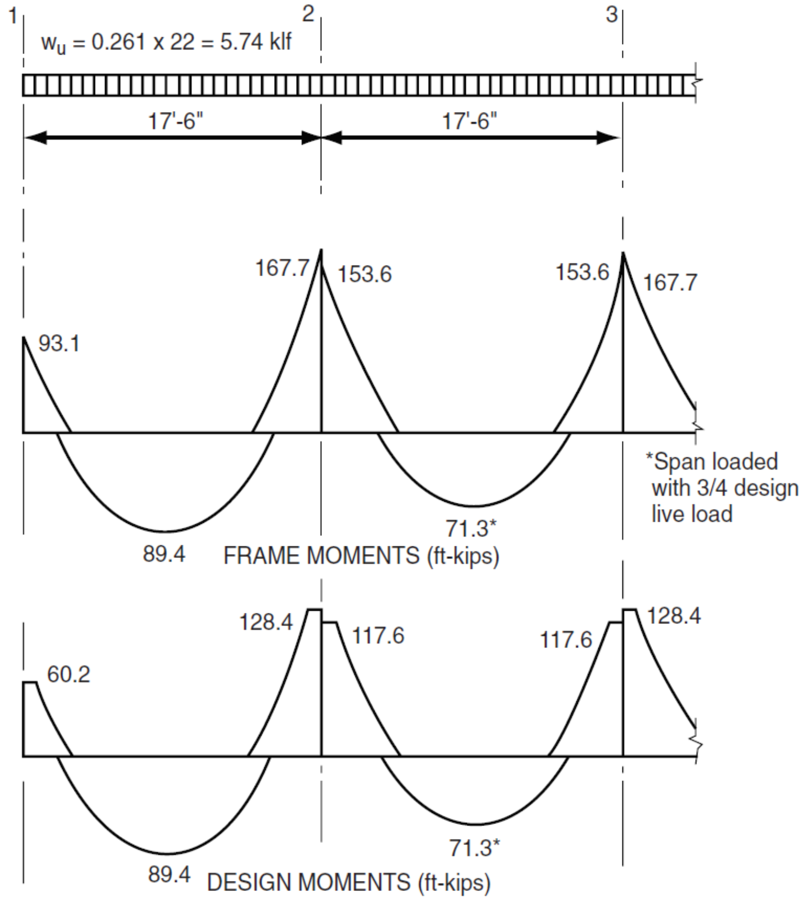


Figure 13 – Positive and Negative Design Moments for Slab-Beam (All Spans Loaded with Full Factored Live Load Except as Noted)

2.5. Distribution of design moments

a. Check whether the moments calculated above can take advantage of the reduction permitted by ***ACI 318-14 (8.11.6.5)***:

Slab systems within the limitations of ***ACI 318-14 (8.10.2)*** may have the resulting reduced in such proportion that the numerical sum of the positive and average negative moments not be greater than the total static moment M_o given by ***Equation 8.10.3.2*** in the ***ACI 318-14***:

ACI 318-14 (8.11.6.5)

Check Applicability of Direct Design Method:

1. There is a minimum of three continuous spans in each direction — ***ACI 318-14 (8.10.2.1)***
2. Successive span lengths are equal — ***ACI 318-14 (8.10.2.2)***
3. Long-to-Short ratio is 22/17.5 = 1.26 < 2.0 — ***ACI 318-14 (8.10.2.3)***
4. Column are not offset — ***ACI 318-14 (8.10.2.4)***
5. Loads are gravity and uniformly distributed with service live-to-dead ratio of 1.33 < 2.0 ***ACI 318-14 (8.10.2.5 and 6)***

6. Check relative stiffness for slab panel: ***ACI 318-14 (8.10.2.7)***

Interior Panel:

$\alpha_{f1} = 3.16$, $l_2 = 22 \times 12 = 264$ in.

$\alpha_{f2} = 3.98$, $l_1 = 17.5 \times 12 = 210$ in.

$$\frac{\alpha_{f1} l_2^2}{\alpha_{f2} l_1^2} = \frac{3.16 \times 264^2}{3.98 \times 210^2} = 1.25 \;\rightarrow\; 0.2 < 1.25 < 5.0$$ ***O.K.*** ***ACI 318-14 (Eq. 8.10.2.7a)***

Interior Panel:

$\alpha_{f1} = 3.16$, $l_2 = 22 \times 12 = 264$ in.

$\alpha_{f2} = 16.45$, $l_1 = 17.5 \times 12 = 210$ in.

$$\frac{\alpha_{f1} l_2^2}{\alpha_{f2} l_1^2} = \frac{3.16 \times 264^2}{16.45 \times 210^2} = 0.30 \;\rightarrow\; 0.2 < 0.30 < 5.0$$ ***O.K.*** ***ACI 318-14 (Eq. 8.10.2.7a)***

All limitation of ***ACI 318-14 (8.10.2)*** are satisfied and the provisions of ***ACI 318-14 (8.11.6.5)*** may be applied:

$$M_o = \frac{q_u \ell_2 \ell_n^{\,2}}{8} = 0.261 \times 22 \times \frac{(16)^2}{8} = 183.7 \text{ ft-kip}$$ ***ACI 318-14 (Eq. 8.10.3.2)***

$$\text{End spans: } 89.4 + \frac{(60.2 + 128.4)}{2} = 183.7 \text{ ft-kip}$$

$$\text{Interior span: } 71.2 + \frac{(117.6 + 117.6)}{2} = 188.8 \text{ ft-kip}$$

To illustrate proper procedure, the interior span factored moments may be reduced as follows:

Permissible reduction = 183.7/188.8 = 0.973

Adjusted negative design moment = 117.6 × 0.973 = 114.3 ft-kip

Adjusted positive design moment = 71.2 × 0.973 = 69.3 ft-kip

M_o = 183.7 ft-kip

b. Distribute factored moments to column and middle strips:

The negative and positive factored moments at critical sections may be distributed to the column strip and the two half-middle strips of the slab-beam according to the Direct Design Method (DDM) in 8.10, provided that ***Eq. 8.10.2.7(a)*** is satisfied. ***ACI 318-14 (8.11.6.6)***

Since the relative stiffness of beams are between 0.2 and 5.0 (see step 2.4.1.6), the moments can be distributed across slab-beams as specified in ***ACI 318-14 (8.10.5 and 6)*** where:

$$\frac{\ell_2}{\ell_1} = \frac{22}{17.5} = 1.257$$

$$\frac{\alpha_{f1}\ell_2}{\ell_1} = 3.16 \times 1.257 = 3.97$$

$$\beta_t = \frac{C}{2I_s} = \frac{17,868}{2 \times 4,752} = 1.88$$

Where $I_S = \dfrac{22 \times 12 \times 6^3}{12} = 4,752 \text{ in.}^4$

$C = 17,868 \text{ in.}^4$ (see Figure 9)

Factored moments at critical sections are summarized in Table 2.

Table 2 - Lateral distribution of factored moments							
		Factored Moments (ft-kips)	Column Strip				Moments in Two Half-Middle Strips** (ft-kips)
			Percent*	Moment (ft-kips)	Beam Strip Moment (ft-kips)	Column Strip Moment (ft-kips)	
End Span	Exterior Negative	60.2	75	45.2	38.4	6.8	15
	Positive	89.4	67	59.9	50.9	9.0	29.5
	Interior Negative	128.4	67	86	73.1	12.9	42.4
Interior Span	Negative	117.6	67	78.8	67.0	11.8	38.8
	Positive	71.3	67	47.8	40.6	7.2	23.5
*Since $\alpha_1 l_2/l_1$ > 1.0 beams must be proportioned to resist 85 percent of column strip per ***ACI 318-14 (8.10.5.7)***							
**That portion of the factored moment not resisted by the column strip is assigned to the two half-middle strips							

2.6. Flexural reinforcement requirements

a. Determine flexural reinforcement required for strip moments

The flexural reinforcement calculation for the column strip of end span – interior negative location is provided below:

$M_u = 12.9$ ft-kip

Assume tension-controlled section (φ = 0.9)

Column strip width, b = (17.5 x 12) / 2 = 91 in.

Use average d = 6 – 0.75 – 0.5/2 = 5 in.

$$A_s = \frac{0.85 \times f_c' \times b}{f_y}\left(d - \sqrt{d^2 - \frac{2 \times M_u}{\varphi \times 0.85 \times f_c' \times b}}\right)$$

$$A_s = \frac{0.85 \times 4000 \times 91}{60,000}\left(5 - \sqrt{5^2 - \frac{2 \times 12.9 \times 12,000}{0.9 \times 0.85 \times 4000 \times 91}}\right) = 0.580 \text{ in.}^2$$

$$A_{s,min} = max\begin{bmatrix} 0.0018 \times b \times h \\ 0.0014 \times b \times h \end{bmatrix} = max\begin{bmatrix} 0.0018\ (14)(19) \\ 0.0014\ (14)(19) \end{bmatrix} = max\begin{bmatrix} 0.983 \\ 0.764 \end{bmatrix} = 0.983 \text{ in.}^2 < 0.580 \text{ in.}^2 \text{ in}^2$$

$$\therefore A_S = 0.983 \text{ in.}^2$$

Maximum spacing $s_{max} = 2h = 2 \times 6 = 12$ in. < 18 in. ***ACI 318-14 (8.7.2.2)***

Provide 8 - #4 bars with A_s = 1.60 in.^{2} and s = 91/8 = 11.37 in. $\leq s_{max}$

<u>The flexural reinforcement calculation for the beam strip of end span – interior negative location is provided below:</u>

$$M_u = 73.1 \text{ ft-kip}$$

Assume tension-controlled section (φ = 0.9)

Beam strip width, b = 14 in.

Use average d = 20 – 0.75 – 0.5/2 = 19 in.

$$A_s = \frac{0.85 \times f_c^{'} \times b}{f_y}\left(d - \sqrt{d^2 - \frac{2 \times M_u}{\varphi \times 0.85 \times f_c^{'} \times b}}\right)$$

$$A_s = \frac{0.85 \times 4000 \times 14}{60,000}\left(18.25 - \sqrt{19^2 - \frac{2 \times 73.1 \times 12,000}{0.9 \times 0.85 \times 4000 \times 14}}\right) = 0.881 \text{ in.}^2$$

$$A_{s,min} = max\begin{bmatrix} \dfrac{3\sqrt{f_c^{'}}}{f_y} bd \\ \dfrac{200}{f_y} bd \end{bmatrix} = max\begin{bmatrix} \dfrac{3\sqrt{4000}}{60,000}(14)(19) \\ \dfrac{200}{60,000}(14)(19) \end{bmatrix} = max\begin{bmatrix} 0.841 \\ 0.887 \end{bmatrix} = 0.887 \text{ in.}^2 < 0.881 \text{ in.}^2$$

$$\therefore A_s = 0.887 \text{ in.}^2$$

Provide 5 - #4 bars with A_s = 1.00 in.^{2}

All the values on Table 3 are calculated based on the procedure outlined above.

Table 3 - Required Slab Reinforcement for Flexure [Equivalent Frame Method (EFM)]								
Span Location		**M_u (ft-kip)**	**b * (in.)**	**d ** (in.)**	**A_s Req'd for flexure (in.^{2})**	**Min A_s† †† (in.^{2})**	**Reinforcement Provided**	**A_s Prov. for flexure (in.^{2})**
End Span								
Beam Strip	Exterior Negative	38.4	14	19.00	0.456	0.608	4 - #4	0.8
	Positive	50.9	14	18.25	0.634	0.852	5 - #4	1.0
	Interior Negative	73.1	14	19.00	0.881	0.887	5 - #4	1.0
Column Strip	Exterior Negative	6.8	91	5.00	0.304	0.983	8 - #4	1.6
	Positive	9.0	91	5.00	0.403	0.983	8 - #4	1.6
	Interior Negative	12.9	91	5.00	0.580	0.983	8 - #4	1.6
Middle Strip	Exterior Negative	15.0	159	5.00	0.672	1.717	14 - #4	2.8
	Positive	29.5	159	5.00	1.331	1.717	14 - #4	2.8
	Interior Negative	42.4	159	5.00	1.926	1.717	14 - #4	2.8
Interior Span								
Beam Strip	Positive	40.6	14	18.25	0.503	0.671	4 - #4	0.8
Column Strip	Positive	7.2	91	5.00	0.322	0.983	8 - #4	1.6
Middle Strip	Positive	23.5	159	5.00	1.057	1.717	14 - #4	2.8

* Column strip width, b = (17.5 × 12)/2 - 14 = 91 in.

* Middle strip width, b = 22*12-(17.5*12)/2 = 159 in.

* Beam strip width, b = 14 in.

** Use average d = 6 – 0.75 – 0.5/2 = 5.00 in. for Column and Middle strips

** Use average d = 20 - 1.5 - 0.5/2 = 18.25 in. for Beam strip Positive moment regions

** Use average d = 20 - 0.75 - 0.5/2 = 19 in. for Beam strip Negative moment regions

† Min. A_s = 0.0018 × b × h = 0.0108 × b for Column and Middle strips **ACI 318-14 (7.6.1.1)**

† Min. A_s = min (3(f_c')^0.5/f_y*b*d , 200/f_y*b*d) for Beam strip **ACI 318-14 (9.6.1.2)**

†† Min. A_s = 1.333 × As Req'd if As provided >= 1.333 × As Req'd for Beam strip **ACI 318-14 (9.6.1.3)**

s_{max} = 2 × h = 12 in. < 18 in. **ACI 318-14 (8.7.2.2)**

b. Calculate additional slab reinforcement at columns for moment transfer between slab and column by flexure

Portion of the unbalanced moment transferred by flexure is γ_f x M_u

Where:

$$\gamma_f = \frac{1}{1+(2/3)\times\sqrt{b_1/b_2}}$$ ***ACI 318-14 (8.4.2.3.2)***

b_1 = Dimension of the critical section b_o measured in the direction of the span for which moments are determined in ACI 318, Chapter 8.

b_2 = Dimension of the critical section b_o measured in the direction perpendicular to b_1 in ACI 318, Chapter 8.

b_o = Perimeter of critical section for two-way shear in slabs and footings.

$b_b = \text{Effective slab width} = c_2 + 3 \times h$ ***ACI 318-14 (8.4.2.3.3)***

For Exterior Column:

$$b_1 = c_1 + \frac{d}{2} = 18 + \frac{5}{2} = 20.5 \text{ in. },\ b_2 = c_2 + d = 18 + 5 = 23 \text{ in. },\ b_b = c_2 + 3\,h = 18 + 3\,(6) = 36 \text{ in.}$$

$$\gamma_f = \frac{1}{1 + (2/3) \times \sqrt{20.5/23}} = 0.614$$

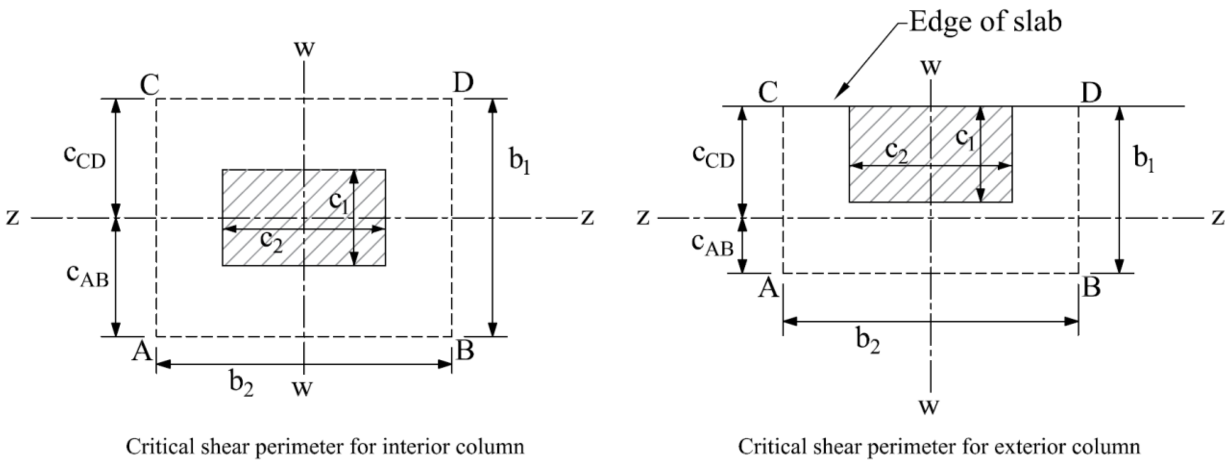


Figure 14 – Critical Shear Perimeters for Columns

$$\gamma_f M_{u,net} = 0.614 \times 93.1 = 57.14 \text{ ft-kip}$$

$$A_{s,req'd} = \frac{0.85 \times f_c' \times b_b}{f_y} \left(d - \sqrt{d^2 - \frac{2 \times \gamma_f M_{u,net}}{\varphi \times 0.85 \times f_c' \times b_b}} \right)$$

$$A_{s,req'd} = \frac{0.85 \times 4000 \times 36}{60,000} \left(5 - \sqrt{5^2 - \frac{2 \times 57.14 \times 12,000}{0.9 \times 0.85 \times 4000 \times 36}} \right) = 2.973 \text{ in.}^2$$

$$A_{s,min} = max \begin{bmatrix} 0.0018 \times b \times h \\ 0.0014 \times b \times h \end{bmatrix} = max \begin{bmatrix} 0.0018\ (14)(19) \\ 0.0014\ (14)(19) \end{bmatrix} = max \begin{bmatrix} 0.983 \\ 0.764 \end{bmatrix} = 0.983 \text{ in.}^2 > 2.973 \text{ in.}^2$$

$$\therefore A_{s,req'd} = 2.973 \text{ in.}^2$$

$$A_{s,provided} = (A_{s,provided})_{(beam)} + (A_{s,provided})_{(b_b - b_{beam})}$$

$$A_{s,provided} = 4 \times 0.2 + 8 \times 0.2 \times \frac{36-14}{91} = 1.187 \text{ in.}^2 < A_{s,req'd} = 2.973 \text{ in.}^2$$

$\therefore$ Additional slab reinforcement at the exterior column is required.

$$A_{req'd,\,add} = 2.973 - 1.187 = 1.786 \text{ in.}^2$$

Use 10 - #4 $\rightarrow A_{provided,\,add} = 10 \times 0.2 = 2.0 \text{ in.}^2 < A_{req'd,\,add} = 1.786 \text{ in.}^2$

Table 4 - Additional Slab Reinforcement at columns for moment transfer between slab and column [Equivalent Frame Method (EFM)]									
Span Location		**Effective slab width, b_b (in.)**	**d (in.)**	**γ_f**	**M_u* (ft-kip)**	**$\gamma_f M_u$ (ft-kip)**	**A_s req'd within b_b (in.^{2})**	**A_s prov. for flexure within b_b (in.^{2})**	**Add'l Reinf.**
End Span									
Column Strip	Exterior Negative	36	5	0.614	93.1	57.14	2.973	1.187	10-#4
	Interior Negative	36	5	0.600	44.5	26.70	1.265	1.387	-
*M_u is taken at the centerline of the support in Equivalent Frame Method solution.									

b. Determine transverse reinforcement required for beam strip shear

The transverse reinforcement calculation for the beam strip of end span – exterior location is provided below.

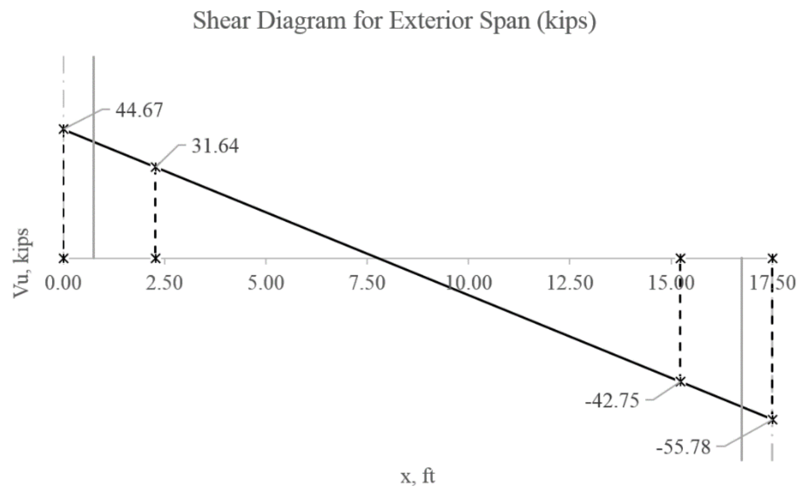


Figure 15 – Shear at critical sections for the end span (at distance d from the face of the column)

$$d = h - c_{clear} - \frac{d_{stirrup}}{2} = 20 - 1.5 - \frac{0.5}{2} = 18.25 \text{ in. (using \#4 stirrups)}$$

The required shear at a distance d from the face of the supporting column V_{u_d}= 31.64 kips (Figure 15).

$$\varphi_v V_c = \varphi_v \times 2 \times \sqrt{f_c^{'}} \times b \times d$$ ***ACI 318-14 (22.5.5.1)***

$$\varphi_v V_c = 0.75 \times 2 \times \sqrt{4000} \times 14 \times 18.25 = 24.25 \text{ kips} < V_{u\_d} = 31.64 \text{ kip}$$ ∴ Stirrups are required.

Distance from the column face beyond which minimum reinforcement is required:

$$V_s = \frac{V_{u\_d} - \varphi_v V_c}{\varphi_v}$$ ***ACI 318-14 (22.5.10.1)***

$$V_s = \frac{31.64 - 24.25}{0.75} = 9.85 \text{ kip} < V_{s,max} = 129.31 \text{ kip}$$ ***O.K.***

$$V_{s,max} = 8\times\sqrt{f_c^{'}}\times b\times d = 8\times\sqrt{4000}\times 14\times 18.25 = 129.31 \text{ kip}$$ **ACI 318-14 (22.5.10.1)**

$$\frac{A_{v,req'd}}{s} = \frac{V_s}{f_{yt}\times d} = \frac{9.85\times 1000}{60,000\times 18.25} = 0.009 \text{ in.}^2\text{/in.}$$ **ACI 318-14 (22.5.10.5.3)**

$$\frac{A_{v,\min}}{s} = max\begin{bmatrix}\dfrac{0.75\sqrt{f_c^{'}}}{f_{yt}}b \\ \dfrac{50}{f_{yt}}b\end{bmatrix}$$ **ACI 318-14 (9.6.3.3)**

$$\frac{A_{v,\min}}{s} = max\begin{bmatrix}\dfrac{0.75\sqrt{4000}}{60,000}(14) \\ \dfrac{50}{60,000}(14)\end{bmatrix} = max\begin{bmatrix}0.0111 \\ 0.0117\end{bmatrix} = 0.0117 \text{ in.}^2\text{/in}$$

$$\frac{A_{v,req'd}}{s} < \frac{A_{v,\min}}{s} \rightarrow \therefore \text{ use } \frac{A_{s,req'd}}{s} = \frac{A_{v,\min}}{s}$$

$$s_{req'd} = \frac{n\times A_{stirrup}}{\dfrac{A_{v,req'd}}{s}} = \frac{2\times 0.2}{0.0117} = 34.28 \text{ in.}$$

$$V_s = 9.85 \text{ kips} < 4\times\sqrt{f_c^{'}}\times b\times d = 4\times\sqrt{4000}\times 14\times 18.25 = 64.66 \text{ kips}$$

$$\therefore s_{max} = Lesser\ of\begin{bmatrix}\dfrac{d}{2} \\ 24\end{bmatrix} = Lesser\ of\begin{bmatrix}\dfrac{18.25}{2} \\ 24\end{bmatrix} = Lesser\ of\begin{bmatrix}9.13 \\ 24\end{bmatrix} = 9.13 \text{ in.}$$ **ACI 318-14 (9.7.6.2.2)**

Since $s_{req'd} > s_{max} \rightarrow$ use s_{max}

Select $s_{provided}$ = 8 in. #4 stirrups with first stirrup located at distance 3 in. from the column face.

The distance where the shear is zero is calculated as follows:

$$x = \frac{l}{V_{u,L}+V_{u,R}}\times V_{u,L} = \frac{17.5}{44.67+55.78}\times 44.67 = 7.78 \text{ ft} = 93.4 \text{ in.}$$

The distance from support beyond which minimum reinforcement is required is calculated as follows:

$$x_1 = x - \frac{x}{V_u}\times\varphi_v V_c = 7.78 - \frac{7.78}{44.67}\times 24.25 = 3.56 \text{ ft} = 43 \text{ in.}$$

The distance at which no shear reinforcement is required is calculated as follows:

$$x_2 = x - \frac{x}{V_u}\times\frac{\varphi_v V_c}{2} = 7.78 - \frac{7.78}{44.67}\times\frac{24.25}{2} = 5.67 \text{ ft} = 68 \text{ in.}$$

$$\#\ of\ stirrups = \frac{x_2 - 3 - \dfrac{c_1}{2} - \dfrac{s_{provided}}{2}}{s_{provided}} + 1 = \frac{68 - 3 - \dfrac{18}{2} - \dfrac{8}{2}}{8} + 1 = 7.5 \rightarrow \text{use 8 stirrups}$$

All the values on Table 5 are calculated based on the procedure outlined above.

Table 5 - Required Beam Reinforcement for Shear					
Span Location	**$A_{v,min}/s$ in^2/in**	**$A_{v,req'd}/s$ in^2/in**	**$s_{req'd}$ in**	**s_{max} in**	**Reinforcement Provided**
End Span					
Exterior	0.0117	0.0090	34.28	9.13	8 - #4 @ 8 in*
Interior	0.0117	0.0225	17.76	9.13	10 - #4 @ 8.6 in
Interior Span					
Interior	0.0117	0.0158	25.37	9.13	9 - #4 @ 8.6 in
* Minimum transverse reinforcement governs					

2.7. Column design moments

The unbalanced moment from the slab-beams at the supports of the equivalent frame are distributed to the actual columns above and below the slab-beam in proportion to the relative stiffness of the actual columns. Referring to Fig. 9, the unbalanced moment at joints 1 and 2 are:

Joint 1 = +93.1 ft-kip

Joint 2 = -119 + 74.5 = -44.5 ft-kip

The stiffness and carry-over factors of the actual columns and the distribution of the unbalanced moments to the exterior and interior columns are shown in Fig 9.

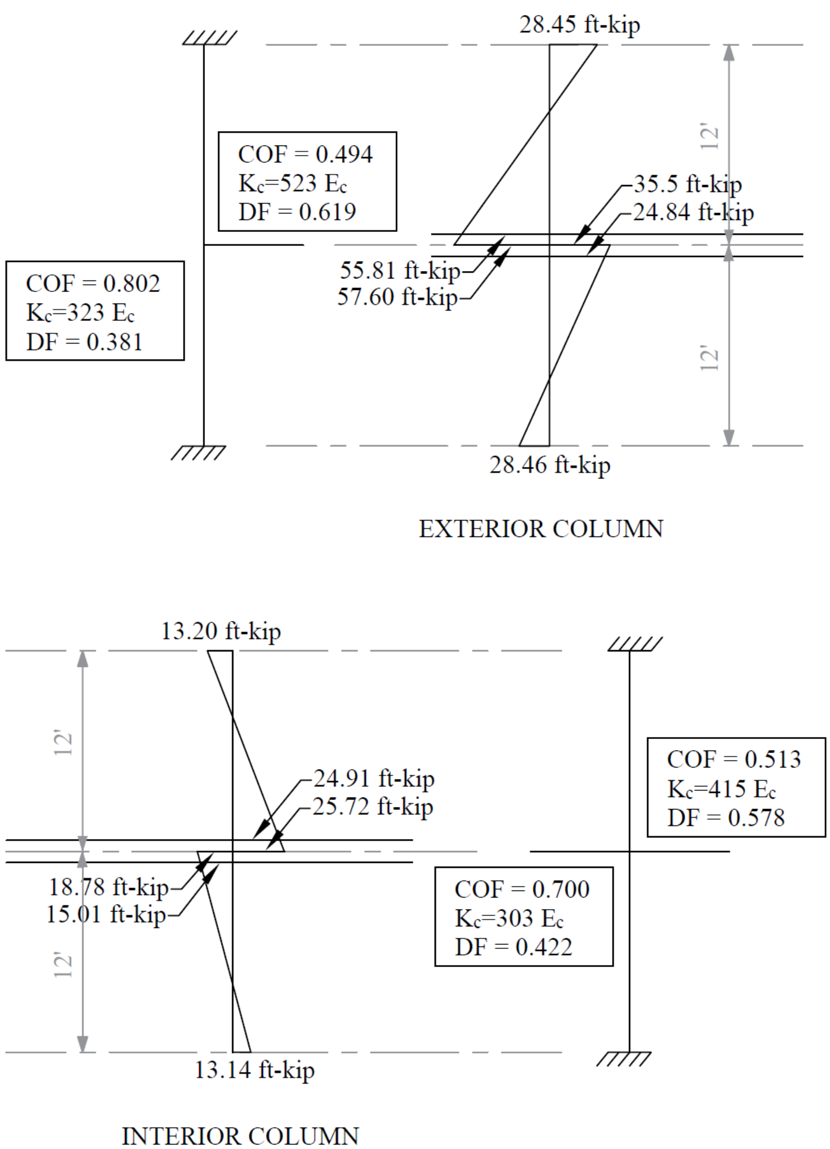


Figure 16 - Column Moments (Unbalanced Moments from Slab-Beam)

In summary:

Design moment in exterior column = 55.81 ft-kip

Design moment in interior column = 24.91 ft-kip

The moments determined above are combined with the factored axial loads (for each story) and factored moments in the transverse direction for design of column sections. A detailed analysis to obtain the moment values at the face of interior, exterior, and corner columns from the unbalanced moment values can be found in the "Two-Way Flat Plate Concrete Floor Slab Design" example.

3. Design of Interior, Edge, and Corner Columns

The design of interior, edge, and corner columns is explained in the "Two-Way Flat Plate Concrete Floor Slab Design" example.

4. Two-Way Slab Shear Strength

Shear strength of the slab in the vicinity of columns/supports includes an evaluation of one-way shear (beam action) and two-way shear (punching) in accordance with ACI 318 Chapter 22.

4.1. One-Way (Beam action) Shear Strength

One-way shear is critical at a distance d from the face of the column. Figure 17 shows the V_u at the critical sections around each column. Since there is no shear reinforcement, the design shear capacity of the section equals to the design shear capacity of the concrete:

$$\varphi V_n = \varphi V_c + \varphi V_s = \varphi V_c$$ ***ACI 318-14 (Eq. 22.5.1.1)***

Where:

$$\varphi V_c = \varphi 2\lambda\sqrt{f'_c}\,b_w d$$ ***ACI 318-14 (Eq. 22.5.5.1)***

$\lambda = 1$ for normal weight concrete

$$\varphi V_c = 0.75\times 2\times 1.0\times\sqrt{4000}\times(22\times 12-14)\times\frac{5}{1000} = 118.59 \text{ kips}$$

Because $\varphi V_c > V_u$ at all the critical sections, the slab is ***o.k.*** in one-way shear.

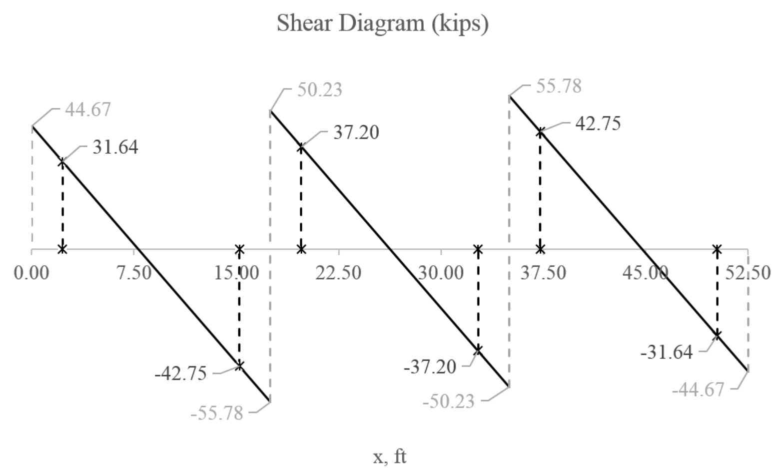


Figure 17 – One-way shear at critical sections (at distance d from the face of the supporting column)

4.2. Two-Way (Punching) Shear Strength

Two-way shear is critical on a rectangular section located at $d_{slab}/2$ away from the face of the column. The factored shear force V_u in the critical section is calculated as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section.

The factored unbalanced moment used for shear transfer, M_{unb}, is calculated as the sum of the joint moments to the left and right. Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account.

For the exterior column:

$$V_u = 44.67 - 0.340\left(\frac{20.5\times 23}{144}\right) = 43.56 \text{ kips}$$

$$M_{unb} = 93.1 - 43.56\left(\frac{20.5 - 9.09 - 18/2}{12}\right) = 84.37 \text{ ft-kip}$$

For the exterior column in Figure 18, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}}$$

$$c_{AB} = \frac{2(14\times 26\times(6.5+14/2)+6.5\times 5\times(6.5/2))}{2\times(14\times 26+6.5\times 5)+14\times 19+2\times 4.5\times 5} = 9.09 \text{ in.}$$

$$A_c = 2\times(14\times 26+6.5\times 5)+14\times 19+2\times 4.5\times 5 = 1104 \text{ in.}^2$$

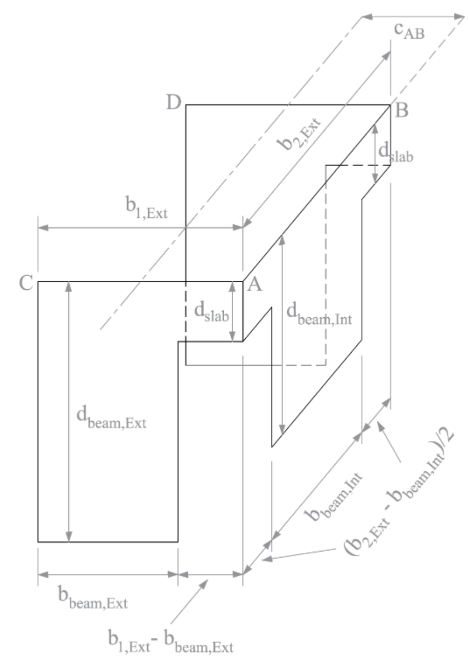


Figure 18 – Critical section of exterior support of interior frame

The polar moment J_c of the shear perimeter is:

$$J_c = 2\left[\frac{b_{beam,Ext}d^3_{beam,Ext}}{12}+\frac{d_{beam,Ext}b_{beam,Ext}{}^3}{12}+\left[b_{beam,Ext}d_{beam,Ext}\right]\left[\frac{b_{beam,Ext}}{2}+\left(b_1-b_{beam,Ext}\right)-c_{AB}\right]^2\right]$$

$$+2\left[\frac{\left(b_1-b_{beam,Ext}\right)d^3_{slab,Ext}}{12}+\frac{d_{slab}\left(b_1-b_{beam,Ext}\right)^3}{12}+\left[\left(b_1-b_{beam,Ext}\right)d_{slab}\right]\left[c_{AB}-\frac{b_1-b_{beam,Ext}}{2}\right]^2\right]$$

$$+\left[b_{beam,Int}\ d_{beam.Int}+\left(b_2-b_{beam,Int}\right)\ d_{slab}\right]c^2_{AB}$$

$$J_c = 2\left[\frac{14\times26^3}{12}+\frac{26\times14^3}{12}+\left[14\times26\right]\left[\frac{14}{2}+\left(20.5-14\right)-9.09\right]^2\right]$$

$$+2\left[\frac{\left(20.5-14\right)\times5^3}{12}+\frac{5\times\left(20.5-14\right)^3}{12}+\left[\left(20.5-14\right)\times5\right]\left[9.09-\frac{20.5-14}{2}\right]^2\right]$$

$$+\left[14\times19+\left(23-14\right)\ \times5\right]\ \times9.09^2$$

$$J_c = 95{,}338\ \text{in.}^4$$

$\gamma_v = 1-\gamma_f = 1-0.614 = 0.386$ ***ACI 318-14 (Eq. 8.4.4.2.2)***

The length of the critical perimeter for the exterior column:

$$b_o = 2\times(18+5/2)+(18+5) = 64\ \text{in.}$$

$v_u = \dfrac{V_u}{A_c}+\dfrac{\gamma_v M_{unb} c_{AB}}{J_c}$ ***ACI 318-14 (R.8.4.4.2.3)***

$$v_u = \frac{43.56\times1000}{1104}+\frac{0.386\times84.37\times12\times1000\times9.09}{95{,}338} = 76.8\ \text{psi}$$

$v_c = \min\left[4\lambda\sqrt{f_c'}\ ,\ \left(2+\dfrac{4}{\beta}\right)\lambda\sqrt{f_c'}\ ,\ \left(\dfrac{\alpha_s d}{b_o}+2\right)\lambda\sqrt{f_c'}\right]$ ***ACI 318-14 (Table 22.6.5.2)***

$$v_c = \min\left[4\times1\times\sqrt{4000}\ ,\ \left(2+\frac{4}{1}\right)\times1\times\sqrt{4000}\ ,\ \left(\frac{30\times5}{64}+2\right)\times1\times\sqrt{4000}\right]$$

$$v_c = \min\left[253\ ,\ 379.5\ ,\ 274.7\right]\ \text{psi} = 253\ \text{psi}$$

$\varphi v_c = 0.75\times253 = 189.7\ \text{psi}\ > v_u = 76.8\ \text{psi}$ ***O.K.***

For the interior column:

$$V_u = 55.78 + 50.23 - 0.340\left(\frac{23\times 23}{144}\right) = 104.76 \text{ kips}$$

$$M_{unb} = 167.7 - 153.6 - 104.76(0) = 14.10 \text{ ft-kip}$$

For the interior column in Figure 19, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{b_{1,Int}}{2} = \frac{23}{2} = 11.5 \text{ in.}$$

$$A_c = 4\times(14\times 19 + 9\times 5) = 1244 \text{ in.}^2$$

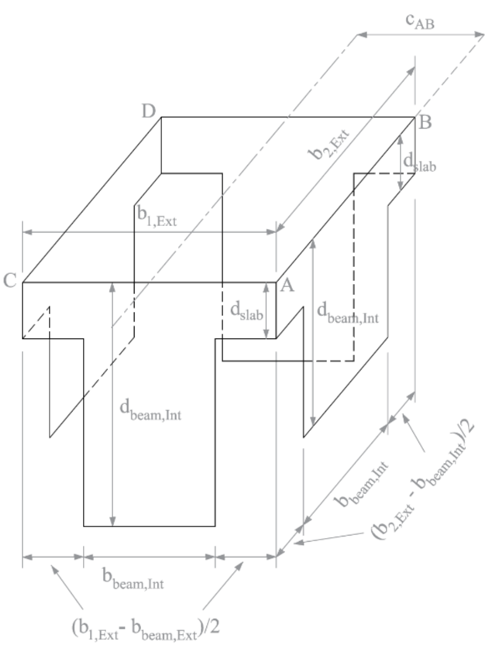


Figure 19 – Critical section of interior support of interior frame

The polar moment J_c of the shear perimeter is:

$$J_c = 2\left[\frac{b_{beam,Int}d_{beam,Int}^3}{12} + \frac{d_{beam,Int}b_{beam,Int}^{\;3}}{12} + \left[b_{beam,Int}d_{beam,Int}\right]\left[\frac{b_{beam,Int}}{2} + \left(\frac{b_1 - b_{beam,Int}}{2}\right) - c_{AB}\right]^2\right]$$

$$+4\left[\frac{\left(\frac{b_1 - b_{beam,Int}}{2}\right)d_{slab,Int}^3}{12} + \frac{d_{slab}\left(\frac{b_1 - b_{beam,Int}}{2}\right)^3}{12} + \left[\left(\frac{b_1 - b_{beam,Int}}{2}\right)d_{slab}\right]\left[c_{AB} - \frac{b_1 - b_{beam,Int}}{2\times 2}\right]^2\right]$$

$$+\left[b_{beam,Int}\ d_{beam.Int} + \left(b_2 - b_{beam,Int}\right)\ d_{slab}\right]\ c_{AB}^2$$

$$J_c = 2\left[\frac{14\times 19^3}{12} + \frac{19\times 14^3}{12} + \left[14\times 19\right]\left[\frac{14}{2} + \left(\frac{23-14}{2}\right) - 11.5\right]^2\right]$$

$$+4\left[\frac{\left(\frac{23-14}{2}\right)\times 5^3}{12} + \frac{5\times\left(\frac{23-14}{2}\right)^3}{12} + \left[\left(\frac{23-14}{2}\right)\times 5\right]\left[11.5 - \frac{23-14}{2\times 2}\right]^2\right]$$

$$+\left[14\times 19 + \left(23-14\right)\ \times 5\right]\ \times 11.5^2$$

$J_c = 114{,}993 \text{ in.}^4$

$\gamma_v = 1 - \gamma_f = 1 - 0.600 = 0.400$ **ACI 318-14 (Eq. 8.4.4.2.2)**

The length of the critical perimeter for the exterior column:

$b_o = 4 \times (18 + 5) = 92 \text{ in.}$

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v M_{unb} c_{AB}}{J_c}$$

$$v_u = \frac{104.76 \times 1000}{1244} + \frac{0.4 \times 14.10 \times 12 \times 1000 \times 11.5}{114{,}993} = 91.0 \text{ psi}$$

$$v_c = \min\left[4\lambda\sqrt{f_c'}\ ,\ \left(2+\frac{4}{\beta}\right)\lambda\sqrt{f_c'}\ ,\ \left(\frac{\alpha_s d}{b_o}+2\right)\lambda\sqrt{f_c'}\right]$$ **ACI 318-14 (Table 22.6.5.2)**

$$v_c = \min\left[4 \times 1 \times \sqrt{4000}\ ,\ \left(2+\frac{4}{1}\right) \times 1 \times \sqrt{4000}\ ,\ \left(\frac{40 \times 5}{92}+2\right) \times 1 \times \sqrt{4000}\right]$$

$v_c = \min[253\ ,\ 379.5\ ,\ 274.7] \text{ psi} = 253 \text{ psi}$

$\varphi v_c = 0.75 \times 253 = 189.7 \text{ psi} > v_u = 91.0 \text{ psi}$ **O.K.**

5. Two-Way Slab Deflection Control (Serviceability Requirements)

Since the slab thickness was selected based on the minimum slab thickness tables in ACI 318-14, the deflection calculations are not required. However, the calculations of immediate and time-dependent deflections are covered in this section for illustration and comparison with spSlab model results.

5.1. Immediate (Instantaneous) Deflections

The calculation of deflections for two-way slabs is challenging even if linear elastic behavior can be assumed. Elastic analysis for three service load levels (D, $D + L_{sustained}$, $D+L_{Full}$) is used to obtain immediate deflections of the two-way slab in this example. However, other procedures may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests. **ACI 318-14 (24.2.3)**

The effective moment of inertia (I_e) is used to account for the cracking effect on the flexural stiffness of the slab. I_e for uncracked section ($M_{cr} > M_a$) is equal to I_g. When the section is cracked ($M_{cr} < M_a$), then the following equation should be used:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g$$ **ACI 318-14 (Eq. 24.2.3.5a)**

Where:

M_a = Maximum moment in member due to service loads at stage deflection is calculated.

The values of the maximum moments for the three service load levels are calculated from structural analysis as shown previously in this document. These moments are shown in Figure 20.

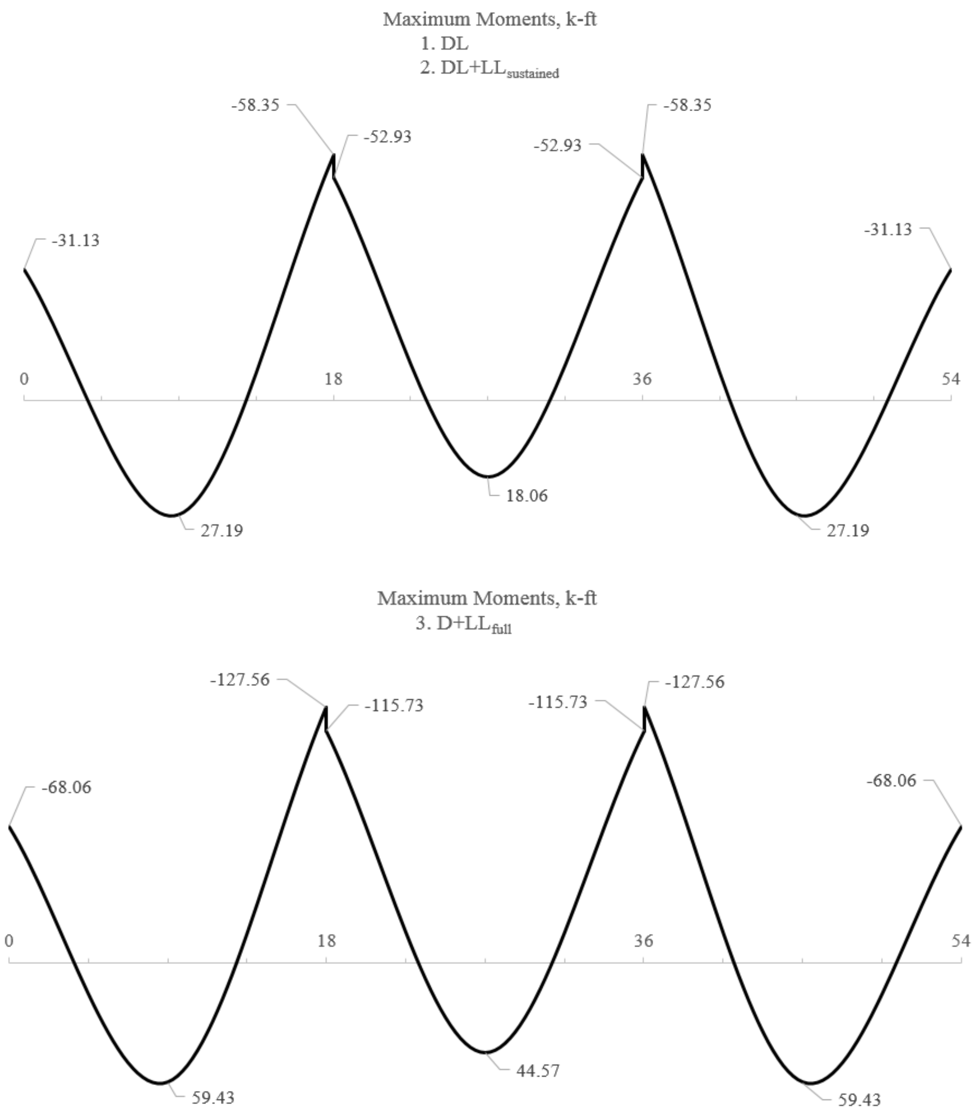


Figure 20 – Maximum Moments for the Three Service Load Levels

For positive moment (midspan) section of the exterior span:

M_{cr} = Cracking moment.

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{474.3 \times 25395}{5} \times \frac{1}{12 \times 1000} = 63.14 \text{ ft-kips}$$ ***ACI 318-14 (Eq. 24.2.3.5b)***

f_r = Modulus of rapture of concrete.

$$f_r = 7.5\lambda\sqrt{f_c'} = 7.5 \times 1.0 \times \sqrt{4000} = 474.3 \text{ psi}$$ ***ACI 318-14 (Eq. 19.2.3.1)***

I_g = Moment of inertia of the gross uncracked concrete section.

$I_g = 25395 \text{ in.}^2$ for T-section (see Figure 21)

y_t = Distance from centroidal axis of gross section, neglecting reinforcement, to tension face, in.

$y_t = 15.9$ in. (see Figure 21)

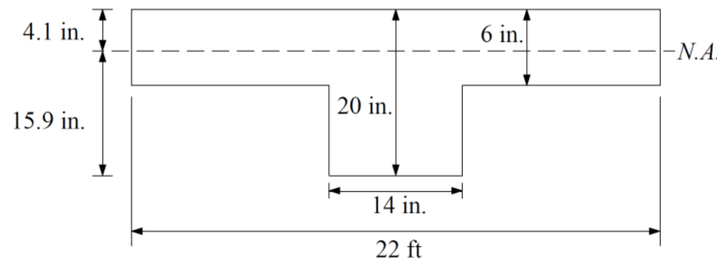


Figure 21 – I_g calculations for slab section near support

I_{cr} = Moment of inertia of the cracked section transformed to concrete. ***PCA Notes on ACI 318-11 (9.5.2.2)***

As calculated previously, the positive reinforcement for the end span frame strip is 22 #4 bars located at 1.0 in. along the slab section from the bottom of the slab and 4 #4 bars located at 1.75 in. along the beam section from the bottom of the beam. Five of the slab section bars are not continuous and will be excluded from the calculation of I_{cr}. Figure 22 shows all the parameters needed to calculate the moment of inertia of the cracked section transformed to concrete at midspan.

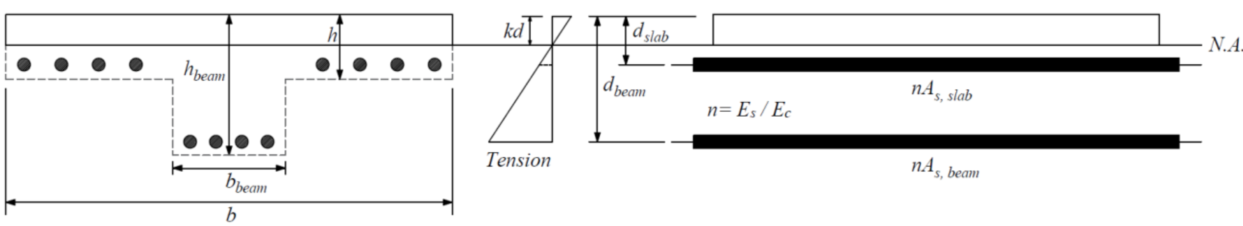


Figure 22 – Cracked Transformed Section (positive moment section)

E_{cs} = Modulus of elasticity of slab concrete.

$$E_{cs} = w_c^{1.5} 33\sqrt{f_c'} = 150^{1.5} \times 33 \times \sqrt{4000} = 3834 \times 10^3 \text{ psi}$$ ***ACI 318-14 (19.2.2.1.a)***

$$n = \frac{E_s}{E_{cs}} = \frac{29000000}{3834000} = 7.56$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$a = \frac{b}{2} = \frac{22 \times 12}{2} = 132 \text{ in.}$$

$$b = n\,A_{s,beam} + n\,A_{s,slab} = 7.56 \times (4 \times 0.20) + 7.56 \times (17 \times 0.20) = 31.77 \text{ in.}^2$$

$$c = -1 \times \left(n\,A_{s,beam} d_{s,beam} + n\,A_{s,slab} d_{s,slab}\right) = -1 \times \left(7.56 \times (4 \times 0.20) \times 18.25 + 7.56 \times (17 \times 0.20) \times 5.0\right) = -239 \text{ in.}^3$$

$$kd=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-31.77\pm\sqrt{31.77^2-4\times132\times-239}}{2\times132}=3.999\text{ in.}$$

$$I_{cr}=\frac{b(kd)^3}{3}+nA_{s,slab}(d_{slab}-kd)^2+nA_{s,beam}(d_{beam}-kd)^2$$

$$I_{cr}=\frac{22\times12\times(3.999)^3}{3}+7.56\times(17\times0.20)(5-3.999)^2+7.56\times(4\times0.20)(18.25-3.999)^2=2282\text{ in.}^4$$

<u>For negative moment section (near the interior support of the end span):</u>

The negative reinforcement for the end span frame strip near the interior support is 27 #4 bars located at 1.0 in. along the section from the top of the slab.

$$M_{cr}=\frac{f_rI_g}{y_t}=\frac{474.3\times9333}{10.0}\times\frac{1}{12\times1000}=36.89\text{ ft-kips}$$ ***ACI 318-14 (Eq. 24.2.3.5b)***

$$f_r=7.5\lambda\sqrt{f_c'}=7.5\times1.0\times\sqrt{4000}=474.3\text{ psi}$$ ***ACI 318-14 (Eq. 19.2.3.1)***

$$I_g=9333\text{ in.}^2$$

$$y_t=10.0\text{ in.}$$

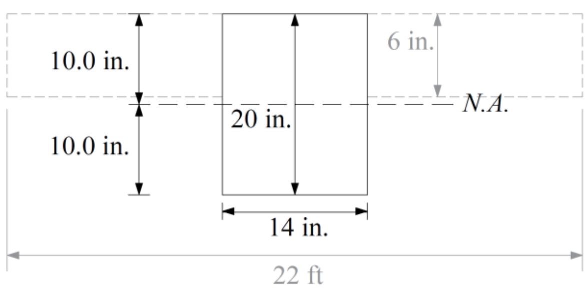


Figure 23 – I_g calculations for slab section near support

$$E_{cs}=w_c^{1.5}33\sqrt{f_c'}=150^{1.5}\times33\times\sqrt{4000}=3834\times10^3\text{ psi}$$ ***ACI 318-14 (19.2.2.1.a)***

$$n=\frac{E_s}{E_{cs}}=\frac{29000000}{3834000}=7.56$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$B=\frac{b_{beam}}{n\,A_{s,total}}=\frac{14}{7.56\times(27\times0.20)}=0.34\text{ in.}^{-1}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$kd=\frac{\sqrt{2dB+1}-1}{B}=\frac{\sqrt{2\times19\times0.34+1}-1}{0.34}=8.03\text{ in.}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr}=\frac{b_{beam}(kd)^3}{3}+nA_{s,total}(d-kd)^2$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr}=\frac{14\times(8.03)^3}{3}+7.56\times(27\times0.20)(19-8.03)^{2}=7331\text{ in.}^4$$

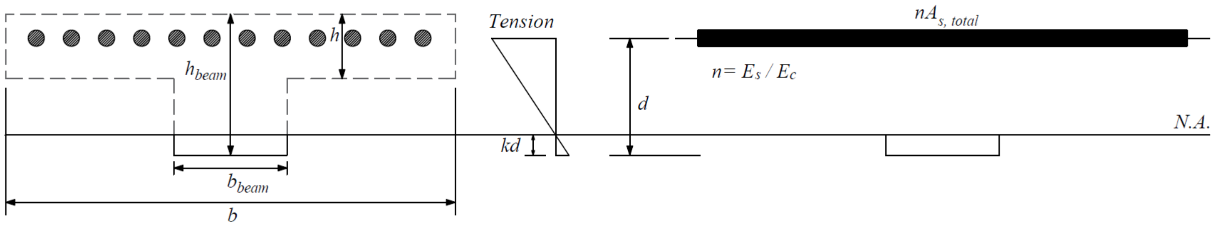


Figure 24 – Cracked Transformed Section (interior negative moment section for end span)

The effective moment of inertia procedure described in the Code is considered sufficiently accurate to estimate deflections. The effective moment of inertia, I_e, was developed to provide a transition between the upper and lower bounds of I_g and I_{cr} as a function of the ratio M_{cr}/M_a. For conventionally reinforced (nonprestressed) members, the effective moment of inertia, I_e, shall be calculated by Eq. (24.2.3.5a) unless obtained by a more comprehensive analysis.

I_e shall be permitted to be taken as the value obtained from Eq. (24.2.3.5a) at midspan for simple and continuous spans, and at the support for cantilevers. ***ACI 318-14 (24.2.3.7)***

For continuous one-way slabs and beams. I_e shall be permitted to be taken as the average of values obtained from Eq. (24.2.3.5a) for the critical positive and negative moment sections. ***ACI 318-14 (24.2.3.6)***

For the exterior span (span with one end continuous) with service load level ($D+LL_{full}$):

$$I_e^-=\left(\frac{M_{cr}}{M_a}\right)^3 I_g+\left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right]I_{cr}\text{ , since }M_{cr}=36.89\text{ ft-kips}<M_a=127.56\text{ ft-kips}$$

ACI 318-14 (24.2.3.5a)

Where I_e^- is the effective moment of inertia for the critical negative moment section (near the support).

$$I_e^-=\left(\frac{36.89}{127.56}\right)^3\times9333+\left[1-\left(\frac{36.89}{127.56}\right)^3\right]\times7331=7380\text{ in.}^4$$

$$I_e^+=I_g=25395\text{ in.}^4\text{ , since }M_{cr}=63.14\text{ ft-kips}>M_a=59.43\text{ ft-kips}$$

Where I_e^+ is the effective moment of inertia for the critical positive moment section (midspan).

Since midspan stiffness (including the effect of cracking) has a dominant effect on deflections, midspan section is heavily represented in calculation of I_e and this is considered satisfactory in approximate deflection calculations. The averaged effective moment of inertia ($I_{e,avg}$) is given by:

$$I_{e,avg} = 0.85\, I_e^+ + 0.15\, I_e^- \text{ for end span}$$ ***PCA Notes on ACI 318-11 (9.5.2.4(1))***

$$I_{e,avg} = 0.85\,(25395) + 0.15\,(7380) = 22693 \text{ in.}^4$$

Where:

I_e^- = The effective moment of inertia for the critical negative moment section near the support.

I_e^+ = The effective moment of inertia for the critical positive moment section (midspan).

<u>For the interior span (span with both ends continuous) with service load level ($D+LL_{full}$):</u>

$$I_e^- = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \text{, since } M_{cr} = 36.89 \text{ ft-kips} < M_a = 115.73 \text{ ft-kips}$$

ACI 318-14 (24.2.3.5a)

$$I_e^- = \left(\frac{36.89}{115.73}\right)^3 \times 9333 + \left[1-\left(\frac{36.89}{115.73}\right)^3\right] \times 7331 = 7396 \text{ in.}^4$$

$$I_e^+ = I_g = 25395 \text{ in.}^4 \text{, since } M_{cr} = 63.14 \text{ ft-kips} > M_a = 44.57 \text{ ft-kips}$$

The averaged effective moment of inertia ($I_{e,avg}$) is given by:

$$I_{e,avg} = 0.70\, I_e^+ + 0.15\left(I_{e,l}^- + I_{e,r}^-\right) \text{ for interior span}$$ ***PCA Notes on ACI 318-11 (9.5.2.4(2))***

$$I_{e,avg} = 0.70\,(25395) + 0.15\,(7396 + 7396) = 19995 \text{ in.}^4$$

Where:

$I_{e,l}^-$ = The effective moment of inertia for the critical negative moment section near the left support.

$I_{e,R}^-$ = The effective moment of inertia for the critical negative moment section near the right support.

Table 6 provides a summary of the required parameters and calculated values needed for deflections for exterior and interior equivalent frame. It also provides a summary of the same values for column strip and middle strip to facilitate calculation of panel deflection.

Table 6 – Averaged Effective Moment of Inertia Calculations													
For Frame Strip													
Span	zone	I_g, in.^{4}	I_{cr}, in.^{4}	M_a, ft-kip			M_{cr}, k-ft	I_e, in.^{4}			$I_{e,avg}$, in.^{4}		
				D	D + LL_{Sus}	D + L_{full}		D	D + LL_{Sus}	D + L_{full}	D	D + LL_{Sus}	D + L_{full}
Ext	Left	9333	7147	-30.61	-30.61	-66.92	36.89	9333	9333	7513	22761	22761	22693
	Midspan	25395	2282	27.19	27.19	59.43	63.14	25395	25395	25395			
	Right	9333	7331	-58.35	-58.35	-127.56	36.89	7837	7837	7380			
Int	Left	9333	7331	-52.93	-52.93	-115.73	36.89	8009	8009	7396	20179	20179	19995
	Mid	25395	1553	18.06	18.06	44.57	63.14	25395	25395	25395			
	Right	9333	7331	-52.93	-52.93	-115.73	36.89	8009	8009	7396			

Deflections in two-way slab systems shall be calculated taking into account size and shape of the panel, conditions of support, and nature of restraints at the panel edges. For immediate deflections two-way slab systems the midpanel deflection is computed as the sum of deflection at midspan of the column strip or column line in one direction (Δ_{cx} or Δ_{cy}) and deflection at midspan of the middle strip in the orthogonal direction (Δ_{mx} or Δ_{my}). Figure 25 shows the deflection computation for a rectangular panel. The average Δ for panels that have different properties in the two direction is calculated as follows:

$$\Delta = \frac{(\Delta_{cx} + \Delta_{my}) + (\Delta_{cy} + \Delta_{mx})}{2}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 8)

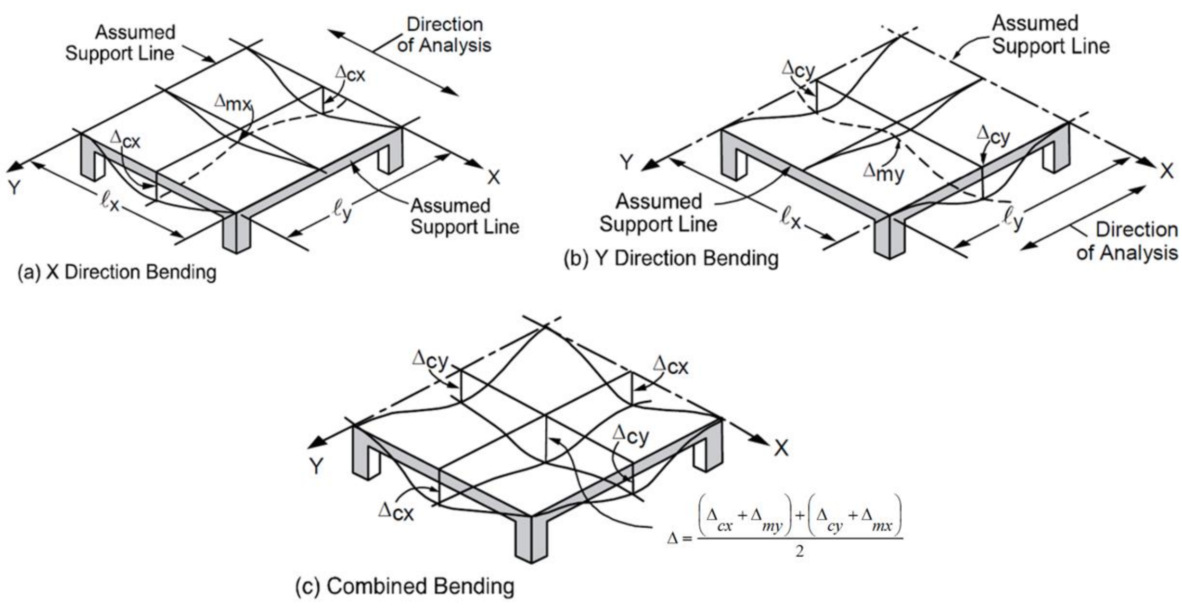


Figure 25 – Deflection Computation for a rectangular Panel

To calculate each term of the previous equation, the following procedure should be used. Figure 26 shows the procedure of calculating the term Δ_{cx}. same procedure can be used to find the other terms.

$$\Delta_{cx} = \Delta_{cx,fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$

$$\Delta_{c,fixed} = LDF_c \times \Delta_{frame,fixed} \times \left(\frac{I_{frame}}{I_c}\right)_g$$

$$\Delta\theta_{c,R} = \theta_{c,R}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)_{column}$$

$$\Delta\theta_{c,L} = \theta_{c,L}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)_{column}$$

$$LDF_c = \frac{LDF^+ + \dfrac{LDF_l^- + LDF_R^-}{2}}{2}$$

$$\theta_{c,R} = \frac{(M_{net,R})_{column}}{K_{ec}}$$

$$\theta_{c,L} = \frac{(M_{net,L})_{column}}{K_{ec}}$$

$$\Delta_{frame,fixed} = \frac{wl^4}{384E_c I_{frame,averaged}}$$

Figure 26 – Δ_{cx} calculation procedure

For exterior span - service dead load case:

$$\Delta_{frame,\,fixed} = \frac{wl^4}{384E_c I_{frame,\,averaged}}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 10)***

Where:

$\Delta_{frame,\,fixed}$ = Deflection of column strip assuing fixed end condition.

$$w = \text{slab weight + beam weight} = \left(\frac{150\times 6}{12} + \frac{150\times(20-6)\times 14}{22\times 144}\right)(22) = 1854 \text{ lb/ft}$$

$$E_c = w_c^{1.5}33\sqrt{f_c'} = 150^{1.5}\times 33\times\sqrt{4000} = 3834\times 10^3 \text{ psi}$$ ***ACI 318-14 (19.2.2.1.a)***

$I_{frame,averaged}$ = The averaged effective moment of inertia ($I_{e,avg}$) for the frame strip for service dead load case from Table 6 = 22761 in.^{4}

$$\Delta_{frame,\,fixed} = \frac{(1854)(17.5-18/12)^4(12)^3}{384(3834\times 10^3)(22761)} = 0.0063 \text{ in.}$$

$$\Delta_{c,\,fixed} = LDF_c \times \Delta_{frame,\,fixed} \times \left(\frac{I_{frame}}{I_c}\right)_g$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 11)***

Where LDF_c is the load distribution factor for the column strip. The load distribution factor for the column strip can be found from the following equation:

$$LDF_c = \frac{LDF^+ + \frac{LDF_l^- + LDF_R^-}{2}}{2}$$

And the load distribution factor for the middle strip can be found from the following equation:

$$LDF_m = 1 - LDF_c$$

For the end span, LDF for exterior negative region (LDF_L^-), interior negative region (LDF_R^-), and positive region (LDF_L^+) are 0.75, 0.67, and 0.67, respectively (From Table 2 of this document). Thus, the load distribution factor for the column strip for the end span is given by:

$$LDF_c = \frac{0.67 + \frac{0.75 + 0.67}{2}}{2} = 0.690$$

$I_{c,g}$ = The gross moment of inertia (I_g) for the column strip (for T section) = 20040 in.^{4}

$I_{frame,g}$ = The gross moment of inertia (I_g) for the frame strip (for T section) = 25395 in.^{4}

$$\Delta_{c,\,fixed} = 0.690 \times 0.0063 \times \frac{25395}{20040} = 0.0055 \text{ in.}$$

$$\theta_{c,L} = \frac{(M_{net,\,L})_{frame}}{K_{ec}}$$ **PCA Notes on ACI 318-11 (9.5.3.4 Eq. 12)**

Where:

$\theta_{c,L}$ = Rotation of the span left support.

$(M_{net,L})_{frame} = 31.13$ ft-kips = Net frame strip negative moment of the left support.

K_{ec} = effective column stiffness for exterior column.

= 764 x E_c = 2929 x 10^6 in.-lb (calculated previously).

$$\theta_{c,L} = \frac{31.13 \times 12 \times 1000}{2929 \times 10^6} = 0.00012 \text{ rad}$$

$$\Delta\theta_{c,L} = \theta_{c,L}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)_{frame}$$ **PCA Notes on ACI 318-11 (9.5.3.4 Eq. 14)**

Where:

$\Delta\theta_{c,L}$ = Midspan deflection due to rotation of left support.

$\left(\frac{I_g}{I_e}\right)_{frame}$ = Gross-to-effective moment of inertia ratio for frame strip.

$$\Delta\theta_{c,L} = 0.00012 \times \frac{(17.5 - 18/12) \times 12}{8} \times \frac{25395}{22761} = 0.0033 \text{ in.}$$

$$\theta_{c,R} = \frac{(M_{net,R})_{frame}}{K_{ec}} = \frac{(58.35 - 52.93) \times 12 \times 1000}{2419 \times 10^6} = 0.00003 \text{ rad}$$

Where

$\theta_{c,R}$ = Rotation of the end span right support.

$(M_{net,R})_{frame}$ = Net frame strip negative moment of the right support.

K_{ec} = effective column stiffness for interior column.

= 631 x E_c = 2419 x 10^6 in.-lb (calculated previously).

$$\Delta\theta_{c,R} = \theta_{c,R}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)_{frame} = 0.00003 \times \frac{(17.5 - 18/12) \times 12}{8} \times \frac{25395}{22761} = 0.00072 \text{ in.}$$

Where:

$\Delta\theta_{c,R}$ = Midspan delfection due to rotation of right support.

$$\Delta_{cx} = \Delta_{cx,\,fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 9)***

$$\Delta_{cx} = 0.0055 + 0.0033 + 0.00072 = 0.009 \text{ in.}$$

Following the same procedure, Δ_{mx} can be calculated for the middle strip. This procedure is repeated for the equivalent frame in the orthogonal direction to obtain Δ_{cy}, and Δ_{my} for the end and middle spans for the other load levels (*D+LL$_{sus}$ and D+LL$_{full}$*).

Assuming square panel, $\Delta_{cx} = \Delta_{cy}$= 0.009 in. and $\Delta_{mx} = \Delta_{my}$= 0.021 in.

The average Δ for the corner panel is calculated as follows:

$$\Delta = \frac{(\Delta_{cx} + \Delta_{my}) + (\Delta_{cy} + \Delta_{mx})}{2} = \left(\Delta_{cx} + \Delta_{my}\right) = \left(\Delta_{cy} + \Delta_{mx}\right) = 0.009 + 0.021 = 0.030 \text{ in.}$$

Table 7 - Instantaneous Deflections

Column Strip

Span	LDF	D						
		$\Delta_{frame-fixed}$, in	$\Delta_{c-fixed}$, in	θ_{c1}, rad	θ_{c2}, rad	$\Delta\theta_{c1}$, in	$\Delta\theta_{c2}$, in	Δ_{cx}, in
Ext	0.69	0.0063	0.0055	0.00012	0.00003	0.0033	0.0007	0.009
Int	0.67	0.0071	0.0060	0.00003	0.00003	-0.0008	-0.0008	0.004

Span	LDF	D+LL$_{sus}$						
		$\Delta_{frame-fixed}$, in	$\Delta_{c-fixed}$, in	θ_{c1}, rad	θ_{c2}, rad	$\Delta\theta_{c1}$, in	$\Delta\theta_{c2}$, in	Δ_{cx}, in
Ext	0.69	0.0063	0.0055	0.00012	0.00003	0.0033	0.0007	0.009
Int	0.67	0.0071	0.0060	0.00003	0.00003	-0.0008	-0.0008	0.004

Span	LDF	D+LL$_{full}$						
		$\Delta_{frame-fixed}$, in	$\Delta_{c-fixed}$, in	θ_{c1}, rad	θ_{c2}, rad	$\Delta\theta_{c1}$, in	$\Delta\theta_{c2}$, in	Δ_{cx}, in
Ext	0.69	0.0137	0.0120	0.00027	0.00006	0.0072	0.0016	0.021
Int	0.67	0.0156	0.0132	0.00006	0.00006	-0.0018	-0.0018	0.010

Span	LDF	LL
		Δ_{cx}, in
Ext	0.69	0.011
Int	0.67	0.005

Middle Strip

LDF	D						
	$\Delta_{frame-fixed}$, in	$\Delta_{m-fixed}$, in	θ_{m1}, rad	θ_{m2}, rad	$\Delta\theta_{m1}$, in	$\Delta\theta_{m2}$, in	Δ_{mx}, in
0.31	0.0063	0.0172	0.00012	0.00003	0.0033	0.0007	0.021
0.33	0.0071	0.0207	0.00003	0.00003	-0.0008	-0.0008	0.019

LDF	D+LL$_{sus}$						
	$\Delta_{frame-fixed}$, in	$\Delta_{m-fixed}$, in	θ_{m1}, rad	θ_{m2}, rad	$\Delta\theta_{m1}$, in	$\Delta\theta_{m2}$, in	Δ_{mx}, in
0.31	0.00627	0.01724	0.00012	0.00003	0.00330	0.00072	0.021
0.33	0.00707	0.02069	0.00003	0.00003	-0.00081	-0.00081	0.019

LDF	D+LL$_{full}$						
	$\Delta_{frame-fixed}$, in	$\Delta_{m-fixed}$, in	θ_{m1}, rad	θ_{m2}, rad	$\Delta\theta_{m1}$, in	$\Delta\theta_{m2}$, in	Δ_{mx}, in
0.31	0.01374	0.03780	0.00027	0.00006	0.00724	0.00158	0.047
0.33	0.01559	0.04566	0.00006	0.00006	-0.00179	-0.00179	0.042

LDF	LL
	Δ_{mx}, in
0.31	0.025
0.33	0.023

5.2. Time-Dependent (Long-Term) Deflections (Δ_{lt})

The additional time-dependent (long-term) deflection resulting from creep and shrinkage (Δ_{cs}) may be estimated as follows:

$$\Delta_{cs} = \lambda_{\Delta} \times (\Delta_{sust})_{Inst}$$ ***PCA Notes on ACI 318-11 (9.5.2.5 Eq. 4)***

The total time-dependent (long-term) deflection is calculated as:

$$(\Delta_{total})_{lt} = (\Delta_{sust})_{Inst} \times (1+\lambda_{\Delta}) + [(\Delta_{total})_{Inst} - (\Delta_{sust})_{Inst}]$$ ***CSA A23.3-04 (N9.8.2.5)***

Where:

$(\Delta_{sust})_{Inst} =$ Immediate (instantaneous) deflection due to sustained load, in.

$$\lambda_{\Delta} = \frac{\xi}{1+50\rho'}$$ ***ACI 318-14 (24.2.4.1.1)***

$(\Delta_{total})_{lt} =$ Time-dependent (long-term) total delfection, in.

$(\Delta_{total})_{Inst} =$ Total immediate (instantaneous) deflection, in.

For the exterior span

ξ = 2, consider the sustained load duration to be 60 months or more. ***ACI 318-14 (Table 24.2.4.1.3)***

ρ' = 0, conservatively.

$$\lambda_{\Delta} = \frac{2}{1+50\times 0} = 2$$

$$\Delta_{cs} = 2\times 0.009 = 0.019 \text{ in.}$$

$$(\Delta_{total})_{lt} = 0.009\times(1+2)+(0.021-0.009) = 0.040 \text{ in.}$$

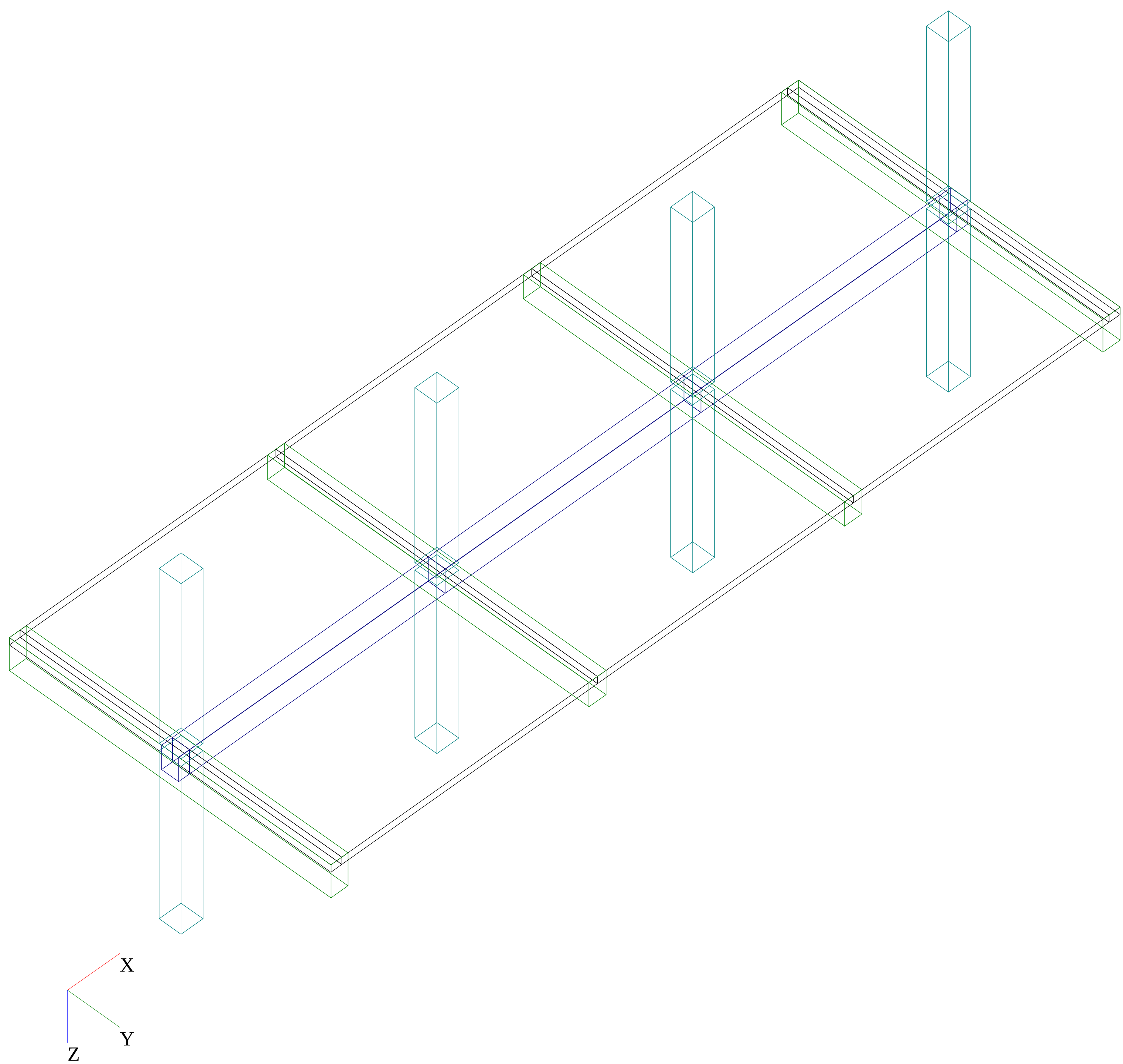
Table 8 shows long-term deflections for the exterior and interior spans for the analysis in the x-direction, for column and middle strips.

Table 8 - Long-Term Deflections					
Column Strip					
Span	**$(\Delta_{sust})_{Inst}$, in**	**λ_{Δ}**	**Δ_{cs}, in**	**$(\Delta_{total})_{Inst}$, in**	**$(\Delta_{total})_{lt}$, in**
Exterior	0.009	2.000	0.019	0.021	0.040
Interior	0.004	2.000	0.009	0.010	0.018
Middle Strip					
Exterior	0.021	2.000	0.043	0.047	0.089
Interior	0.019	2.000	0.038	0.042	0.080

6. spSlab Software Program Model Solution

spSlab program utilizes the Equivalent Frame Method described and illustrated in details here for modeling, analysis and design of two-way concrete floor slab systems. spSlab uses the exact geometry and boundary conditions provided as input to perform an elastic stiffness (matrix) analysis of the equivalent frame taking into account the torsional stiffness of the slabs framing into the column. It also takes into account the complications introduced by a large number of parameters such as vertical and torsional stiffness of transverse beams, the stiffening effect of drop panels, column capitals, and effective contribution of columns above and below the floor slab using the of equivalent column concept ***(ACI 318-14 (R8.11.4))***.

spSlab Program models the equivalent frame as a design strip. The design strip is, then, separated by spSlab into column and middle strips. The program calculates the internal forces (Shear Force & Bending Moment), moment and shear capacity vs. demand diagrams for column and middle strips, instantaneous and long-term deflection results, and required flexural reinforcement for column and middle strips. The graphical and text results will be provided from the spSlab model in a future revision to this document. For a sample output refer to "Two-Way Flat Plate Concrete Floor Slab Design" example.



spSlab v5.00. Licensed to: StructurePoint. License ID: 66184-1055152-4-2C6B6-2C6B6

File: C:\...\TSDA-spSlab-Two-Way Slab with Beams Spanning Between Supports.slb

Project: Two-Way Slab With Beams Spanning Between Supports

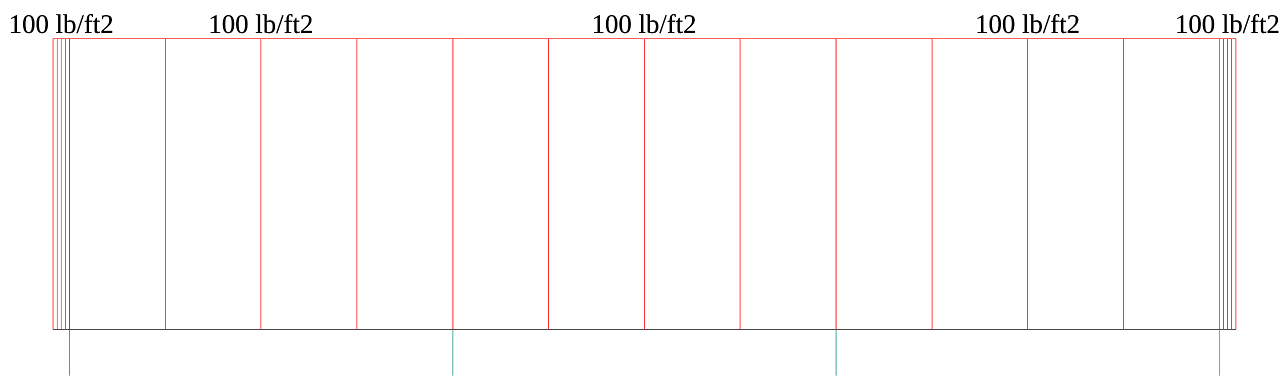
Frame: Interior Frame

Engineer: SP

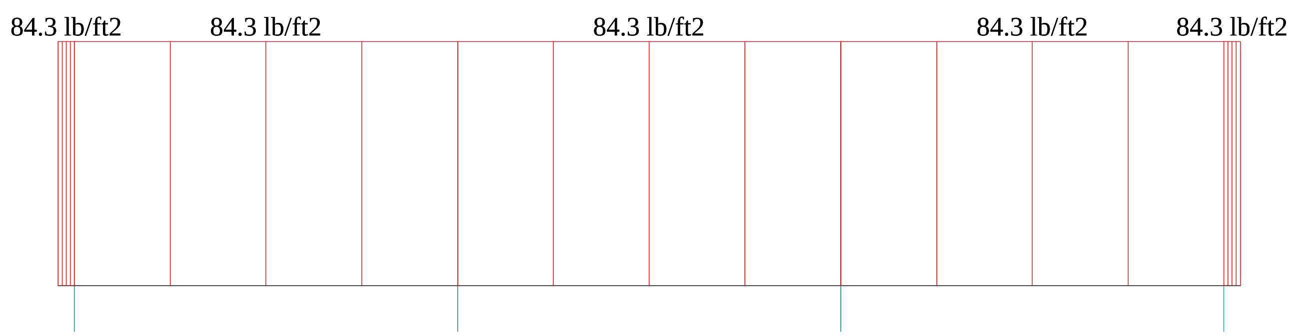
Code: ACI 318-14

Date: 03/31/17

Time: 14:47:17



CASE/PATTERN: Live/All



spSlab v5.00. Licensed to: StructurePoint. License ID: 66184-1055152-4-2C6B6-2C6B6

File: C:\...\TSDA-spSlab-Two-Way Slab with Beams Spanning Between Supports.slb

Project: Two-Way Slab With Beams Spanning Between Supports

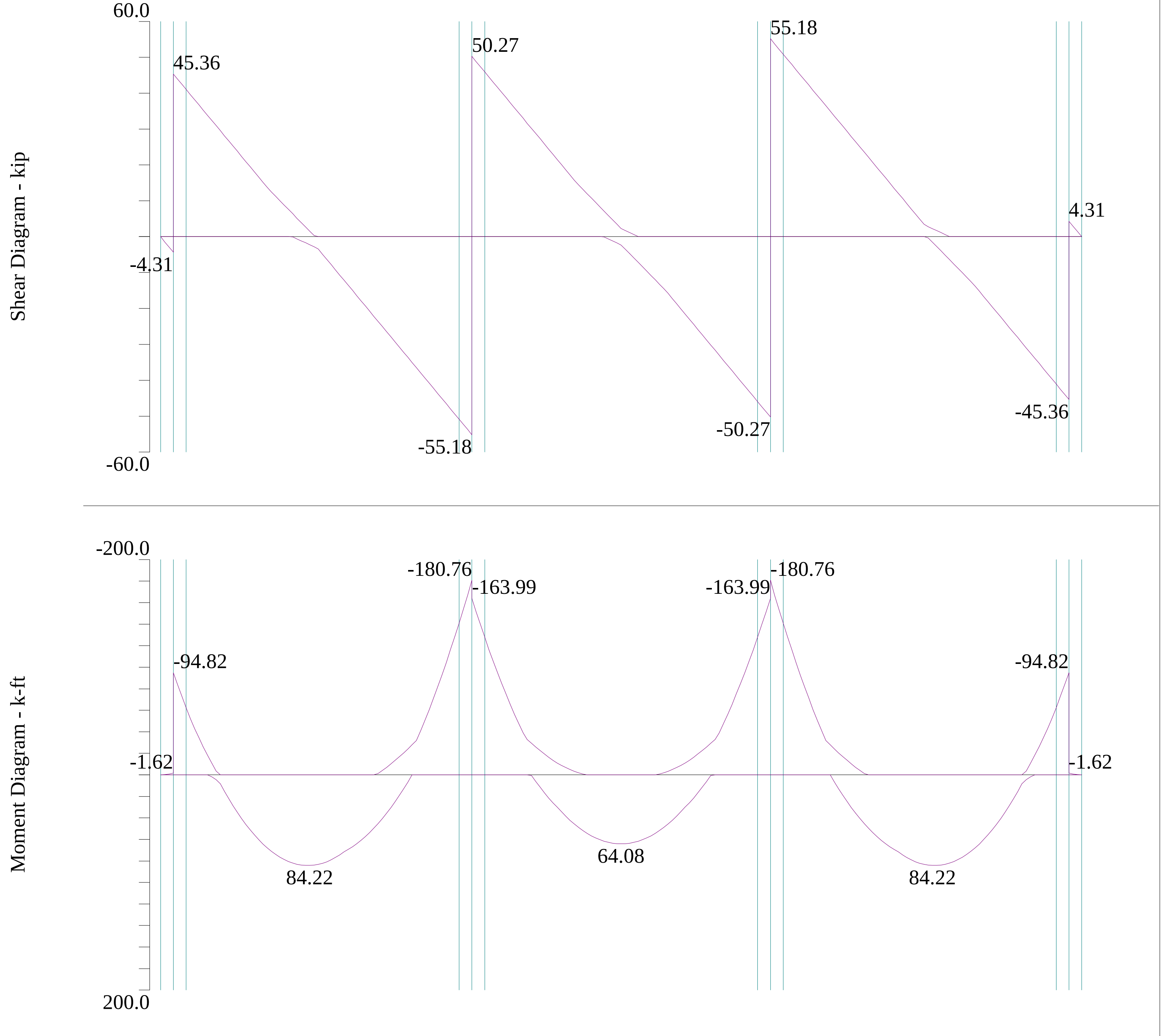
Frame: Interior Frame

Engineer: SP

Code: ACI 318-14

Date: 03/31/17

Time: 14:48:17



spSlab v5.00. Licensed to: StructurePoint. License ID: 66184-1055152-4-2C6B6-2C6B6

File: C:\...\TSDA-spSlab-Two-Way Slab with Beams Spanning Between Supports.slb

Project: Two-Way Slab With Beams Spanning Between Supports

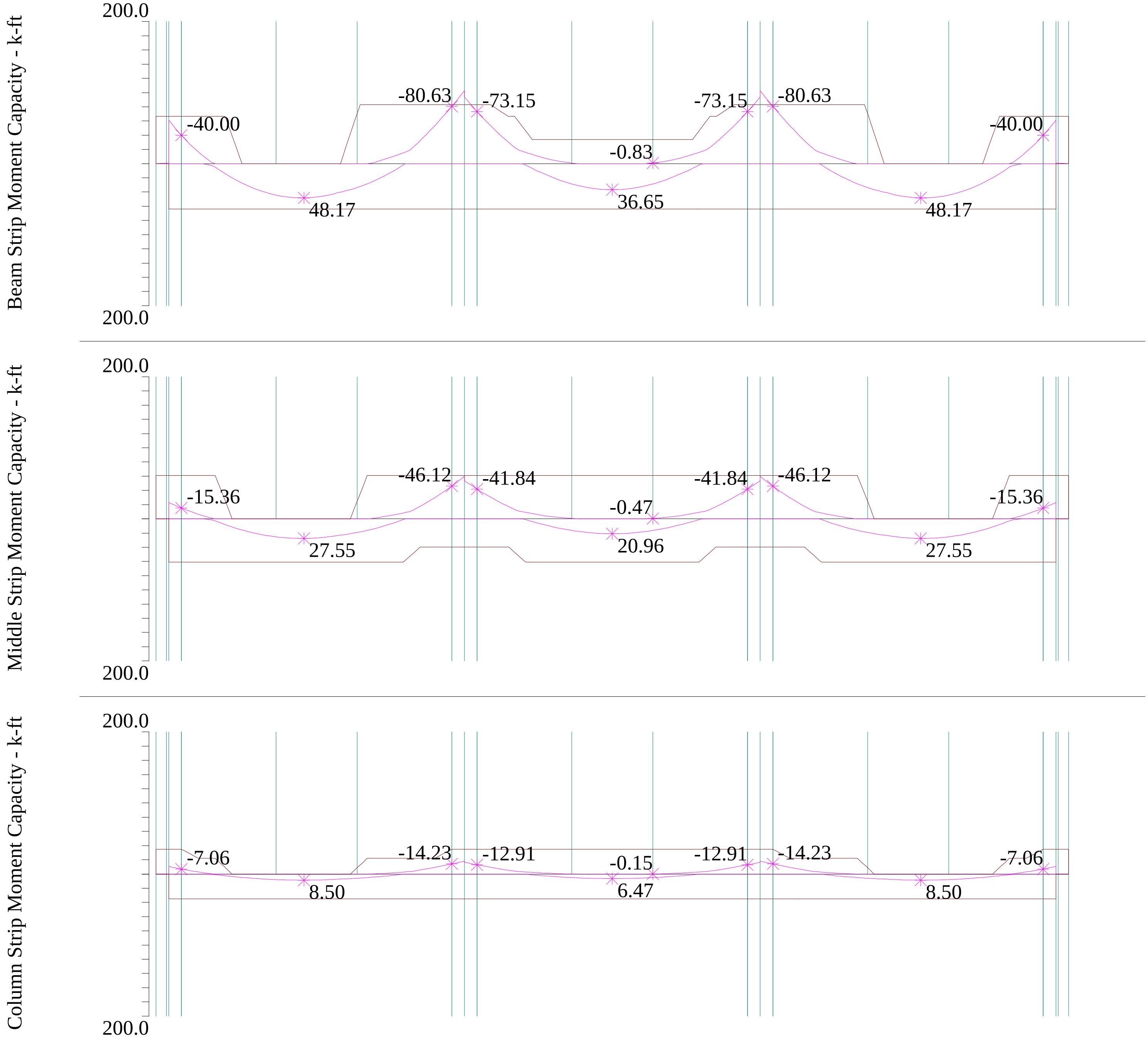
Frame: Interior Frame

Engineer: SP

Code: ACI 318-14

Date: 03/31/17

Time: 14:49:25



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File: C:\...\TSDA-spSlab-Two-Way Slab with Beams Spanning Between Supports.slb

Project: Two-Way Slab With Beams Spanning Between Supports

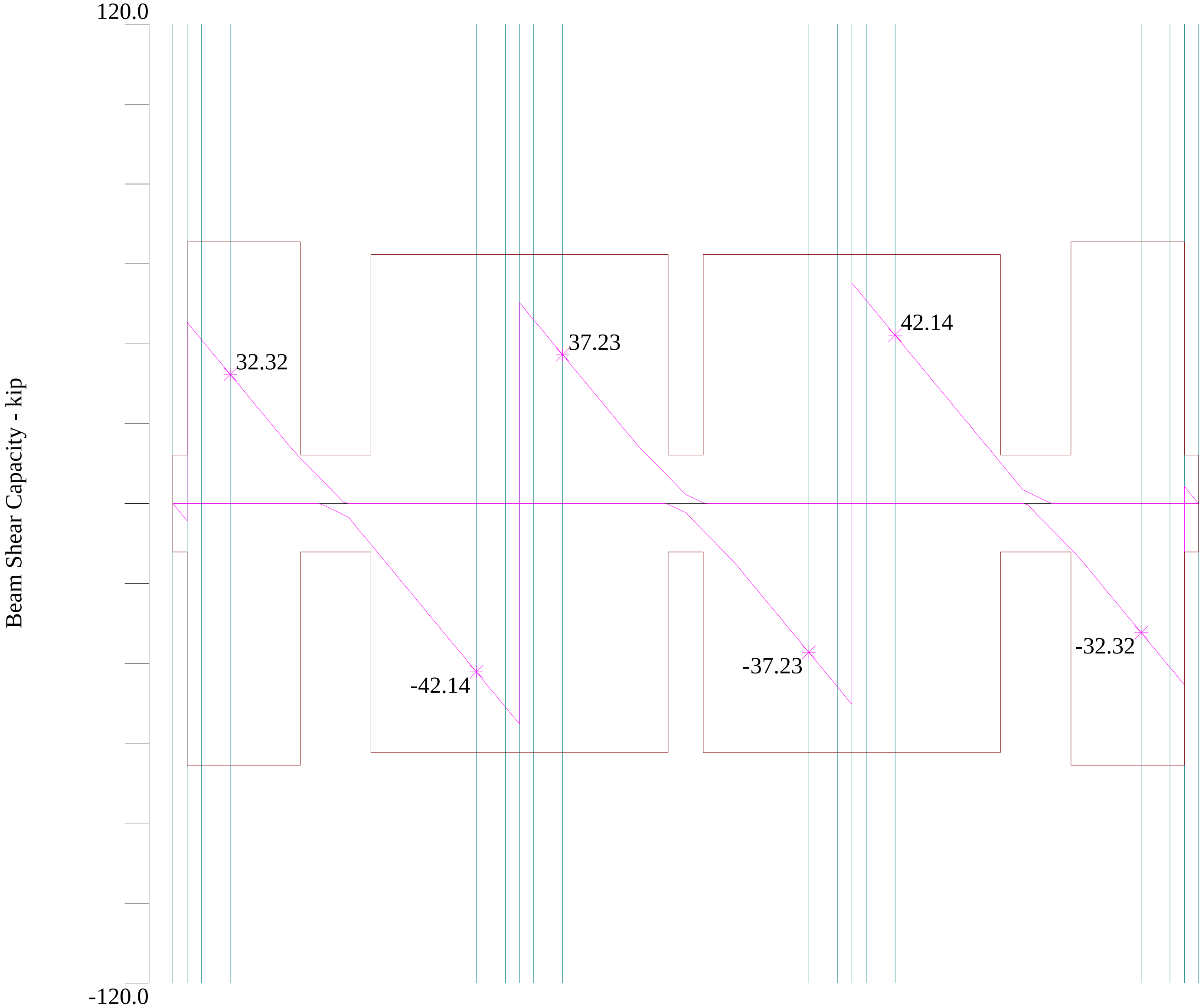
Frame: Interior Frame

Engineer: SP

Code: ACI 318-14

Date: 03/31/17

Time: 14:51:28



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Project: Two-Way Slab With Beams Spanning Between Supports

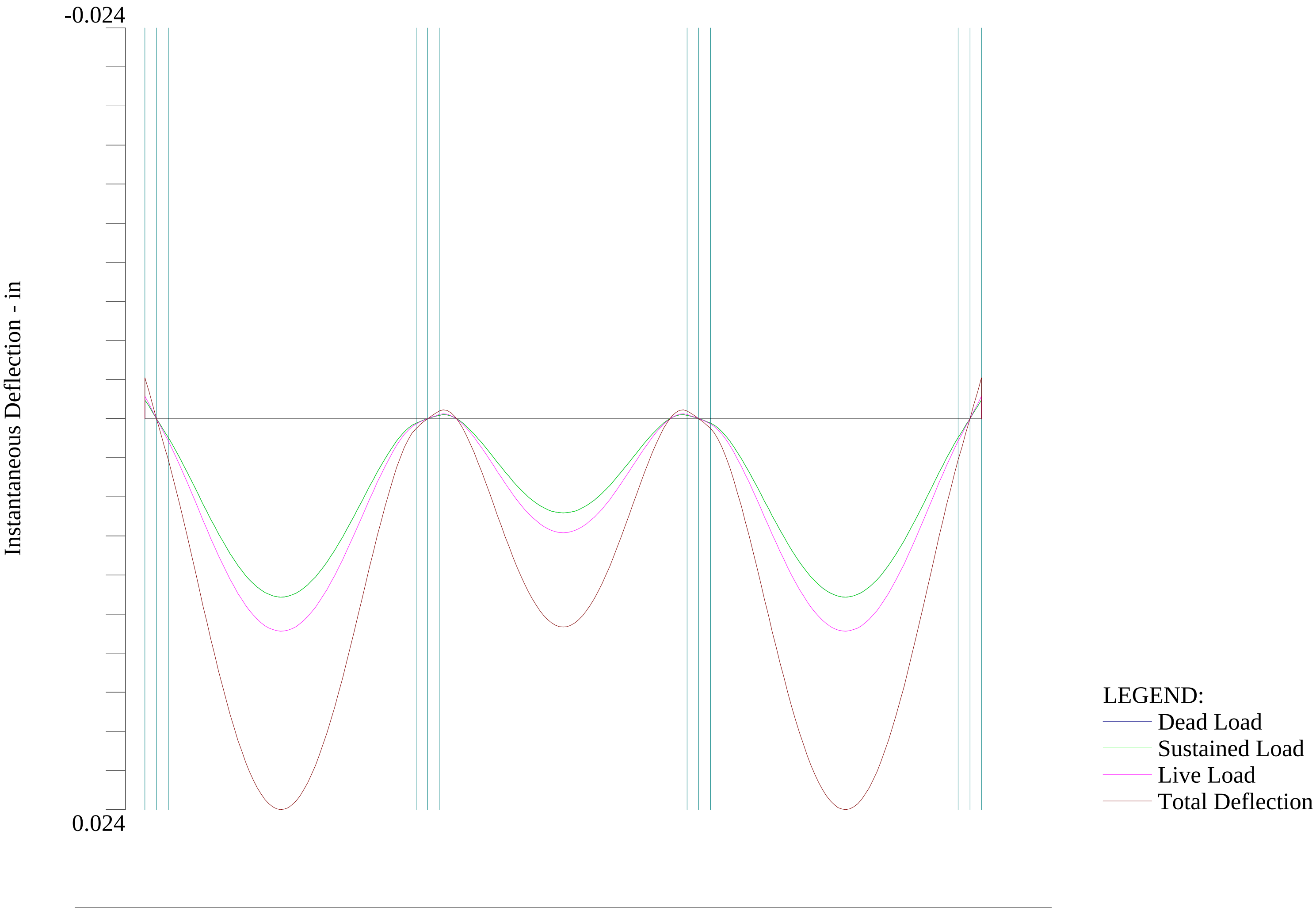
Frame: Interior Frame

Engineer: SP

Code: ACI 318-14

Date: 03/31/17

Time: 14:54:08



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File: C:\...\TSDA-spSlab-Two-Way Slab with Beams Spanning Between Supports.slb

Project: Two-Way Slab With Beams Spanning Between Supports

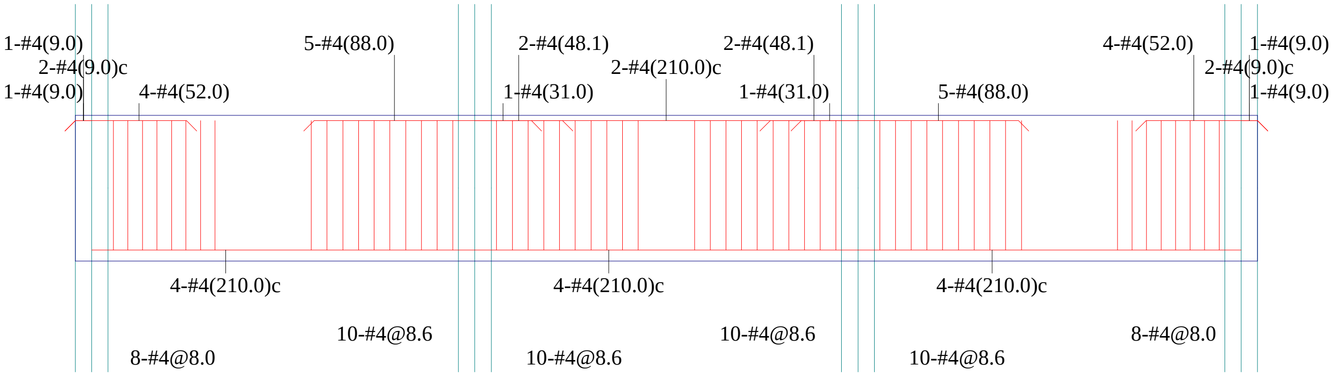
Frame: Interior Frame

Engineer: SP

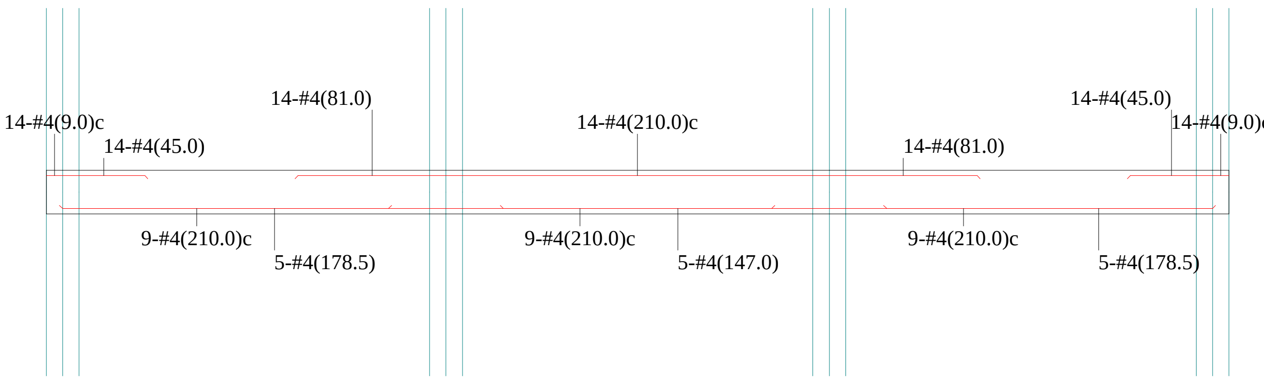
Code: ACI 318-14

Date: 03/31/17

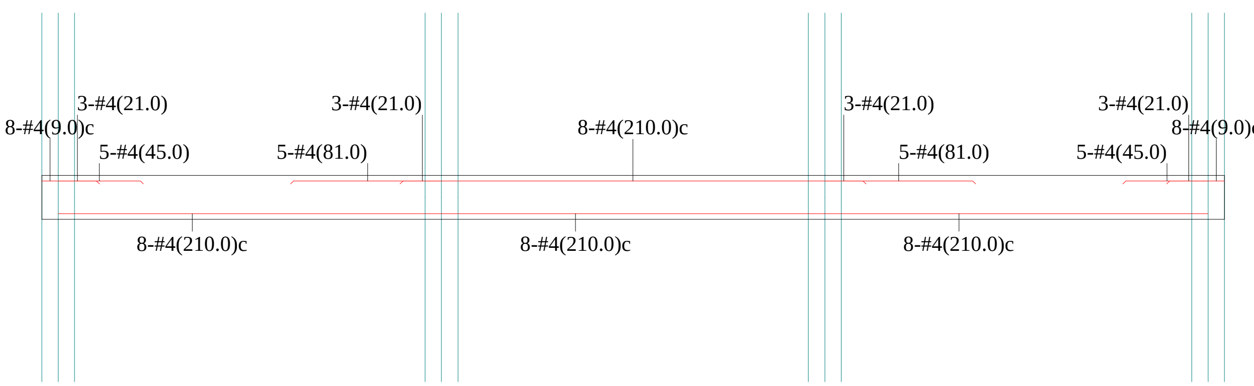
Time: 14:54:54



Beam Strip Flexural and Transverse Reinforcement



Middle Strip Flexural Reinforcement



Column Strip Flexural Reinforcement

spSlab v5.00. Licensed to: StructurePoint. License ID: 66184-1055152-4-2C6B6-2C6B6

File: C:\TSDA\TSDA-spSlab-Two-Way Slab with Beams Spanning Between Supports.slb

Project: Two-Way Slab With Beams Spanning Between Supports

Frame: Interior Frame

Engineer: SP

Code: ACI 318-14

Date: 03/31/17

Time: 14:56:22

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           ooooo    oooooo   oo         oo       ooooo   oo
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               oo   oooooo         oo   oo      oo   oo  oo   oo
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           ooooo    oo         oooooo    ooo     ooooo o  ooooo   (TM)
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spSlab v5.00 (TM)
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams, One-way and Two-way Slab Systems

[1] INPUT ECHO

General Information

File name: C:\TSDA\TSDA-spSlab-Two-Way Slab with Beams Spanning Between Supports.slb
Project: Two-Way Slab With Beams Spanning Between Supports
Frame: Interior Frame
Engineer: SP
Code: ACI 318-14
Reinforcement Database: ASTM A615
Mode: Design
Number of supports = 4 + Left cantilever + Right cantilever
Floor System: Two-Way

Live load pattern ratio = 75%
Minimum free edge distance for punching shear = 4 times slab thickness.
Circular critical section around circular supports used (if possible).
Deflections are based on cracked section properties.
In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
User-defined slab strip widths NOT selected.
User-defined distribution factors NOT selected.
One-way shear in drop panel NOT selected.
Distribution of shear to strips NOT selected.
Beam T-section design NOT selected.
Longitudinal beam contribution in negative reinforcement design over support NOT selected.
Transverse beam contribution in negative reinforcement design over support NOT selected.

Material Properties

		Slabs|Beams	Columns	
wc	=	150	150	lb/ft3
f'c	=	4	4	ksi
Ec	=	3834.3	3834.3	ksi
fr	=	0.47434	0.47434	ksi

fy = 60 ksi, Bars are not epoxy-coated
fyt = 60 ksi
Es = 29000 ksi

Reinforcement Database

Units: Db (in), Ab (in^2), Wb (lb/ft)

Size	Db	Ab	Wb	Size	Db	Ab	Wb
#3	0.38	0.11	0.38	#4	0.50	0.20	0.67
#5	0.63	0.31	1.04	#6	0.75	0.44	1.50
#7	0.88	0.60	2.04	#8	1.00	0.79	2.67
#9	1.13	1.00	3.40	#10	1.27	1.27	4.30

#11	1.41	1.56	5.31	#14	1.69	2.25	7.65
#18	2.26	4.00	13.60				

Span Data

Slabs

Units: L1, wL, wR, L2L, L2R (ft); t, Hmin (in)

Span	Loc	L1	t	wL	wR	L2L	L2R	Hmin	
1	Int	0.750	6.00	11.000	11.000	22.000	22.000	---	LC *i
2	Int	17.500	6.00	11.000	11.000	22.000	22.000	5.81	
3	Int	17.500	6.00	11.000	11.000	22.000	22.000	5.79	
4	Int	17.500	6.00	11.000	11.000	22.000	22.000	5.81	
5	Int	0.750	6.00	11.000	11.000	22.000	22.000	---	RC *i

NOTES:
Deflection check required for panels where code-specified Hmin for two-way construction doesn't apply due to:
*i - cantilever end span (LC, RC) support condition

Ribs and Longitudinal Beams

Units: b, h, Sp (in)

	Ribs			Beams		
Span	b	h	Sp	b	h	Offset
1	0.00	0.00	0.00	14.00	20.00	0.00
2	0.00	0.00	0.00	14.00	20.00	0.00
3	0.00	0.00	0.00	14.00	20.00	0.00
4	0.00	0.00	0.00	14.00	20.00	0.00
5	0.00	0.00	0.00	14.00	20.00	0.00

Support Data

Columns

Units: c1a, c2a, c1b, c2b (in); Ha, Hb (ft)

Supp	c1a	c2a	Ha	c1b	c2b	Hb	Red%
1	18.00	18.00	12.000	18.00	18.00	12.000	100
2	18.00	18.00	12.000	18.00	18.00	12.000	100
3	18.00	18.00	12.000	18.00	18.00	12.000	100
4	18.00	18.00	12.000	18.00	18.00	12.000	100

Transverse Beams

Units: b, h, Ecc (in)

Supp	b	h	Ecc
1	14.00	27.00	-2.00
2	14.00	20.00	0.00
3	14.00	20.00	0.00
4	14.00	27.00	2.00

Boundary Conditions

Units: Kz (kip/in); Kry (kip-in/rad)

Supp	Spring Kz	Spring Kry	Far End A	Far End B
1	0	0	Fixed	Fixed
2	0	0	Fixed	Fixed
3	0	0	Fixed	Fixed
4	0	0	Fixed	Fixed

Load Data

Load Cases and Combinations

Case	Dead	Live
Type	DEAD	LIVE
U1	1.200	1.600

Area Loads

Units: Wa (lb/ft2)

Case/Patt	Span	Wa
Dead	1	84.30
	2	84.30
	3	84.30
	4	84.30
	5	84.30
Live	1	100.00
	2	100.00
	3	100.00
	4	100.00
	5	100.00

Live/Odd	1	75.00
	3	75.00
	5	75.00
Live/Even	2	75.00
	4	75.00
Live/S1	1	75.00
	2	75.00
Live/S2	2	75.00
	3	75.00
Live/S3	3	75.00
	4	75.00
Live/S4	4	75.00
	5	75.00

Reinforcement Criteria

Slabs and Ribs

	Top bars Min	Top bars Max	Bottom bars Min	Bottom bars Max	
Bar Size	#4	#8	#4	#8	
Bar spacing	1.00	18.00	1.00	18.00	in
Reinf ratio	0.14	5.00	0.14	5.00	%
Cover	0.75		0.75		in

There is NOT more than 12 in of concrete below top bars.

Beams

	Top bars Min	Top bars Max	Bottom bars Min	Bottom bars Max		Stirrups Min	Stirrups Max	
Bar Size	#4	#8	#4	#8		#4	#5	
Bar spacing	1.00	18.00	1.00	18.00		6.00	18.00	in
Reinf ratio	0.14	5.00	0.14	5.00	%			
Cover	0.75		1.51		in			
Layer dist.	1.00		1.00		in			
No. of legs						2	6	
Side cover						1.50		in
1st Stirrup						3.00		in

There is NOT more than 12 in of concrete below top bars.

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                         oooooo     o                 o
                        oo    oo   oo                oo
  ooooo    oooooo       oo         oo        ooooo   oo
 oo    o   oo   oo       oo        oo       o    oo  oo
 oo        oo   oo        ooo      oo        oooooo  oooooo
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 o    oo   oo           oo    oo   oo  o    oo   oo  oo   oo
  ooooo    oo            oooooo     ooo      ooooo o  ooooo   (TM)
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spSlab v5.00 (TM)
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams, One-way and Two-way Slab Systems

[2] DESIGN RESULTS*

*Unless otherwise noted, all results are in the direction of analysis only. Another analysis in the perpendicular direction has to be carried out for two-way slab systems.

Strip Widths and Distribution Factors

Units: Width (ft).

Span	Strip	Width Left**	Width Right**	Width Bottom*	Moment Factor Left**	Moment Factor Right**	Moment Factor Bottom*
1	Column	7.58	7.58	7.58	0.122	0.122	0.113
	Middle	13.25	13.25	13.25	0.188	0.188	0.250
	Beam	1.17	1.17	1.17	0.690	0.690	0.637
2	Column	7.58	7.58	7.58	0.113	0.101	0.101
	Middle	13.25	13.25	13.25	0.246	0.327	0.327
	Beam	1.17	1.17	1.17	0.641	0.572	0.572
3	Column	7.58	7.58	7.58	0.101	0.101	0.101
	Middle	13.25	13.25	13.25	0.327	0.327	0.327
	Beam	1.17	1.17	1.17	0.572	0.572	0.572
4	Column	7.58	7.58	7.58	0.101	0.113	0.101
	Middle	13.25	13.25	13.25	0.327	0.246	0.327
	Beam	1.17	1.17	1.17	0.572	0.641	0.572
5	Column	7.58	7.58	7.58	0.122	0.122	0.113
	Middle	13.25	13.25	13.25	0.188	0.188	0.250
	Beam	1.17	1.17	1.17	0.690	0.690	0.637

*Used for bottom reinforcement. **Used for top reinforcement.

Top Reinforcement

Units: Width (ft), Mmax (k-ft), Xmax (ft), As (in^2), Sp (in)

Span	Strip	Zone	Width	Mmax	Xmax	AsMin	AsMax	AsReq	SpProv	Bars
1	Column	Left	7.58	0.02	0.217	0.983	8.218	0.001	11.375	8-#4 *3 *5
		Midspan	7.58	0.06	0.402	0.983	8.218	0.003	11.375	8-#4 *3 *5
		Right	7.58	0.14	0.619	0.983	8.218	0.006	11.375	8-#4 *3 *5
	Middle	Left	13.25	0.03	0.217	1.717	14.360	0.001	11.357	14-#4 *3 *5
		Midspan	13.25	0.10	0.402	1.717	14.360	0.004	11.357	14-#4 *3 *5
		Right	13.25	0.22	0.619	1.717	14.360	0.010	11.357	14-#4 *3 *5
	Beam	Left	1.17	0.11	0.217	0.372	4.805	0.001	8.664	2-#4 *3
		Midspan	1.17	0.35	0.402	0.372	4.805	0.004	8.664	2-#4 *3
		Right	1.17	0.79	0.619	0.372	4.805	0.009	2.888	4-#4 *3
2	Column	Left	7.58	7.06	0.750	0.983	8.218	0.316	11.375	8-#4 *3 *5
		Midspan	7.58	0.00	8.750	0.000	8.218	0.000	0.000	---
		Right	7.58	14.23	16.750	0.983	8.218	0.640	11.375	8-#4 *3 *5

	Middle	Left	13.25	15.36	0.750	1.717	14.360	0.688	11.357	14-#4 *3 *5
		Midspan	13.25	0.00	8.750	0.000	14.360	0.000	0.000	---
		Right	13.25	46.12	16.750	1.717	14.360	2.099	11.357	14-#4 *5
	Beam	Left	1.17	40.00	0.750	0.632	4.805	0.475	2.888	4-#4 *3
		Midspan	1.17	0.00	8.750	0.000	4.805	0.000	0.000	---
		Right	1.17	80.63	16.750	0.887	4.805	0.975	2.166	5-#4
3	Column	Left	7.58	12.91	0.750	0.983	8.218	0.580	11.375	8-#4 *3 *5
		Midspan	7.58	0.15	11.150	0.983	8.218	0.007	11.375	8-#4 *3 *5
		Right	7.58	12.91	16.750	0.983	8.218	0.580	11.375	8-#4 *3 *5
	Middle	Left	13.25	41.84	0.750	1.717	14.360	1.900	11.357	14-#4 *5
		Midspan	13.25	0.47	11.150	1.717	14.360	0.021	11.357	14-#4 *3 *5
		Right	13.25	41.84	16.750	1.717	14.360	1.900	11.357	14-#4 *5
	Beam	Left	1.17	73.15	0.750	0.887	4.805	0.881	2.166	5-#4 *3
		Midspan	1.17	0.83	11.150	0.372	4.805	0.010	8.664	2-#4 *3
		Right	1.17	73.15	16.750	0.887	4.805	0.881	2.166	5-#4 *3
4	Column	Left	7.58	14.23	0.750	0.983	8.218	0.640	11.375	8-#4 *3 *5
		Midspan	7.58	0.00	8.750	0.000	8.218	0.000	0.000	---
		Right	7.58	7.06	16.750	0.983	8.218	0.316	11.375	8-#4 *3 *5
	Middle	Left	13.25	46.12	0.750	1.717	14.360	2.099	11.357	14-#4 *5
		Midspan	13.25	0.00	8.750	0.000	14.360	0.000	0.000	---
		Right	13.25	15.36	16.750	1.717	14.360	0.688	11.357	14-#4 *3 *5
	Beam	Left	1.17	80.63	0.750	0.887	4.805	0.975	2.166	5-#4
		Midspan	1.17	0.00	8.750	0.000	4.805	0.000	0.000	---
		Right	1.17	40.00	16.750	0.632	4.805	0.475	2.888	4-#4 *3
5	Column	Left	7.58	0.14	0.131	0.983	8.218	0.006	11.375	8-#4 *3 *5
		Midspan	7.58	0.06	0.348	0.983	8.218	0.003	11.375	8-#4 *3 *5
		Right	7.58	0.02	0.533	0.983	8.218	0.001	11.375	8-#4 *3 *5
	Middle	Left	13.25	0.22	0.131	1.717	14.360	0.010	11.357	14-#4 *3 *5
		Midspan	13.25	0.10	0.348	1.717	14.360	0.004	11.357	14-#4 *3 *5
		Right	13.25	0.03	0.533	1.717	14.360	0.001	11.357	14-#4 *3 *5
	Beam	Left	1.17	0.79	0.131	0.372	4.805	0.009	2.888	4-#4 *3
		Midspan	1.17	0.35	0.348	0.372	4.805	0.004	8.664	2-#4 *3
		Right	1.17	0.11	0.533	0.372	4.805	0.001	8.664	2-#4 *3

NOTES:
*3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

Top Bar Details

Units: Length (ft)

		Left				Continuous		Right			
Span	Strip	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
1	Column	---		---		8-#4	0.75	---		---	
	Middle	---		---		14-#4	0.75	---		---	
	Beam	---		---		2-#4	0.75	1-#4	0.75	1-#4	0.75
2	Column	5-#4	3.75	3-#4	1.75	---		5-#4	6.75	3-#4	1.75
	Middle	14-#4	3.75	---		---		14-#4	6.75	---	
	Beam	4-#4	4.33	---		---		5-#4	7.33	---	
3	Column	---		---		8-#4	17.50	---		---	
	Middle	---		---		14-#4	17.50	---		---	
	Beam	2-#4*	4.01	1-#4*	2.59	2-#4	17.50	2-#4*	4.01	1-#4*	2.59
4	Column	5-#4	6.75	3-#4	1.75	---		5-#4	3.75	3-#4	1.75
	Middle	14-#4	6.75	---		---		14-#4	3.75	---	
	Beam	5-#4	7.33	---		---		4-#4	4.33	---	
5	Column	---		---		8-#4	0.75	---		---	
	Middle	---		---		14-#4	0.75	---		---	
	Beam	1-#4	0.75	1-#4	0.75	2-#4	0.75	---		---	

NOTES:
* - Bar cut-off location does not meet ACI 318, 12.10.5.1. Revise location,
unless the requirements of either 12.10.5.2 or 12.10.5.3 are manually checked and satisfied.

Top Bar Development Lengths

Units: Length (in)

		Left				Continuous		Right			
Span	Strip	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen
1	Column	---		---		8-#4	12.00	---		---	
	Middle	---		---		14-#4	12.00	---		---	
	Beam	---		---		2-#4	12.00	1-#4	12.00	1-#4	12.00
2	Column	5-#4	12.00	3-#4	12.00	---		5-#4	12.00	3-#4	12.00

	Middle	14-#4	12.00	---		---		14-#4	12.00	---	
	Beam	4-#4	12.00	---		---		5-#4	13.87	---	
3	Column	---		---		8-#4	12.00	---		---	
	Middle	---		---		14-#4	12.00	---		---	
	Beam	2-#4	12.54	1-#4	12.54	2-#4	12.00	2-#4	12.54	1-#4	12.54
4	Column	5-#4	12.00	3-#4	12.00	---		5-#4	12.00	3-#4	12.00
	Middle	14-#4	12.00	---		---		14-#4	12.00	---	
	Beam	5-#4	13.87	---		---		4-#4	12.00	---	
5	Column	---		---		8-#4	12.00	---		---	
	Middle	---		---		14-#4	12.00	---		---	
	Beam	1-#4	12.00	1-#4	12.00	2-#4	12.00	---		---	

Bottom Reinforcement

Units: Width (ft), Mmax (k-ft), Xmax (ft), As (in^2), Sp (in)

Span	Strip	Width	Mmax	Xmax	AsMin	AsMax	AsReq	SpProv	Bars
1	Column	7.58	0.00	0.309	0.000	8.218	0.000	0.000	---
	Middle	13.25	0.00	0.309	0.000	14.360	0.000	0.000	---
	Beam	1.17	0.00	0.309	0.000	4.613	0.000	0.000	---
2	Column	7.58	8.50	8.000	0.983	8.218	0.381	11.375	8-#4 *3 *5
	Middle	13.25	27.55	8.000	1.717	14.360	1.242	11.357	14-#4 *3 *5
	Beam	1.17	48.17	8.000	0.797	4.613	0.599	2.888	4-#4 *3
3	Column	7.58	6.47	8.750	0.983	8.218	0.289	11.375	8-#4 *3 *5
	Middle	13.25	20.96	8.750	1.717	14.360	0.942	11.357	14-#4 *3 *5
	Beam	1.17	36.65	8.750	0.603	4.613	0.454	2.888	4-#4 *3
4	Column	7.58	8.50	9.500	0.983	8.218	0.381	11.375	8-#4 *3 *5
	Middle	13.25	27.55	9.500	1.717	14.360	1.242	11.357	14-#4 *3 *5
	Beam	1.17	48.17	9.500	0.797	4.613	0.599	2.888	4-#4 *3
5	Column	7.58	0.00	0.441	0.000	8.218	0.000	0.000	---
	Middle	13.25	0.00	0.441	0.000	14.360	0.000	0.000	---
	Beam	1.17	0.00	0.441	0.000	4.613	0.000	0.000	---

NOTES:
*3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

Bottom Bar Details

Units: Start (ft), Length (ft)

		Long Bars			Short Bars		
Span	Strip	Bars	Start	Length	Bars	Start	Length
1	Column	---			---		
	Middle	---			---		
	Beam	---			---		
2	Column	8-#4	0.00	17.50	---		
	Middle	9-#4	0.00	17.50	5-#4	0.00	14.88
	Beam	4-#4	0.00	17.50	---		
3	Column	8-#4	0.00	17.50	---		
	Middle	9-#4	0.00	17.50	5-#4	2.63	12.25
	Beam	4-#4	0.00	17.50	---		
4	Column	8-#4	0.00	17.50	---		
	Middle	9-#4	0.00	17.50	5-#4	2.63	14.88
	Beam	4-#4	0.00	17.50	---		
5	Column	---			---		
	Middle	---			---		
	Beam	---			---		

Bottom Bar Development Lengths

Units: DevLen (in)

		Long Bars		Short Bars	
Span	Strip	Bars	DevLen	Bars	DevLen
1	Column	---		---	
	Middle	---		---	
	Beam	---		---	
2	Column	8-#4	12.00	---	
	Middle	9-#4	12.00	5-#4	12.00
	Beam	4-#4	12.00	---	
3	Column	8-#4	12.00	---	
	Middle	9-#4	12.00	5-#4	12.00
	Beam	4-#4	12.00	---	

4	Column	8-#4	12.00	---	
	Middle	9-#4	12.00	5-#4	12.00
	Beam	4-#4	12.00	---	
5	Column	---		---	
	Middle	---		---	
	Beam	---		---	

Flexural Capacity
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Units: x (ft), As (in^2), PhiMn, Mu (k-ft)

			Top						Bottom					
Span	Strip	x	AsTop	PhiMn-	Mu-	Comb	Pat	Status	AsBot	PhiMn+	Mu+	Comb	Pat	Status
1	Column	0.000	1.60	-34.88	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.217	1.60	-34.88	-0.02	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.375	1.60	-34.88	-0.05	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.402	1.60	-34.88	-0.06	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.619	1.60	-34.88	-0.14	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.750	1.60	-34.88	-0.20	U1	All	---	0.00	0.00	0.00	U1	All	---
	Middle	0.000	2.80	-61.04	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.217	2.80	-61.04	-0.03	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.375	2.80	-61.04	-0.08	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.402	2.80	-61.04	-0.10	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.619	2.80	-61.04	-0.22	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.750	2.80	-61.04	-0.30	U1	All	---	0.00	0.00	0.00	U1	All	---
	Beam	0.000	0.80	-66.58	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.217	0.80	-66.58	-0.11	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.375	0.80	-66.58	-0.31	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.402	0.80	-66.58	-0.35	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.619	0.80	-66.58	-0.79	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.750	0.80	-66.58	-1.12	U1	All	---	0.00	0.00	0.00	U1	All	---
2	Column	0.000	1.60	-34.88	-10.78	U1	All	---	1.60	34.88	0.00	U1	All	---
		0.750	1.60	-34.88	-7.06	U1	All	OK	1.60	34.88	0.00	U1	All	OK
		1.750	1.00	-22.06	-2.95	U1	Even	OK	1.60	34.88	0.00	U1	All	OK
		2.750	1.00	-22.06	0.00	U1	All	OK	1.60	34.88	0.83	U1	All	OK
		3.750	0.00	0.00	0.00	U1	All	OK	1.60	34.88	3.52	U1	All	OK
		6.350	0.00	0.00	0.00	U1	All	OK	1.60	34.88	7.81	U1	All	OK
		8.000	0.00	0.00	0.00	U1	All	OK	1.60	34.88	8.50	U1	All	OK
		8.750	0.00	0.00	0.00	U1	All	OK	1.60	34.88	8.29	U1	All	OK
		10.750	0.00	0.00	0.00	U1	All	OK	1.60	34.88	6.46	U1	Even	OK
		11.150	0.40	-8.93	0.00	U1	All	OK	1.60	34.88	5.94	U1	Even	OK
		11.750	1.00	-22.06	0.00	U1	All	OK	1.60	34.88	5.02	U1	Even	OK
		15.750	1.00	-22.06	-9.46	U1	All	OK	1.60	34.88	0.00	U1	All	OK
		16.750	1.60	-34.88	-14.23	U1	All	OK	1.60	34.88	0.00	U1	All	OK
		17.500	1.60	-34.88	-18.14	U1	All	---	1.60	34.88	0.00	U1	All	---
	Middle	0.000	2.80	-61.04	-22.97	U1	All	---	2.80	61.04	0.00	U1	All	---
		0.750	2.80	-61.04	-15.36	U1	All	OK	2.80	61.04	0.00	U1	All	OK
		2.750	2.80	-61.04	0.00	U1	All	OK	2.80	61.04	2.68	U1	All	OK
		3.750	0.00	0.00	0.00	U1	All	OK	2.80	61.04	11.41	U1	All	OK
		6.350	0.00	0.00	0.00	U1	All	OK	2.80	61.04	25.30	U1	All	OK
		8.000	0.00	0.00	0.00	U1	All	OK	2.80	61.04	27.55	U1	All	OK
		8.750	0.00	0.00	0.00	U1	All	OK	2.80	61.04	26.88	U1	All	OK
		10.750	0.00	0.00	0.00	U1	All	OK	2.80	61.04	20.94	U1	Even	OK
		11.150	1.12	-24.89	0.00	U1	All	OK	2.80	61.04	19.25	U1	Even	OK
		11.750	2.80	-61.04	0.00	U1	All	OK	2.80	61.04	16.26	U1	Even	OK
		13.875	2.80	-61.04	-8.12	U1	Odd	OK	2.80	61.04	1.02	U1	Even	OK
		14.875	2.80	-61.04	-17.71	U1	All	OK	1.80	39.69	0.00	U1	All	OK
		16.750	2.80	-61.04	-46.12	U1	All	OK	1.80	39.69	0.00	U1	All	OK
		17.500	2.80	-61.04	-59.82	U1	All	---	1.80	39.69	0.00	U1	All	---
	Beam	0.000	0.80	-66.58	-61.08	U1	All	---	0.80	63.86	0.00	U1	All	---
		0.750	0.80	-66.58	-40.00	U1	All	OK	0.80	63.86	0.00	U1	All	OK
		3.333	0.80	-66.58	0.00	U1	All	OK	0.80	63.86	13.97	U1	All	OK
		4.333	0.00	0.00	0.00	U1	All	OK	0.80	63.86	27.32	U1	All	OK
		6.350	0.00	0.00	0.00	U1	All	OK	0.80	63.86	44.24	U1	All	OK
		8.000	0.00	0.00	0.00	U1	All	OK	0.80	63.86	48.17	U1	All	OK
		8.750	0.00	0.00	0.00	U1	All	OK	0.80	63.86	46.99	U1	All	OK
		10.167	0.00	0.00	0.00	U1	All	OK	0.80	63.86	40.07	U1	Even	OK
		11.150	0.85	-70.69	0.00	U1	All	OK	0.80	63.86	33.65	U1	Even	OK
		11.322	1.00	-82.66	0.00	U1	All	OK	0.80	63.86	32.25	U1	Even	OK
		16.750	1.00	-82.66	-80.63	U1	All	OK	0.80	63.86	0.00	U1	All	OK
		17.000	1.00	-82.66	-87.85	U1	All	---	0.80	63.86	0.00	U1	All	---
		17.500	1.00	-82.66	-102.80	U1	All	---	0.80	63.86	0.00	U1	All	---
3	Column	0.000	1.60	-34.88	-16.55	U1	All	---	1.60	34.88	0.00	U1	All	---
		0.750	1.60	-34.88	-12.91	U1	All	OK	1.60	34.88	0.00	U1	All	OK
		6.350	1.60	-34.88	-0.15	U1	Even	OK	1.60	34.88	5.05	U1	Odd	OK
		8.750	1.60	-34.88	0.00	U1	All	OK	1.60	34.88	6.47	U1	Odd	OK
		11.150	1.60	-34.88	-0.15	U1	Even	OK	1.60	34.88	5.05	U1	Odd	OK
		16.750	1.60	-34.88	-12.91	U1	All	OK	1.60	34.88	0.00	U1	All	OK
		17.500	1.60	-34.88	-16.55	U1	All	---	1.60	34.88	0.00	U1	All	---
	Middle	0.000	2.80	-61.04	-53.65	U1	All	---	1.80	39.69	0.00	U1	All	---
		0.750	2.80	-61.04	-41.84	U1	All	OK	1.80	39.69	0.00	U1	All	OK
		2.625	2.80	-61.04	-16.97	U1	All	OK	1.80	39.69	0.00	U1	All	OK
		3.625	2.80	-61.04	-9.01	U1	S1	OK	2.80	61.04	1.12	U1	S3	OK
		6.350	2.80	-61.04	-0.47	U1	Even	OK	2.80	61.04	16.37	U1	Odd	OK

	8.750	2.80	-61.04	0.00	U1 All	OK	2.80	61.04	20.96	U1 Odd	OK
	11.150	2.80	-61.04	-0.47	U1 Even	OK	2.80	61.04	16.37	U1 Odd	OK
	13.875	2.80	-61.04	-9.01	U1 S4	OK	2.80	61.04	1.12	U1 S2	OK
	14.875	2.80	-61.04	-16.97	U1 All	OK	1.80	39.69	0.00	U1 All	OK
	16.750	2.80	-61.04	-41.84	U1 All	OK	1.80	39.69	0.00	U1 All	OK
	17.500	2.80	-61.04	-53.65	U1 All	---	1.80	39.69	0.00	U1 All	---
Beam	0.000	1.00	-82.66	-93.79	U1 All	---	0.80	63.86	0.00	U1 All	---
	0.250	1.00	-82.66	-86.71	U1 All	---	0.80	63.86	0.00	U1 All	---
	0.750	1.00	-82.66	-73.15	U1 All	OK	0.80	63.86	0.00	U1 All	OK
	1.542	1.00	-82.66	-53.37	U1 All	OK	0.80	63.86	0.00	U1 All	OK
	2.587	0.80	-66.58	-30.42	U1 All	OK	0.80	63.86	0.00	U1 All	OK
	2.963	0.80	-66.58	-23.03	U1 All	OK	0.80	63.86	0.00	U1 All	OK
	4.008	0.40	-33.75	-12.84	U1 S1	OK	0.80	63.86	6.74	U1 S3	OK
	6.350	0.40	-33.75	-0.83	U1 Even	OK	0.80	63.86	28.62	U1 Odd	OK
	8.750	0.40	-33.75	0.00	U1 All	OK	0.80	63.86	36.65	U1 Odd	OK
	11.150	0.40	-33.75	-0.83	U1 Even	OK	0.80	63.86	28.62	U1 Odd	OK
	13.492	0.40	-33.75	-12.84	U1 S4	OK	0.80	63.86	6.74	U1 S2	OK
	14.537	0.80	-66.58	-23.03	U1 All	OK	0.80	63.86	0.00	U1 All	OK
	14.913	0.80	-66.58	-30.42	U1 All	OK	0.80	63.86	0.00	U1 All	OK
	15.958	1.00	-82.66	-53.37	U1 All	OK	0.80	63.86	0.00	U1 All	OK
	16.750	1.00	-82.66	-73.15	U1 All	OK	0.80	63.86	0.00	U1 All	OK
	17.250	1.00	-82.66	-86.71	U1 All	---	0.80	63.86	0.00	U1 All	---
	17.500	1.00	-82.66	-93.79	U1 All	---	0.80	63.86	0.00	U1 All	---
4 Column	0.000	1.60	-34.88	-18.14	U1 All	---	1.60	34.88	0.00	U1 All	---
	0.750	1.60	-34.88	-14.23	U1 All	OK	1.60	34.88	0.00	U1 All	OK
	1.750	1.00	-22.06	-9.46	U1 All	OK	1.60	34.88	0.00	U1 All	OK
	5.750	1.00	-22.06	0.00	U1 All	OK	1.60	34.88	5.02	U1 Even	OK
	6.350	0.40	-8.93	0.00	U1 All	OK	1.60	34.88	5.94	U1 Even	OK
	6.750	0.00	0.00	0.00	U1 All	OK	1.60	34.88	6.46	U1 Even	OK
	8.750	0.00	0.00	0.00	U1 All	OK	1.60	34.88	8.29	U1 All	OK
	9.500	0.00	0.00	0.00	U1 All	OK	1.60	34.88	8.50	U1 All	OK
	11.150	0.00	0.00	0.00	U1 All	OK	1.60	34.88	7.81	U1 All	OK
	13.750	0.00	0.00	0.00	U1 All	OK	1.60	34.88	3.52	U1 All	OK
	14.750	1.00	-22.06	0.00	U1 All	OK	1.60	34.88	0.83	U1 All	OK
	15.750	1.00	-22.06	-2.95	U1 Even	OK	1.60	34.88	0.00	U1 All	OK
	16.750	1.60	-34.88	-7.06	U1 All	OK	1.60	34.88	0.00	U1 All	OK
	17.500	1.60	-34.88	-10.78	U1 All	---	1.60	34.88	0.00	U1 All	---
Middle	0.000	2.80	-61.04	-59.82	U1 All	---	1.80	39.69	0.00	U1 All	---
	0.750	2.80	-61.04	-46.12	U1 All	OK	1.80	39.69	0.00	U1 All	OK
	2.625	2.80	-61.04	-17.71	U1 All	OK	1.80	39.69	0.00	U1 All	OK
	3.625	2.80	-61.04	-8.12	U1 Odd	OK	2.80	61.04	1.02	U1 Even	OK
	5.750	2.80	-61.04	0.00	U1 All	OK	2.80	61.04	16.26	U1 Even	OK
	6.350	1.12	-24.89	0.00	U1 All	OK	2.80	61.04	19.25	U1 Even	OK
	6.750	0.00	0.00	0.00	U1 All	OK	2.80	61.04	20.94	U1 Even	OK
	8.750	0.00	0.00	0.00	U1 All	OK	2.80	61.04	26.88	U1 All	OK
	9.500	0.00	0.00	0.00	U1 All	OK	2.80	61.04	27.55	U1 All	OK
	11.150	0.00	0.00	0.00	U1 All	OK	2.80	61.04	25.30	U1 All	OK
	13.750	0.00	0.00	0.00	U1 All	OK	2.80	61.04	11.41	U1 All	OK
	14.750	2.80	-61.04	0.00	U1 All	OK	2.80	61.04	2.68	U1 All	OK
	16.750	2.80	-61.04	-15.36	U1 All	OK	2.80	61.04	0.00	U1 All	OK
	17.500	2.80	-61.04	-22.97	U1 All	---	2.80	61.04	0.00	U1 All	---
Beam	0.000	1.00	-82.66	-102.80	U1 All	---	0.80	63.86	0.00	U1 All	---
	0.500	1.00	-82.66	-87.85	U1 All	---	0.80	63.86	0.00	U1 All	---
	0.750	1.00	-82.66	-80.63	U1 All	OK	0.80	63.86	0.00	U1 All	OK
	6.178	1.00	-82.66	0.00	U1 All	OK	0.80	63.86	32.25	U1 Even	OK
	6.350	0.85	-70.69	0.00	U1 All	OK	0.80	63.86	33.65	U1 Even	OK
	7.333	0.00	0.00	0.00	U1 All	OK	0.80	63.86	40.07	U1 Even	OK
	8.750	0.00	0.00	0.00	U1 All	OK	0.80	63.86	46.99	U1 All	OK
	9.500	0.00	0.00	0.00	U1 All	OK	0.80	63.86	48.17	U1 All	OK
	11.150	0.00	0.00	0.00	U1 All	OK	0.80	63.86	44.24	U1 All	OK
	13.167	0.00	0.00	0.00	U1 All	OK	0.80	63.86	27.32	U1 All	OK
	14.167	0.80	-66.58	0.00	U1 All	OK	0.80	63.86	13.97	U1 All	OK
	16.750	0.80	-66.58	-40.00	U1 All	OK	0.80	63.86	0.00	U1 All	OK
	17.500	0.80	-66.58	-61.08	U1 All	---	0.80	63.86	0.00	U1 All	---
5 Column	0.000	1.60	-34.88	-0.20	U1 All	---	0.00	0.00	0.00	U1 All	---
	0.131	1.60	-34.88	-0.14	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.348	1.60	-34.88	-0.06	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.375	1.60	-34.88	-0.05	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.533	1.60	-34.88	-0.02	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.750	1.60	-34.88	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
Middle	0.000	2.80	-61.04	-0.30	U1 All	---	0.00	0.00	0.00	U1 All	---
	0.131	2.80	-61.04	-0.22	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.348	2.80	-61.04	-0.10	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.375	2.80	-61.04	-0.08	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.533	2.80	-61.04	-0.03	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.750	2.80	-61.04	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
Beam	0.000	0.80	-66.58	-1.12	U1 All	---	0.00	0.00	0.00	U1 All	---
	0.131	0.80	-66.58	-0.79	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.348	0.80	-66.58	-0.35	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.375	0.80	-66.58	-0.31	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.533	0.80	-66.58	-0.11	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.750	0.80	-66.58	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK

Longitudinal Beam Transverse Reinforcement Demand and Capacity

Section Properties

Units: d (in), Av/s (in^2/in), PhiVc (kip)

Span	d	(Av/s)min	PhiVc
1	18.24	0.0117	24.23
2	18.24	0.0117	24.23
3	18.24	0.0117	24.23
4	18.24	0.0117	24.23
5	18.24	0.0117	24.23

Beam Transverse Reinforcement Demand

Units: Start, End, Xu (in), Vu (ft), Av/s (kip/in^2)

Span	Start	End	Required Xu	Required Vu	Required Comb/Patt	Required Av/s	Demand Av/s	
1	0.000	0.000	0.000	0.00	U1/All	0.0000	0.0000	
2	1.000	4.122	2.270	32.32	U1/All	0.0099	0.0117	*8
	4.122	5.973	4.122	21.68	U1/All	0.0000	0.0117	*8
	5.973	7.824	5.973	11.37	U1/Even	0.0000	0.0000	
	7.824	9.676	9.676	10.23	U1/All	0.0000	0.0000	
	9.676	11.527	11.527	20.87	U1/All	0.0000	0.0117	*8
	11.527	13.378	13.378	31.50	U1/All	0.0089	0.0117	*8
	13.378	16.500	15.230	42.14	U1/All	0.0218	0.0218	
3	1.000	4.122	2.270	37.23	U1/All	0.0158	0.0158	
	4.122	5.973	4.122	26.59	U1/All	0.0029	0.0117	*8
	5.973	7.824	5.973	15.96	U1/All	0.0000	0.0117	*8
	7.824	9.676	9.676	6.78	U1/S3	0.0000	0.0000	
	9.676	11.527	11.527	15.96	U1/All	0.0000	0.0117	*8
	11.527	13.378	13.378	26.59	U1/All	0.0029	0.0117	*8
	13.378	16.500	15.230	37.23	U1/All	0.0158	0.0158	
4	1.000	4.122	2.270	42.14	U1/All	0.0218	0.0218	
	4.122	5.973	4.122	31.50	U1/All	0.0089	0.0117	*8
	5.973	7.824	5.973	20.87	U1/All	0.0000	0.0117	*8
	7.824	9.676	7.824	10.23	U1/All	0.0000	0.0000	
	9.676	11.527	11.527	11.37	U1/Even	0.0000	0.0000	
	11.527	13.378	13.378	21.68	U1/All	0.0000	0.0117	*8
	13.378	16.500	15.230	32.32	U1/All	0.0099	0.0117	*8
5	0.750	0.750	0.750	0.00	U1/All	0.0000	0.0000	

NOTES:
*8 - Minimum transverse (stirrup) reinforcement governs.

Beam Transverse Reinforcement Details

Units: spacing & distance (in).

Span	Size	Stirrups (2 legs each unless otherwise noted)
1	#5	--- None ---
2	#4	8 @ 8.0 + <-- 44.4 --> + 10 @ 8.6
3	#4	10 @ 8.6 + <-- 22.2 --> + 10 @ 8.6
4	#4	10 @ 8.6 + <-- 44.4 --> + 8 @ 8.0
5	#5	--- None ---

Beam Transverse Reinforcement Capacity

Units: Start, End, Xu (ft), Vu, PhiVn (kip), Av/s (in^2/in), Av (in^2), Sp (in)

Span	Start	End	Required Xu	Required Vu	Required Comb/Patt	Required Av/s	Provided Av	Provided Sp	Provided Av/s	Provided PhiVn	
1	0.000	0.750	0.000	0.00	U1/All	-----	-----	-----	-----	-----	
2	0.000	1.000	2.270	32.32	U1/All	-----	-----	-----	-----	-----	
	1.000	5.973	2.270	32.32	U1/All	0.0099	0.40	8.0	0.0503	65.50	*8
	5.973	9.676	5.973	11.37	U1/Even	0.0000	-----	-----	-----	12.11	
	9.676	16.500	15.230	42.14	U1/All	0.0218	0.40	8.6	0.0464	62.32	
	16.500	17.500	15.230	42.14	U1/All	-----	-----	-----	-----	-----	
3	0.000	1.000	2.270	37.23	U1/All	-----	-----	-----	-----	-----	
	1.000	7.824	2.270	37.23	U1/All	0.0158	0.40	8.6	0.0464	62.32	
	7.824	9.676	9.676	6.78	U1/S3	0.0000	-----	-----	-----	12.11	
	9.676	16.500	15.230	37.23	U1/All	0.0158	0.40	8.6	0.0464	62.32	
	16.500	17.500	15.230	37.23	U1/All	-----	-----	-----	-----	-----	
4	0.000	1.000	2.270	42.14	U1/All	-----	-----	-----	-----	-----	
	1.000	7.824	2.270	42.14	U1/All	0.0218	0.40	8.6	0.0464	62.32	
	7.824	11.527	11.527	11.37	U1/Even	0.0000	-----	-----	-----	12.11	
	11.527	16.500	15.230	32.32	U1/All	0.0099	0.40	8.0	0.0503	65.50	*8
	16.500	17.500	15.230	32.32	U1/All	-----	-----	-----	-----	-----	
5	0.000	0.750	0.750	0.00	U1/All	-----	-----	-----	-----	-----	

NOTES:
*8 - Minimum transverse (stirrup) reinforcement governs.

Slab Shear Capacity

Units: b, d (in), Xu (ft), PhiVc, Vu(kip)

Span	b	d	Vratio	PhiVc	Vu	Xu
1	250.00	5.00	0.000	118.59	0.00	0.00
2	250.00	5.00	0.000	118.59	0.00	16.33
3	250.00	5.00	0.000	118.59	0.00	16.33
4	250.00	5.00	0.000	118.59	0.00	1.17
5	250.00	5.00	0.000	118.59	0.00	0.00

Flexural Transfer of Negative Unbalanced Moment at Supports

Units: Width (in), Munb (k-ft), As (in^2)

Supp	Width	Width-c	d	Munb	Comb	Pat	GammaF	AsReq	AsProv	Add Bars
1	36.00	36.00	5.00	93.21	U1	All	0.687	3.421	1.187	12-#4
2	36.00	36.00	5.00	46.73	U1	Even	0.600	1.333	1.387	---
3	36.00	36.00	5.00	46.73	U1	Even	0.600	1.333	1.387	---
4	36.00	36.00	5.00	93.21	U1	All	0.687	3.421	1.187	12-#4

Punching Shear Around Columns

Critical Section Properties

Units: b1, b2, b0, davg, CG, c(left), c(right) (in), Ac (in^2), Jc (in^4)

Supp	Type	b1	b2	b0	davg	CG	c(left)	c(right)	Ac	Jc
1	Rect	20.50	44.00	64.00	17.25	2.41	11.41	9.09	1104	95338
2	Rect	23.00	23.00	92.00	13.52	0.00	11.50	11.50	1244	1.1499e+005
3	Rect	23.00	23.00	92.00	13.52	0.00	11.50	11.50	1244	1.1499e+005
4	Rect	20.50	44.00	64.00	17.25	-2.41	9.09	11.41	1104	95338

Punching Shear Results

Units: Vu (kip), Munb (k-ft), vu (psi), Phi*vc (psi)

Supp	Vu	vu	Munb	Comb	Pat	GammaV	vu	Phi*vc
1	48.47	43.9	83.49	U1	All	0.313	73.8	189.7
2	104.50	84.0	-16.77	U1	All	0.400	92.1	189.7
3	104.50	84.0	16.77	U1	All	0.400	92.1	189.7
4	48.47	43.9	-83.49	U1	All	0.313	73.8	189.7

Material Takeoff

Reinforcement in the Direction of Analysis

Top Bars:	673.5 lb	<=>	12.47 lb/ft	<=>	0.567 lb/ft^2
Bottom Bars:	876.8 lb	<=>	16.24 lb/ft	<=>	0.738 lb/ft^2
Stirrups:	183.9 lb	<=>	3.41 lb/ft	<=>	0.155 lb/ft^2
Total Steel:	1734.2 lb	<=>	32.11 lb/ft	<=>	1.460 lb/ft^2
Concrete:	817.2 ft^3	<=>	15.13 ft^3/ft	<=>	0.688 ft^3/ft^2

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spSlab v5.00 (TM)
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams, One-way and Two-way Slab Systems

[3] DEFLECTION RESULTS

Section Properties

Frame Section Properties

Units: Ig, Icr (in^4), Mcr (k-ft)

Span	Zone	M+ve Ig	M+ve Icr	M+ve Mcr	M-ve Ig	M-ve Icr	M-ve Mcr
1	Left	25395	0	63.14	9333	6766	-36.89
	Midspan	25395	0	63.14	9333	7147	-36.89
	Right	433026	0	1267.91	433026	23081	-1267.91
2	Left	25395	1552	63.14	9333	7147	-36.89
	Midspan	25395	2280	63.14	9333	0	-36.89
	Right	25395	1552	63.14	9333	7331	-36.89
3	Left	25395	1552	63.14	9333	7331	-36.89
	Midspan	25395	1552	63.14	9333	6766	-36.89
	Right	25395	1552	63.14	9333	7331	-36.89
4	Left	25395	1552	63.14	9333	7331	-36.89
	Midspan	25395	2280	63.14	9333	0	-36.89
	Right	25395	1552	63.14	9333	7147	-36.89
5	Left	433026	0	1267.91	433026	23081	-1267.91
	Midspan	25395	0	63.14	9333	7147	-36.89
	Right	25395	0	63.14	9333	6766	-36.89

NOTES: M+ve values are for positive moments (tension at bottom face).
M-ve values are for negative moments (tension at top face).

Frame Effective Section Properties

Units: Ie, Ie,avg (in^4), Mmax (k-ft)

Span	Zone	Weight	Load Level: Dead Mmax	Dead Ie	Sustained Mmax	Sustained Ie	Dead+Live Mmax	Dead+Live Ie
1	Right	1.000	-0.52	433026	-0.52	433026	-1.14	433026
	Span Avg	----	----	433026	----	433026	----	433026
2	Middle	0.850	27.19	25395	27.19	25395	59.43	25395
	Right	0.150	-58.35	7837	-58.35	7837	-127.56	7380
	Span Avg	----	----	22761	----	22761	----	22693
3	Left	0.150	-52.93	8009	-52.93	8009	-115.73	7396
	Middle	0.700	18.06	25395	18.06	25395	39.49	25395
	Right	0.150	-52.93	8009	-52.93	8009	-115.73	7396
	Span Avg	----	----	20179	----	20179	----	19995
4	Left	0.150	-58.35	7837	-58.35	7837	-127.56	7380
	Middle	0.850	27.19	25395	27.19	25395	59.43	25395
	Span Avg	----	----	22761	----	22761	----	22693
5	Left	1.000	-0.52	433026	-0.52	433026	-1.14	433026
	Span Avg	----	----	433026	----	433026	----	433026

Strip Section Properties at Midspan

Units: Ig (in^4)

	Column Strip			Middle Strip		
Span	Ig	LDF	Ratio	Ig	LDF	Ratio
1	20040.5	0.781	0.990	2862	0.219	1.943
2	20040.5	0.693	0.878	2862	0.307	2.723
3	20040.5	0.673	0.853	2862	0.327	2.903
4	20040.5	0.693	0.878	2862	0.307	2.723
5	20040.5	0.781	0.990	2862	0.219	1.943

NOTES: Load distirubtion factor, LDL, averages moment distribution factors listed in [2] Design Results.
Ratio refers to proportion of strip to frame deflections under fix-end condtions.

Instantaneous Deflections

Extreme Instantaneous Frame Deflections and Corresponding Locations

Units: Def (in), Loc (ft)

				Live			Total	
Span	Direction	Value	Dead	Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.001	---	-0.001	-0.001	-0.001	-0.003
		Loc	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	0.011	---	0.013	0.013	0.011	0.024
		Loc	8.000	---	8.000	8.000	8.000	8.000
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
3	Down	Def	0.006	---	0.007	0.007	0.006	0.013
		Loc	8.750	---	8.750	8.750	8.750	8.750
	Up	Def	-0.000	---	-0.000	-0.000	-0.000	-0.001
		Loc	1.000	---	1.000	1.000	1.000	1.000
4	Down	Def	0.011	---	0.013	0.013	0.011	0.024
		Loc	9.500	---	9.500	9.500	9.500	9.500
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
5	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.001	---	-0.001	-0.001	-0.001	-0.003
		Loc	0.750	---	0.750	0.750	0.750	0.750

Extreme Instantaneous Column Strip Deflections and Corresponding Locations

Units: Def (in), Loc (ft)

				Live			Total	
Span	Direction	Value	Dead	Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.001	---	-0.001	-0.001	-0.001	-0.003
		Loc	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	0.010	---	0.012	0.012	0.010	0.023
		Loc	8.000	---	8.000	8.000	8.000	8.000
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
3	Down	Def	0.005	---	0.006	0.006	0.005	0.011
		Loc	8.750	---	8.750	8.750	8.750	8.750
	Up	Def	-0.000	---	-0.000	-0.000	-0.000	-0.001
		Loc	1.000	---	1.000	1.000	1.000	1.000
4	Down	Def	0.010	---	0.012	0.012	0.010	0.023
		Loc	9.500	---	9.500	9.500	9.500	9.500
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
5	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.001	---	-0.001	-0.001	-0.001	-0.003
		Loc	0.750	---	0.750	0.750	0.750	0.750

Extreme Instantaneous Middle Strip Deflections and Corresponding Locations

Units: Def (in), Loc (ft)

				Live			Total	
Span	Direction	Value	Dead	Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.001	---	-0.001	-0.001	-0.001	-0.003
		Loc	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	0.022	---	0.026	0.026	0.022	0.049
		Loc	8.500	---	8.500	8.500	8.500	8.500
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
3	Down	Def	0.020	---	0.024	0.024	0.020	0.044
		Loc	8.750	---	8.750	8.750	8.750	8.750
	Up	Def	-0.000	---	-0.000	-0.000	-0.000	-0.000
		Loc	0.750	---	0.750	0.750	0.750	0.750

4	Down	Def	0.022	---	0.026	0.026	0.022	0.049
		Loc	9.000	---	9.000	9.000	9.000	9.000
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
5	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.001	---	-0.001	-0.001	-0.001	-0.003
		Loc	0.750	---	0.750	0.750	0.750	0.750

Long-term Deflections

Long-term Column Strip Deflection Factors

Time dependant factor for sustained loads = 2.000
Units: Astop, Asbot (in^2), b, d (in), Rho' (%), Lambda (-)

		M+ve					M-ve				
Span	Zone	Astop	b	d	Rho'	Lambda	Asbot	b	d	Rho'	Lambda
1	Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

NOTES: Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Long-term Middle Strip Deflection Factors

Time dependant factor for sustained loads = 2.000
Units: Astop, Asbot (in^2), b, d (in), Rho' (%), Lambda (-)

		M+ve					M-ve				
Span	Zone	Astop	b	d	Rho'	Lambda	Asbot	b	d	Rho'	Lambda
1	Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

NOTES: Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Extreme Long-term Column Strip Deflections and Corresponding Locations

Units: D (in), x (ft)

Span	Direction	Value	cs	cs+lu	cs+l	Total
1	Down	Def	---	---	---	---
		Loc	---	---	---	---
	Up	Def	-0.002	-0.004	-0.004	-0.005
		Loc	0.000	0.000	0.000	0.000
2	Down	Def	0.021	0.033	0.033	0.043
		Loc	8.000	8.000	8.000	8.000
	Up	Def	---	---	---	---
		Loc	---	---	---	---
3	Down	Def	0.010	0.015	0.015	0.020
		Loc	8.750	8.750	8.750	8.750
	Up	Def	-0.001	-0.001	-0.001	-0.001
		Loc	1.000	1.000	1.000	1.000
4	Down	Def	0.021	0.033	0.033	0.043
		Loc	9.500	9.500	9.500	9.500
	Up	Def	---	---	---	---
		Loc	---	---	---	---
5	Down	Def	---	---	---	---
		Loc	---	---	---	---
	Up	Def	-0.002	-0.004	-0.004	-0.005
		Loc	0.750	0.750	0.750	0.750

NOTES: Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

Extreme Long-term Middle Strip Deflections and Corresponding Locations

Units: D (in), x (ft)

Span	Direction	Value	cs	cs+lu	cs+l	Total
1	Down	Def	---	---	---	---
		Loc	---	---	---	---
	Up	Def	-0.002	-0.004	-0.004	-0.005
		Loc	0.000	0.000	0.000	0.000
2	Down	Def	0.044	0.071	0.071	0.093
		Loc	8.500	8.500	8.500	8.500
	Up	Def	---	---	---	---

		Loc	---	---	---	---
3	Down	Def	0.040	0.064	0.064	0.084
		Loc	8.750	8.750	8.750	8.750
	Up	Def	-0.000	-0.001	-0.001	-0.001
		Loc	0.750	0.750	0.750	0.750
4	Down	Def	0.044	0.071	0.071	0.093
		Loc	9.000	9.000	9.000	9.000
	Up	Def	---	---	---	---
		Loc	---	---	---	---
5	Down	Def	---	---	---	---
		Loc	---	---	---	---
	Up	Def	-0.002	-0.004	-0.004	-0.005
		Loc	0.750	0.750	0.750	0.750

NOTES: Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

7. Summary and Comparison of Design Results

Table 9 - Comparison of Moments obtained from Hand (EFM) and spSlab Solution (ft-kip)			
		Hand (EFM)	**spSlab**
Exterior Span			
Beam Strip	Exterior Negative*	38.4	40
	Positive	50.9	48.17
	Interior Negative*	73.1	80.63
Column Strip	Exterior Negative*	6.8	7.06
	Positive	9	8.5
	Interior Negative*	12.9	14.23
Middle Strip	Exterior Negative*	15	15.36
	Positive	29.5	27.55
	Interior Negative*	42.4	46.12
Interior Span			
Beam Strip	Interior Negative*	67	73.15
	Positive	40.6	36.65
Column Strip	Interior Negative*	11.8	12.91
	Positive	7.2	6.47
Middle Strip	Interior Negative*	38.8	41.84
	Positive	23.5	20.96
* negative moments are taken at the faces of supports			

Table 10 - Comparison of Reinforcement Results							
Span Location		Reinforcement Provided for Flexure		Additional Reinforcement Provided for Unbalanced Moment Transfer*		Total Reinforcement Provided	
		Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior Span							
Beam Strip	Exterior Negative	4 - #4	4 - #4	n/a	n/a	4 - #4	4 - #4
	Positive	5 - #4	4 - #4	n/a	n/a	5 - #4	4 - #4
	Interior Negative	5 - #4	5 - #4	---	---	5 - #4	5 - #4
Column Strip	Exterior Negative	8 - #4	8 - #4	10 - #4	12 - #4	18 - #4	20 - #4
	Positive	8 - #4	8 - #4	n/a	n/a	8 - #4	8 - #4
	Interior Negative	8 - #4	8 - #4	---	---	8 - #4	8 - #4
Middle Strip	Exterior Negative	14 - #4	14 - #4	n/a	n/a	14 - #4	14 - #4
	Positive	14 - #4	14 - #4	n/a	n/a	14 - #4	14 - #4
	Interior Negative	14 - #4	14 - #4	n/a	n/a	14 - #4	14 - #4
Interior Span							
Beam Strip	Positive	4 - #4	4 - #4	n/a	n/a	4 - #4	4 - #4
Column Strip	Positive	8 - #4	8 - #4	n/a	n/a	8 - #4	8 - #4
Middle Strip	Positive	14 - #4	14 - #4	n/a	n/a	14 - #4	14 - #4

Table 11 - Comparison of Beam Shear Reinforcement Results		
Span Location	Reinforcement Provided	
	Hand	spSlab
End Span		
Exterior	8 - #4 @ 8 in	8 - #4 @ 8 in
Interior	10 - #4 @ 8.6 in	10 - #4 @ 8.6 in
Interior Span		
Interior	9 - #4 @ 8.6 in	10 - #4 @ 8.6 in

Table 12 - Comparison of Two-Way (Punching) Shear Check Results (around Columns Faces)										
Support	**b_1, in.**		**b_2, in.**		**b_o, in.**		**V_u, kips**		**c_{AB}, in.**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	20.5	20.5	23.0	44.0	64	64	43.56	48.47	9.09	9.09
Interior	23.0	23.0	23.0	23.0	92	92	104.76	104.50	11.50	11.50
Support	**J_c, in.^{4}**		**γ_v**		**M_{unb}, ft-kips**		**v_u, psi**		**φv_c, psi**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	95338	95338	0.386	0.313	84.37	83.49	76.8	73.8	189.7	189.7
Interior	114993	114990	0.400	0.400	14.10	16.77	91.0	92.1	189.7	189.7

Table 13 - Comparison of Immediate Deflection Results (in.)								
Column Strip								
Span	**D**		**D+LL$_{sus}$**		**D+LL$_{full}$**		**LL**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	0.009	0.010	0.009	0.010	0.021	0.023	0.011	0.012
Interior	0.004	0.005	0.004	0.005	0.010	0.011	0.005	0.006
Middle Strip								
Span	**D**		**D+LL$_{sus}$**		**D+LL$_{full}$**		**LL**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	0.021	0.022	0.021	0.022	0.047	0.049	0.025	0.026
Interior	0.019	0.020	0.019	0.020	0.042	0.044	0.023	0.024

Table 14 - Comparison of Time-Dependent Deflection Results						
Column Strip						
Span	**λ_Δ**		**Δ_{cs}, in.**		**Δ_{total}, in.**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	2.0	2.0	0.019	0.021	0.040	0.043
Interior	2.0	2.0	0.009	0.010	0.018	0.020
Middle Strip						
Span	**λ_Δ**		**Δ_{cs}, in.**		**Δ_{total}, in.**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	2.0	2.0	0.043	0.044	0.089	0.093
Interior	2.0	2.0	0.038	0.040	0.080	0.084

In all of the hand calculations illustrated above, the results are in close or exact agreement with the automated analysis and design results obtained from the spSlab model. Excerpts of spSlab graphical and text output are given below for illustration.

8. Conclusions & Observations

A slab system can be analyzed and designed by any procedure satisfying equilibrium and geometric compatibility. Three established methods are widely used. The requirements for two of them are described in detail in ACI 318-14 Chapter 8 (8.2.1).

Direct Design Method (DDM) is an approximate method and is applicable to two-way slab concrete floor systems that meet the stringent requirements of ***ACI 318-14 (8.10.2)***. In many projects, however, these requirements limit the usability of the Direct Design Method significantly.

The Equivalent Frame Method (EFM) does not have the limitations of Direct Design Method. It requires more accurate analysis methods that, depending on the size and geometry can prove to be long, tedious, and time-consuming.

StucturePoint's spSlab software program solution utilizes the Equivalent Frame Method to automate the process providing considerable time-savings in the analysis and design of two-way slab systems as compared to hand solutions using DDM or EFM.

Finite Element Method (FEM) is another method for analyzing reinforced concrete slabs, particularly useful for irregular slab systems with variable thicknesses, openings, and other features not permissible in DDM or EFM. Many reputable commercial FEM analysis software packages are available on the market today such as spMats. Using FEM requires critical understanding of the relationship between the actual behavior of the structure and the numerical simulation since this method is an approximate numerical method. The method is based on several assumptions and the operator has a great deal of decisions to make while setting up the model and applying loads and boundary conditions. The results obtained from FEM models should be verified to confirm their suitability for design and detailing of concrete structures.

The following table shows a general comparison between the DDM, EFM and FEM. This table covers general limitations, drawbacks, advantages, and cost-time efficiency of each method where it helps the engineer in deciding which method to use based on the project complexity, schedule, and budget.

Applicable ACI 318-14 Provision	Limitations/Applicability	Concrete Slab Analysis Method		
		DDM (Hand)	EFM (Hand//spSlab)	FEM (spMats)
8.10.2.1	Minimum of three continuous spans in each direction	☑		
8.10.2.2	Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span	☑		
8.10.2.3	Panels shall be rectangular, with ratio of longer to shorter panel dimensions, measured center-to-center supports, not exceed 2.	☑	☑	
8.10.2.4	Column offset shall not exceed 10% of the span in direction of offset from either axis between centerlines of successive columns	☑		
8.10.2.5	All loads shall be due to gravity only	☑		
8.10.2.5	All loads shall be uniformly distributed over an entire panel (q_u)	☑		
8.10.2.6	Unfactored live load shall not exceed two times the unfactored dead load	☑		
8.10.2.7	For a panel with beams between supports on all sides, slab-to-beam stiffness ratio shall be satisfied for beams in the two perpendicular directions.	☑		
8.7.4.2	Structural integrity steel detailing	☑	☑	☑
8.5.4	Openings in slab systems	☑	☑	☑
8.2.2	Concentrated loads	Not permitted	☑	☑
8.11.1.2	Live load arrangement (Load Patterning)	Not required	Required	Engineering judgment required based on modeling technique
R8.10.4.5*	Reinforcement for unbalanced slab moment transfer to column (M_{sc})	Moments @ support face	Moments @ support centerline	Engineering judgment required based on modeling technique
	Irregularities (i.e. variable thickness, non-prismatic, partial bands, mixed systems, support arrangement, etc.)	Not permitted	Engineering judgment required	Engineering judgment required
Complexity		Low	Average	Complex to very complex
Design time/costs		Fast	Limited	Unpredictable/Costly
Design Economy		Conservative (see detailed comparison with spSlab output)	Somewhat conservative	Unknown - highly dependent on modeling assumptions: 1. Linear vs. non-linear 2. Isotropic vs non-isotropic 3. Plate element choice 4. Mesh size and aspect ratio 5. Design & detailing features
General (Drawbacks)		Very limited applications	Limited geometry	Limited guidance non-standard application (user dependent). Required significant engineering judgment
General (Advantages)		Very limited analysis is required	Detailed analysis is required or via software (e.g. spSlab)	Unlimited applicability to handle complex situations permissible by the features of the software used (e.g. spMats)

* The unbalanced slab moment transferred to the column M_{sc} (M_{unb}) is the difference in slab moment on either side of a column at a specific joint. In DDM only moments at the face of the support are calculated and are also used to obtain M_{sc} (M_{unb}). In EFM where a frame analysis is used, moments at the column center line are used to obtain M_{sc} (M_{unb}).