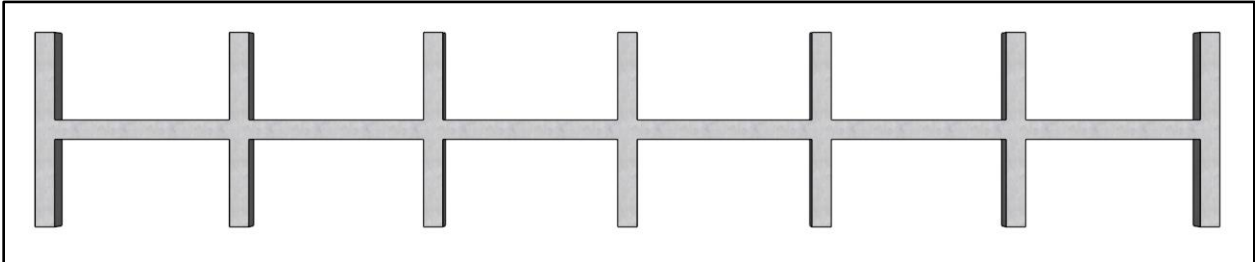
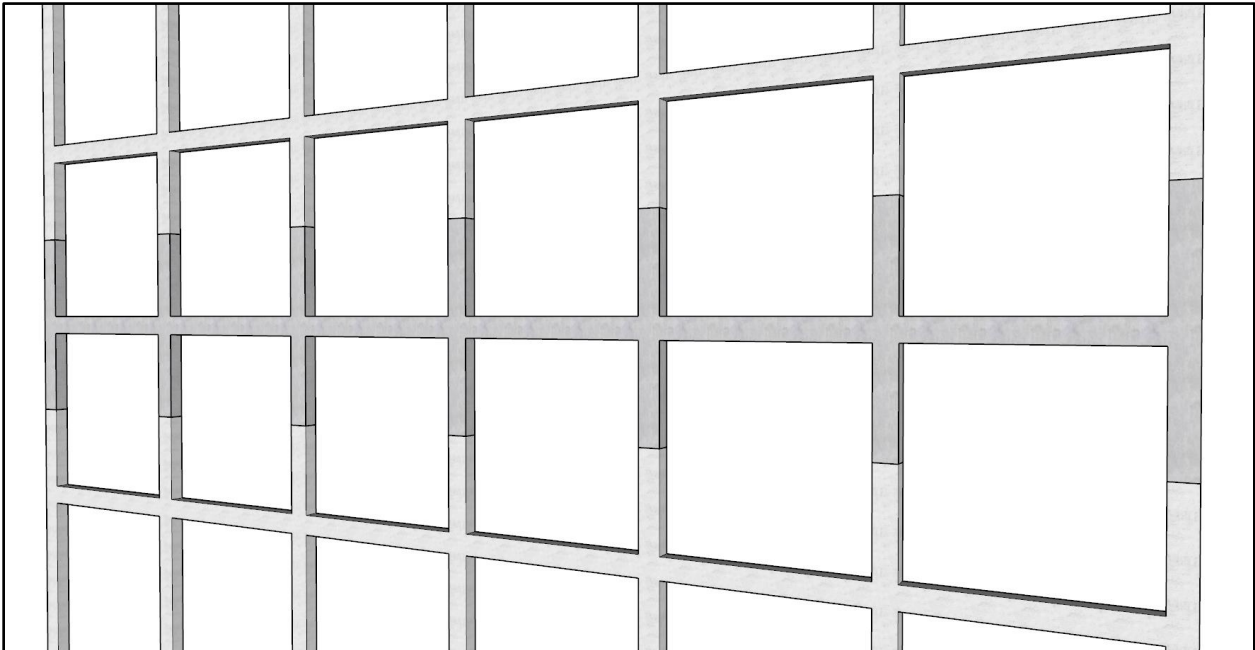


**Arrangement of Live Load Patterns for Moment Envelope (ACI 318-14)**



### Arrangement of Live Load Patterns for Moment Envelope (ACI 318-14)

Determining the moment envelopes at the middle and the ends of each span in a beam-column frame due to uniformly distributed loads. The frame is composed of six equal spans supported by equal height columns as shown in Figure 1. The uniform live load  $w_L$  is twice the uniform dead load  $w_D$  and that the flexural stiffness ( $EI/L$ ) of the columns is twice the stiffness of the beams. Moment envelopes values obtained in the reference and the hand calculations are compared with the values obtained by [spBeam](#) engineering software program from [StructurePoint](#).

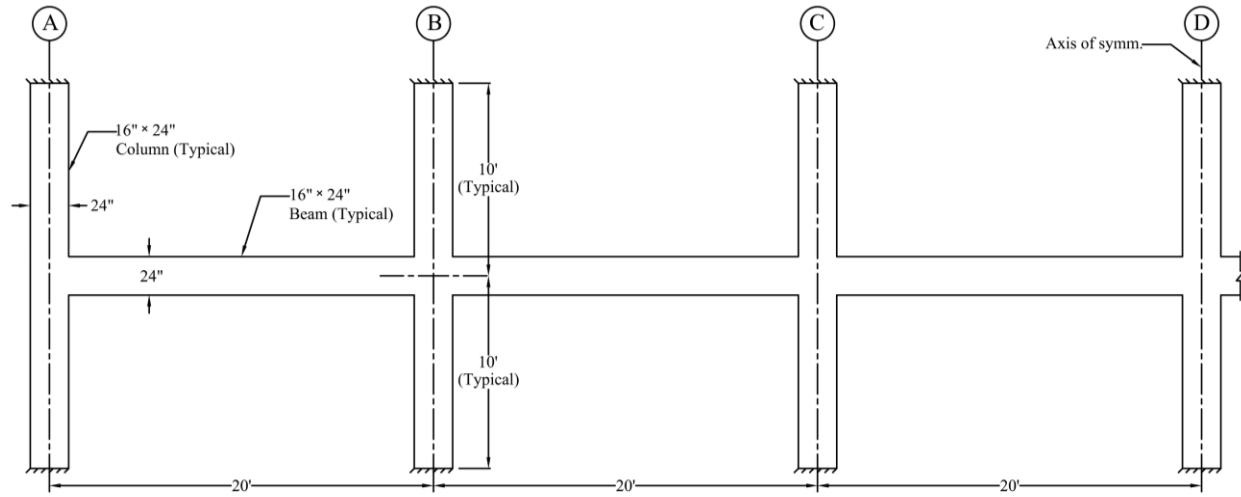


Figure 1 – Continuous Frame Elevation

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## Reference

- [1] Reinforced Concrete Design, 8<sup>th</sup> Edition, 2017, Chu-Kia Wang, Charles G. Salmon, Jose A. Pincheira, Gustavo J. Parra-Montesinos, Oxford University Press, Example 7.3.1.
- [2] [spBeam Engineering Software Program Manual v5.50](#), StructurePoint LLC., 2018.

## Analysis Data

The reference book solved Example 7.3.1 symbolically in term of the span length  $L$  and the total loads  $w$  ( $w_D + w_L$ ). Numerical values are practically assumed in order to compare the analysis output from [spBeam](#) with the reference output when using the same analysis parameter.

$f'_c = 4,000$  psi normal weight concrete for beams and columns.

Span length,  $l = 20$  ft, center-to-center of columns.

Column height = 10 ft, from center of beams to column support.

Beam and column cross-sections are rectangular with dimensions of 16 in.  $\times$  24 in.

Live load,  $w_L = 1.35$  kip/ft.

Dead load,  $w_D = 0.675$  kip/ft.

## Solution

### 1. Span Loading Scenarios

As per the ACI 318-14 code requirements for arrangement of live load on continuous beams, the following arrangements are permitted to obtain maximum positive moment and maximum negative moment, respectively:

- Maximum positive moment near midspan occurs with  $w_L$  applied on the span under consideration and on alternate spans. **ACI 318-14 (6.4.2.a)**
- Maximum negative moment at a support occurs with  $w_L$  applied on adjacent spans only.

**ACI 318-14 (6.4.2.b)**

Seven loading scenarios as shown in Figure 2 are investigated. The frame is analyzed by classical method of structural analysis, namely, the moment distribution method. Moment distribution procedures for each loading scenario are carried out in the next section.

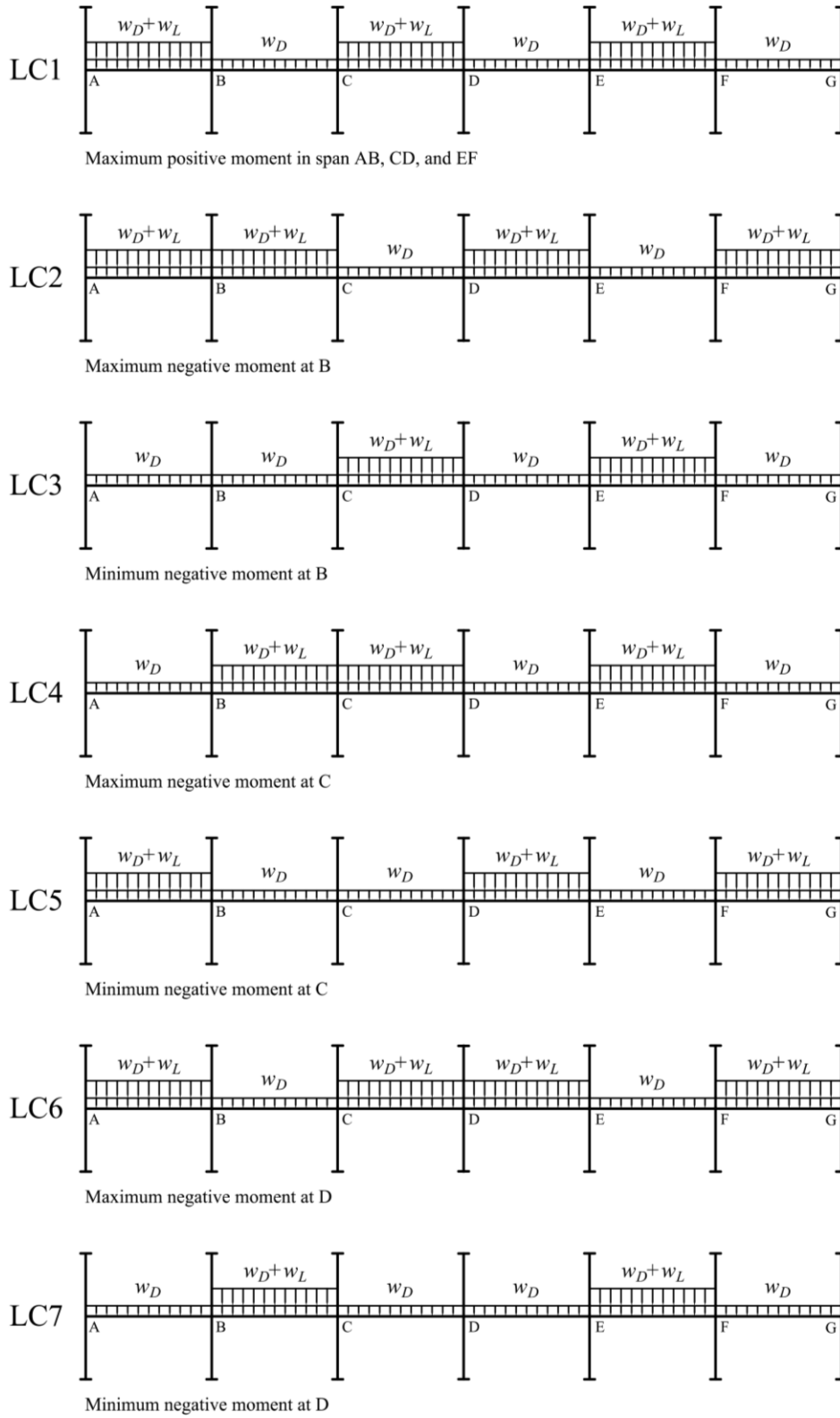


Figure 2 – Loading Scenarios for Six-Span Continuous Frame

## 2. Continuous Beam Analysis – Moment Distribution Method

The moment distribution procedures are used to analyze the frame. Stiffness factors, carry over factors, and fixed-end moment factors for the beams and columns for the seven load scenarios:

Flexural stiffness of beams and columns ends,  $K$

$$K = \frac{4 \times E_c \times I}{l}$$

Where  $K$  is referred to as stiffness factor at beam or column end and can be defined as the amount of moment required to rotate the end of the beam or column 1 rad.

For a rectangular section with dimensions of 16 in.  $\times$  24 in, moment of inertia,  $I$ :

$$I = \frac{b \times h^3}{12} = \frac{16 \text{ in.} \times (24 \text{ in.})^3}{12} = 18,432 \text{ in.}^4$$

Concrete modulus of elasticity for normal weight concrete,  $E_c$ :

$$E_c = 57000 \sqrt{f'_c} = 57000 \sqrt{4000} \text{ psi} = 3605 \text{ ksi} \quad \text{\underline{\underline{ACI 318-14 (19.2.2.1.b)}}$$

For the beams:

$$l = 20 \text{ ft}$$

$$K_{Beam} = \frac{4 \times 3605 \text{ ksi} \times 18,432 \text{ in.}^4}{20 \text{ ft} \times 12 \text{ in./ft}} = 1107.456 \text{ kip-in.}$$

For the columns:

$$K_{Column} = 2 \times K_{Beam} = 2 \times 1107.456 \text{ kip-in.} = 2214.9 \text{ kip-in.}$$

Distribution factor, DF

$$DF = \frac{K}{\sum K}$$

The distribution factor for a member that is connected to a fixed joint is defined as the fraction of the total resisting moment supplied by this member.

For the beams continuous from one end:

$$DF_{Beam} = \frac{K_{Beam}}{K_{Column} + K_{Beam} + K_{Column}} = \frac{1107.456 \text{ kip-in.}}{2214.9 \text{ kip-in.} + 1107.456 \text{ kip-in.} + 2214.9 \text{ kip-in.}} = 0.2$$

For the beams continuous from both ends:

$$DF_{Beam} = \frac{K_{Beam}}{K_{Column} + K_{Beam} + K_{Beam} + K_{Column}}$$

$$DF_{Beam} = \frac{1107.456 \text{ kip-in.}}{2214.9 \text{ kip-in.} + 1107.456 \text{ kip-in.} + 1107.456 \text{ kip-in.} + 2214.9 \text{ kip-in.}} = 0.166$$

Flexural stiffness of beams and columns ends, COF

$$COF = 0.5$$

Where COF is the Carry-Over Factor that represents the fraction of the moment that is “carried over” from the joint to the beam end when the beam far end is fixed.

Fixed-end moments, FEMs

For a beam with uniformly distributed load and fixed ends, FEM can be found using the following equation:

$$FEM = \frac{w \times l^2}{12}$$

For member AB for load scenario LC1:

$$FEM_w = \frac{(w_D + w_L) \times l^2}{12} = \frac{(2.025 \text{ klf}) \times (20 \text{ ft})^2}{12} = 67.5 \text{ kip-ft}$$

$$FEM_{w_D} = \frac{w_D \times l^2}{12} = \frac{(0.675 \text{ klf}) \times (20 \text{ ft})^2}{12} = 22.5 \text{ kip-ft}$$

Beam analysis using moment distribution method

Moment distribution procedures for the seven loading scenarios are shown in Table 1. Counter-clockwise rotational moments acting on member ends are taken as positive. Positive span moments are determined from the following equation:

$$M_{Midspan} = M_o - \frac{(M_L + M_R)}{2}$$

Where  $M_o$  is the moment at the midspan for a simple beam,  $M_L$  and  $M_R$  are the negative moment at the span left and right end, respectively. The typical value of  $M_o$  for all spans due to the dead load  $w_D$  is:

$$M_{o-D} = \frac{w_D \times l^2}{8} = \frac{0.675 \text{ klf} \times (20 \text{ ft})^2}{8} = 33.75 \text{ kip-ft}$$

$$M_{o-D+L} = \frac{(w_D + w_L) \times l^2}{8} = \frac{2.025 \text{ klf} \times (20 \text{ ft})^2}{8} = 101.25 \text{ kip-ft}$$

The moment at the midspan  $M_s$  may be determined by superposition of the effect of end moments with that of the simply supported beam moment due to transverse loading.

$$M_s = M_o - \frac{1}{2} \times (M_L + M_R)$$

Where  $M_o$  is the moment at the midspan for a simply supported beam in the above equation,  $M_o$ ,  $M_L$  and  $M_R$  are taken as positive values. When the beam end moments are not equal, the maximum moment in the span does not occur at midspan, but its value is close to that at midspan.

Joint	A	B		C		D		E		F		G
Member	AB	BA	BC	CB	CD	DC	DE	ED	EF	FE	FG	GF
DF	0.2	0.166	0.166	0.166	0.166	0.166	0.166	0.166	0.166	0.166	0.166	0.2
COF	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Joint	A	B		C		D		E		F		G
Member	AB	BA	BC	CB	CD	DC	DE	ED	EF	FE	FG	GF
FEM	67.5	-67.5	22.5	-22.5	67.5	-67.5	22.5	-22.5	67.5	-67.5	22.5	-22.5
Dist.	-13.50	7.47	7.47	-7.47	-7.47	7.47	7.47	-7.47	-7.47	7.47	7.47	4.50
CO	3.74	-6.75	-3.74	3.74	3.74	-3.74	-3.74	3.74	3.74	-3.74	2.25	3.74
Dist.	-0.75	1.74	1.74	-1.24	-1.24	1.24	1.24	-1.24	-1.24	0.25	0.25	-0.75
CO	0.87	-0.37	-0.62	0.87	0.62	-0.62	-0.62	0.62	0.12	-0.62	-0.37	0.12
Dist.	-0.17	0.16	0.16	-0.25	-0.25	0.21	0.21	-0.12	-0.12	0.16	0.16	-0.02
CO	0.08	-0.09	-0.12	0.08	0.10	-0.12	-0.06	0.10	0.08	-0.06	-0.01	0.08
Dist.	-0.02	0.03	0.03	-0.03	-0.03	0.03	0.03	-0.03	-0.03	0.01	0.01	-0.02
CO	0.02	-0.01	-0.02	0.02	0.02	-0.02	-0.02	0.02	0.01	-0.02	-0.01	0.01
Dist.	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>-0.01</u>	<u>-0.01</u>	<u>0.01</u>	<u>0.01</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>
<b>M<sup>-</sup></b> (kip-ft)	57.76	-65.30	27.42	-26.79	62.98	-63.04	27.02	-26.89	62.58	-64.03	32.25	-14.84
<b>M<sup>+</sup></b> (kip-ft)	39.72		6.65		38.24		6.79		37.94		10.20	



LC2: Span AB, BC, DE, and FG are loaded with live load												
LC2												
	<b>Joint</b>	<b>A</b>	<b>B</b>		<b>C</b>		<b>D</b>		<b>E</b>		<b>F</b>	
<b>Member</b>	<b>AB</b>	<b>BA</b>	<b>BC</b>	<b>CB</b>	<b>CD</b>	<b>DC</b>	<b>DE</b>	<b>ED</b>	<b>EF</b>	<b>FE</b>	<b>FG</b>	<b>GF</b>
FEM	67.5	-67.5	67.5	-67.5	22.5	-22.5	67.5	-67.5	22.5	-22.5	67.5	-67.5
Dist.	-13.50	0.00	0.00	7.47	7.47	-7.47	-7.47	7.47	7.47	-7.47	-7.47	13.50
CO	0.00	-6.75	3.74	0.00	-3.74	3.74	3.74	-3.74	-3.74	3.74	6.75	-3.74
Dist.	0.00	0.50	0.50	0.62	0.62	-1.24	-1.24	1.24	1.24	-1.74	-1.74	0.75
CO	0.25	0.00	0.31	0.25	-0.62	0.31	0.62	-0.62	-0.87	0.62	0.37	-0.87
Dist.	-0.05	-0.05	-0.05	0.06	0.06	-0.15	-0.15	0.25	0.25	-0.16	-0.16	0.17
CO	-0.03	-0.03	0.03	-0.03	-0.08	0.03	0.12	-0.08	-0.08	0.12	0.09	-0.08
Dist.	0.01	0.00	0.00	0.02	0.02	-0.03	-0.03	0.03	0.03	-0.03	-0.03	0.02
CO	0.00	0.00	0.01	0.00	-0.01	0.01	0.01	-0.01	-0.02	0.01	0.01	-0.02
Dist.	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.01</u>	<u>0.01</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>
<b>M<sup>-</sup></b> (kip-ft)	54.18	-73.83	72.03	-59.11	26.23	-27.31	63.10	-62.96	26.78	-27.42	65.30	-57.76
<b>M<sup>+</sup></b> (kip-ft)	37.25		35.68		6.98		38.22		6.65		39.72	

LC3: Span CD and EF are loaded with live load												
LC3												
	<b>Joint</b>	<b>A</b>	<b>B</b>		<b>C</b>		<b>D</b>		<b>E</b>		<b>F</b>	
<b>Member</b>	<b>AB</b>	<b>BA</b>	<b>BC</b>	<b>CB</b>	<b>CD</b>	<b>DC</b>	<b>DE</b>	<b>ED</b>	<b>EF</b>	<b>FE</b>	<b>FG</b>	<b>GF</b>
FEM	22.5	-22.5	22.5	-22.5	67.5	-67.5	22.5	-22.5	67.5	-67.5	22.5	-22.5
Dist.	-4.50	0.00	0.00	-7.47	-7.47	7.47	7.47	-7.47	-7.47	7.47	7.47	4.50
CO	0.00	-2.25	-3.74	0.00	3.74	-3.74	-3.74	3.74	3.74	-3.74	2.25	3.74
Dist.	0.00	0.99	0.99	-0.62	-0.62	1.24	1.24	-1.24	-1.24	0.25	0.25	-0.75
CO	0.50	0.00	-0.31	0.50	0.62	-0.31	-0.62	0.62	0.12	-0.62	-0.37	0.12
Dist.	-0.10	0.05	0.05	-0.19	-0.19	0.15	0.15	-0.12	-0.12	0.16	0.16	-0.02
CO	0.03	-0.05	-0.09	0.03	0.08	-0.09	-0.06	0.08	0.08	-0.06	-0.01	0.08
Dist.	-0.01	0.02	0.02	-0.02	-0.02	0.03	0.03	-0.03	-0.03	0.01	0.01	-0.02
CO	0.01	0.00	-0.01	0.01	0.01	-0.01	-0.01	0.01	0.01	-0.01	-0.01	0.01
Dist.	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>
<b>M<sup>-</sup></b> (kip-ft)	18.43	-23.73	19.42	-30.26	63.65	-62.75	26.96	-26.92	62.58	-64.03	32.25	-14.84
<b>M<sup>+</sup></b> (kip-ft)	12.67		8.91		38.05		6.81		37.94		10.20	

LC4: Span BC, CD and EF are loaded with live load												
LC4												
	<b>Joint</b>	<b>A</b>	<b>B</b>		<b>C</b>		<b>D</b>		<b>E</b>		<b>F</b>	
<b>Member</b>	<b>AB</b>	<b>BA</b>	<b>BC</b>	<b>CB</b>	<b>CD</b>	<b>DC</b>	<b>DE</b>	<b>ED</b>	<b>EF</b>	<b>FE</b>	<b>FG</b>	<b>GF</b>
FEM	22.5	-22.5	67.5	-67.5	67.5	-67.5	22.5	-22.5	67.5	-67.5	22.5	-22.5
Dist.	-4.50	-7.47	-7.47	0.00	0.00	7.47	7.47	-7.47	-7.47	7.47	7.47	4.50
CO	-3.74	-2.25	0.00	-3.74	3.74	0.00	-3.74	3.74	3.74	-3.74	2.25	3.74
Dist.	0.75	0.37	0.37	0.00	0.00	0.62	0.62	-1.24	-1.24	0.25	0.25	-0.75
CO	0.19	0.37	0.00	0.19	0.31	0.00	-0.62	0.31	0.12	-0.62	-0.37	0.12
Dist.	-0.04	-0.06	-0.06	-0.08	-0.08	0.10	0.10	-0.07	-0.07	0.16	0.16	-0.02
CO	-0.03	-0.02	-0.04	-0.03	0.05	-0.04	-0.04	0.05	0.08	-0.04	-0.01	0.08
Dist.	0.01	0.01	0.01	0.00	0.00	0.01	0.01	-0.02	-0.02	0.01	0.01	-0.02
CO	0.00	0.00	0.00	0.00	0.01	0.00	-0.01	0.01	0.00	-0.01	-0.01	0.00
Dist.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<b>M<sup>-</sup></b> (kip-ft)	15.14	-31.54	60.31	-71.16	71.52	-59.34	26.31	-27.20	62.64	-64.01	32.25	-14.84
<b>M<sup>+</sup></b> (kip-ft)	10.41		35.51		35.82		7.00		37.93		10.20	

LC5: Span AB, DE and FG are loaded with live load												
LC5												
	<b>Joint</b>	<b>A</b>	<b>B</b>		<b>C</b>		<b>D</b>		<b>E</b>		<b>F</b>	
<b>Member</b>	<b>AB</b>	<b>BA</b>	<b>BC</b>	<b>CB</b>	<b>CD</b>	<b>DC</b>	<b>DE</b>	<b>ED</b>	<b>EF</b>	<b>FE</b>	<b>FG</b>	<b>GF</b>
FEM	67.5	-67.5	22.5	-22.5	22.5	-22.5	67.5	-67.5	22.5	-22.5	67.5	-67.5
Dist.	-13.50	7.47	7.47	0.00	0.00	-7.47	-7.47	7.47	7.47	-7.47	-7.47	13.50
CO	3.74	-6.75	0.00	3.74	-3.74	0.00	3.74	-3.74	-3.74	3.74	6.75	-3.74
Dist.	-0.75	1.12	1.12	0.00	0.00	-0.62	-0.62	1.24	1.24	-1.74	-1.74	0.75
CO	0.56	-0.37	0.00	0.56	-0.31	0.00	0.62	-0.31	-0.87	0.62	0.37	-0.87
Dist.	-0.11	0.06	0.06	-0.04	-0.04	-0.10	-0.10	0.20	0.20	-0.16	-0.16	0.17
CO	0.03	-0.06	-0.02	0.03	-0.05	-0.02	0.10	-0.05	-0.08	0.10	0.09	-0.08
Dist.	-0.01	0.01	0.01	0.00	0.00	-0.01	-0.01	0.02	0.02	-0.03	-0.03	0.02
CO	0.01	0.00	0.00	0.01	-0.01	0.00	0.01	-0.01	-0.02	0.01	0.01	-0.02
Dist.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<b>M<sup>-</sup></b> (kip-ft)	57.47	-66.02	31.15	-18.21	18.36	-30.73	63.76	-62.67	26.73	-27.45	65.31	-57.76
<b>M<sup>+</sup></b> (kip-ft)	39.51		9.07		9.21		38.04		6.66		39.71	

LC6: Span AB, CD, DE and FG are loaded with live load												
LC6												
	<b>Joint</b>	<b>A</b>	<b>B</b>		<b>C</b>		<b>D</b>		<b>E</b>		<b>F</b>	
<b>Member</b>	<b>AB</b>	<b>BA</b>	<b>BC</b>	<b>CB</b>	<b>CD</b>	<b>DC</b>	<b>DE</b>	<b>ED</b>	<b>EF</b>	<b>FE</b>	<b>FG</b>	<b>GF</b>
FEM	67.5	-67.5	22.5	-22.5	67.5	-67.5	67.5	-67.5	22.5	-22.5	67.5	-67.5
Dist.	-13.50	7.47	7.47	-7.47	-7.47	0.00	0.00	7.47	7.47	-7.47	-7.47	13.50
CO	3.74	-6.75	-3.74	3.74	0.00	-3.74	3.74	0.00	-3.74	3.74	6.75	-3.74
Dist.	-0.75	1.74	1.74	-0.62	-0.62	0.00	0.00	0.62	0.62	-1.74	-1.74	0.75
CO	0.87	-0.37	-0.31	0.87	0.00	-0.31	0.31	0.00	-0.87	0.31	0.37	-0.87
Dist.	-0.17	0.11	0.11	-0.14	-0.14	0.00	0.00	0.14	0.14	-0.11	-0.11	0.17
CO	0.06	-0.09	-0.07	0.06	0.00	-0.07	0.07	0.00	-0.06	0.07	0.09	-0.06
Dist.	-0.01	0.03	0.03	-0.01	-0.01	0.00	0.00	0.01	0.01	-0.03	-0.03	0.01
CO	0.01	-0.01	0.00	0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.01	-0.01
Dist.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<b>M<sup>-</sup></b> (kip-ft)	57.74	-65.36	27.73	-26.07	59.25	-71.62	71.62	-59.25	26.07	-27.73	65.36	-57.74
<b>M<sup>+</sup></b> (kip-ft)	39.70		6.85		35.81		35.81		6.85		39.70	

LC7: Span BC and EF are loaded with live load												
LC7												
	<b>Joint</b>	<b>A</b>	<b>B</b>		<b>C</b>		<b>D</b>		<b>E</b>		<b>F</b>	
<b>Member</b>	<b>AB</b>	<b>BA</b>	<b>BC</b>	<b>CB</b>	<b>CD</b>	<b>DC</b>	<b>DE</b>	<b>ED</b>	<b>EF</b>	<b>FE</b>	<b>FG</b>	<b>GF</b>
FEM	22.5	-22.5	67.5	-67.5	22.5	-22.5	22.5	-22.5	67.5	-67.5	22.5	-22.5
Dist.	-4.50	-7.47	-7.47	7.47	7.47	0.00	0.00	-7.47	-7.47	7.47	7.47	4.50
CO	-3.74	-2.25	3.74	-3.74	0.00	3.74	-3.74	0.00	3.74	-3.74	2.25	3.74
Dist.	0.75	-0.25	-0.25	0.62	0.62	0.00	0.00	-0.62	-0.62	0.25	0.25	-0.75
CO	-0.12	0.37	0.31	-0.12	0.00	0.31	-0.31	0.00	0.12	-0.31	-0.37	0.12
Dist.	0.02	-0.11	-0.11	0.02	0.02	0.00	0.00	-0.02	-0.02	0.11	0.11	-0.02
CO	-0.06	0.01	0.01	-0.06	0.00	0.01	-0.01	0.00	0.06	-0.01	-0.01	0.06
Dist.	0.01	0.00	0.00	0.01	0.01	0.00	0.00	-0.01	-0.01	0.00	0.00	-0.01
CO	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
Dist.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<b>M<sup>-</sup></b> (kip-ft)	14.87	-32.19	63.72	-63.30	30.62	-18.44	18.44	-30.62	63.30	-63.72	32.19	-14.87
<b>M<sup>+</sup></b> (kip-ft)	10.22		37.74		9.22		9.22		37.74		10.22	

Moment envelopes at each span ends and positive moment near midspan are summarized in Table 2. Positive moment values are underlined.

Table 2 - Moment Envelopes Results												
Span	AB		BC		CD		DE		EF		FG	
	M <sub>L</sub>	M <sub>R</sub>	M <sub>L</sub>	M <sub>R</sub>	M <sub>L</sub>	M <sub>R</sub>	M <sub>L</sub>	M <sub>R</sub>	M <sub>L</sub>	M <sub>R</sub>	M <sub>L</sub>	M <sub>R</sub>
<b>Negative Moment</b> (kip-ft)	57.67	-73.79	71.93	-71.12	71.44	-71.52	71.52	-71.44	71.12	-71.93	73.79	-57.67
<b>Positive Moment</b> (kip-ft)	39.85		37.99		38.31		38.31		37.99		39.85	

### 3. Arrangement of Live Load for Moment Envelope –[spBeam](#) Software

[spBeam](#) is widely used for analysis, design and investigation of beams, and one-way slab systems (including standard and wide module joist systems) per latest American (ACI 318) and Canadian (CSA A23.3) codes. [spBeam](#) can be used for new designs or investigation of existing structural members subjected to flexure, shear, and torsion loads. With capacity to integrate up to 20 spans and two cantilevers of wide variety of floor system types, [spBeam](#) is equipped to provide cost-effective, accurate, and fast solutions to engineering challenges.

[spBeam](#) provides top and bottom bar details including development lengths and material quantities, as well as live load patterning and immediate and long-term deflection results. Using the moment redistribution feature engineers can deliver safe designs with savings in materials and labor. Engaging this feature allows up to 20% reduction of negative moments over supports reducing reinforcement congestions in these areas.

Beam analysis and design requires engineering judgment in most situations to properly simulate the behavior of the targeted beam and take into account important design considerations such as: designing the beam as rectangular or T-shaped sections; using the effective flange width or the center-to-center distance between the beam and the adjacent beams. Regardless which of these options is selected, [spBeam](#) provide users with options and flexibility to:

1. Design the beam as a rectangular cross-section or a T-shaped section.
2. Use the effective or full beam flange width.
3. Include the flanges effects in the deflection calculations.
4. Invoke moment redistribution to lower negative moments
5. Using gross (uncracked) or effective (cracked) moment of inertia
6. Design the beam as singly or doubly reinforced section.

For illustration and comparison purposes, the following figures provide a sample of the input modules and results obtained from an [spBeam](#) model created for the beam covered in this design example.

The screenshot displays the spBeam software interface. The main window title is "spBeam - [C:\StructurePoint\Arrangement of Live Load Patterns for Moment Envelope.slb -- Isometric View]". The menu bar includes File, Input, Solve, View, Options, Window, and Help. The toolbar contains various icons for file operations and solving. The 3D model shows a beam with several columns. Two dialog boxes are open:

- General Information**: The "Design Options" tab is active. The "Live load pattern ratio" is set to 100%. Other options include checkboxes for "Compression Reinforcement", "Decremental Reinf. Design", "Torsion Analysis and Design", "Effective flange width", "Rigid beam-column joint", and "Moment Redistribution". There are also radio buttons for "Torsion type" (Equilibrium, Compatibility) and "Stirrups in flanges" (No, Yes). Under "Deflection calculation options", "Effective (cracked)" is selected. "Calculate long-term deflections" is checked, with a duration of 60 months and 0% sustained part of live load.
- Span Loads**: The "Current Case" list shows "Dead" and "Live". The "Live" case is selected. The "Span" is set to 1, "Magnitude" is 1350 lb/ft, and "Location" is from 0 to 20 ft. A table below lists the load data for spans 1 through 6.

A red arrow points from the text below to the "Live load pattern ratio" field in the "General Information" dialog.

The default live load pattern ratio selected by the program equals 75% as permitted by the code. The user has the ability to select different value for the pattern ratio within the range 0-100% (100% in this example for comparison purposes). If 0% is selected then load patterning effects will be neglected.

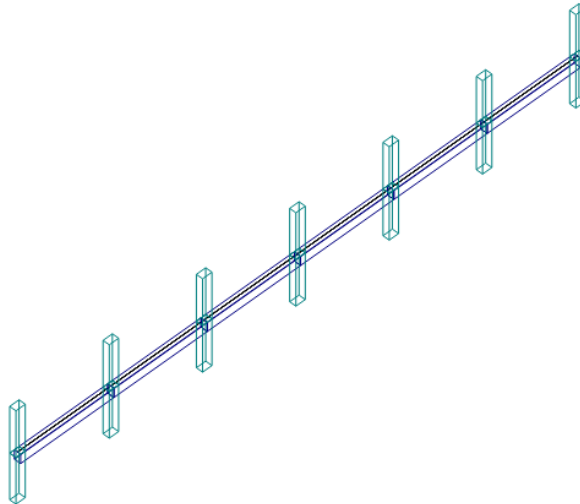
Span No.	Type	Wa	La	Wb	Lb
1	Line Load	1350	0	1350	20
2	Line Load	1350	0	1350	20
3	Line Load	1350	0	1350	20
4	Line Load	1350	0	1350	20
5	Line Load	1350	0	1350	20
6	Line Load	1350	0	1350	20



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spBeam v5.50  
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Reinforced Concrete Beams and One-way Slab Systems  
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## 1. Input Echo

### 1.1. General Information

File Name	...\Arrangement of Live Load for Moment Envelop...
Project	Arrangement of Live Load for Moment Envelope
Frame	
Engineer	SP
Code	ACI 318-14
Reinforcement Database	ASTM A615
Mode	Investigation
Number of supports =	7
Floor System	One-Way/Beam

### 1.2. Solve Options

Live load pattern ratio = 100%
Deflections are based on cracked section properties.
In negative moment regions, $I_g$ and $M_{cr}$ DO NOT include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
Moment redistribution NOT selected.
Effective flange width calculations selected.
Rigid beam-column joint NOT selected.
Torsion analysis and design NOT selected.

### 1.3. Material Properties

#### 1.3.1. Concrete: Slabs / Beams

$w_c$	150 lb/ft <sup>3</sup>
$f'_c$	4 ksi
$E_c$	3605 ksi
$f_r$	0.47434 ksi

#### 1.3.2. Concrete: Columns

$w_c$	150 lb/ft <sup>3</sup>
$f'_c$	4 ksi
$E_c$	3605 ksi
$f_r$	0.47434 ksi

#### 1.3.3. Reinforcing Steel

$f_y$	60 ksi
$f_{yt}$	60 ksi
$E_s$	29000 ksi
Epoxy coated bars	No

#### 1.4. Span Data

##### 1.4.1. Slabs

Span	Loc	L1 ft	t in	wL ft	wR ft	bE <sub>ff</sub> in	H <sub>min</sub> in
1	Int	20.000	0.00	0.667	0.667	16.00	0.00
2	Int	20.000	0.00	0.667	0.667	16.00	0.00
3	Int	20.000	0.00	0.667	0.667	16.00	0.00
4	Int	20.000	0.00	0.667	0.667	16.00	0.00
5	Int	20.000	0.00	0.667	0.667	16.00	0.00
6	Int	20.000	0.00	0.667	0.667	16.00	0.00

##### 1.4.2. Ribs and Longitudinal Beams

Span	Ribs			Beams		Span
	b in	h in	Sp in	b in	h in	H <sub>min</sub> in
1	0.00	0.00	0.00	16.00	24.00	12.97
2	0.00	0.00	0.00	16.00	24.00	11.43
3	0.00	0.00	0.00	16.00	24.00	11.43
4	0.00	0.00	0.00	16.00	24.00	11.43
5	0.00	0.00	0.00	16.00	24.00	11.43
6	0.00	0.00	0.00	16.00	24.00	12.97

#### 1.5. Support Data

##### 1.5.1. Columns

Support	c1a in	c2a in	Ha ft	c1b in	c2b in	Hb ft	Red %
1	24.00	16.00	10.000	24.00	16.00	10.000	100
2	24.00	16.00	10.000	24.00	16.00	10.000	100
3	24.00	16.00	10.000	24.00	16.00	10.000	100
4	24.00	16.00	10.000	24.00	16.00	10.000	100
5	24.00	16.00	10.000	24.00	16.00	10.000	100
6	24.00	16.00	10.000	24.00	16.00	10.000	100
7	24.00	16.00	10.000	24.00	16.00	10.000	100

##### 1.5.2. Boundary Conditions

Support	Spring		Far End	
	K <sub>z</sub> kip/in	K <sub>ry</sub> kip-in/rad	Above	Below
1	0	0	Fixed	Fixed
2	0	0	Fixed	Fixed
3	0	0	Fixed	Fixed
4	0	0	Fixed	Fixed
5	0	0	Fixed	Fixed
6	0	0	Fixed	Fixed
7	0	0	Fixed	Fixed

#### 1.6. Load Data

##### 1.6.1. Load Cases and Combinations

Case Type	Dead DEAD	Live LIVE
U1	1.000	1.000

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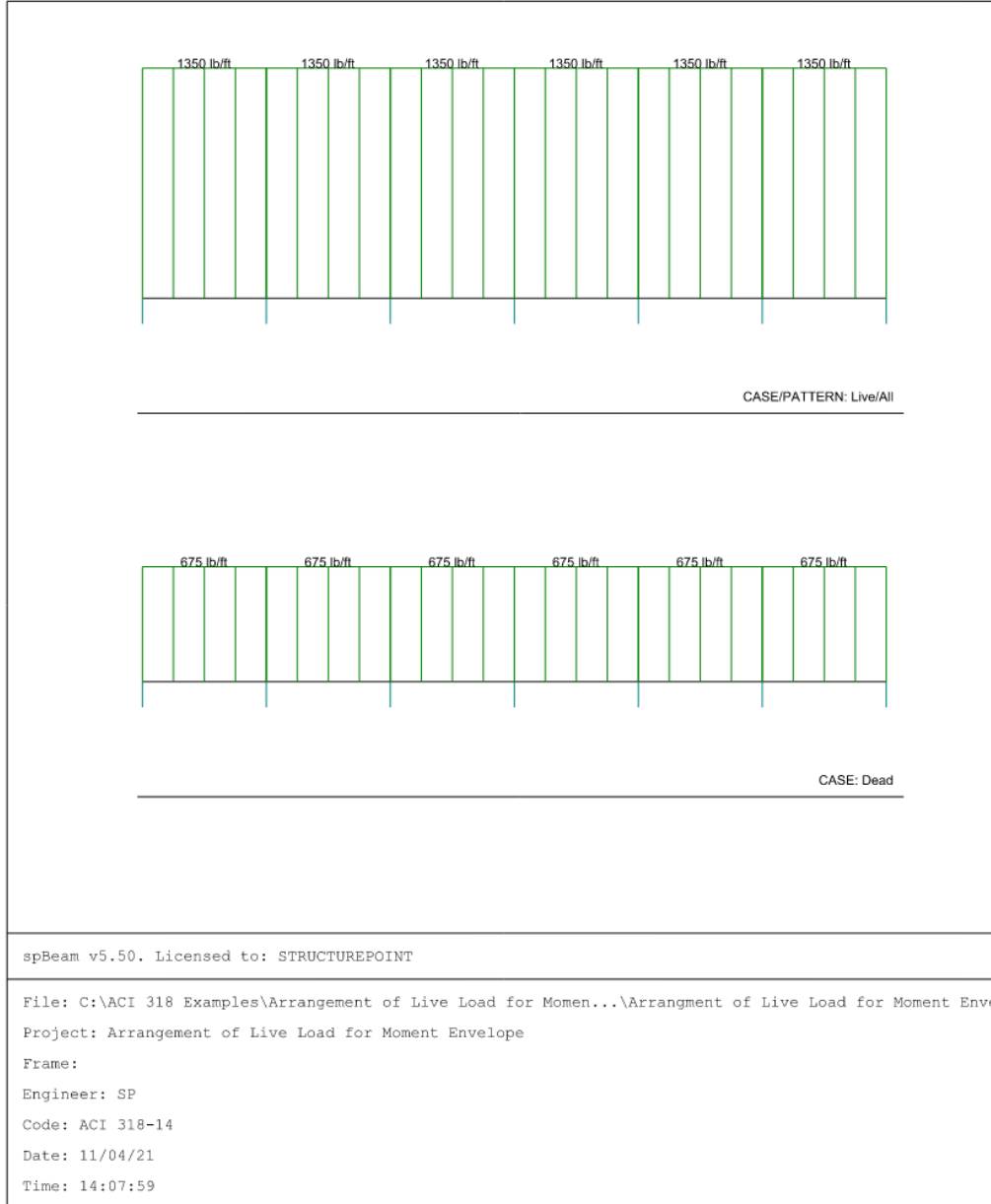
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**1.6.2. Line Loads**

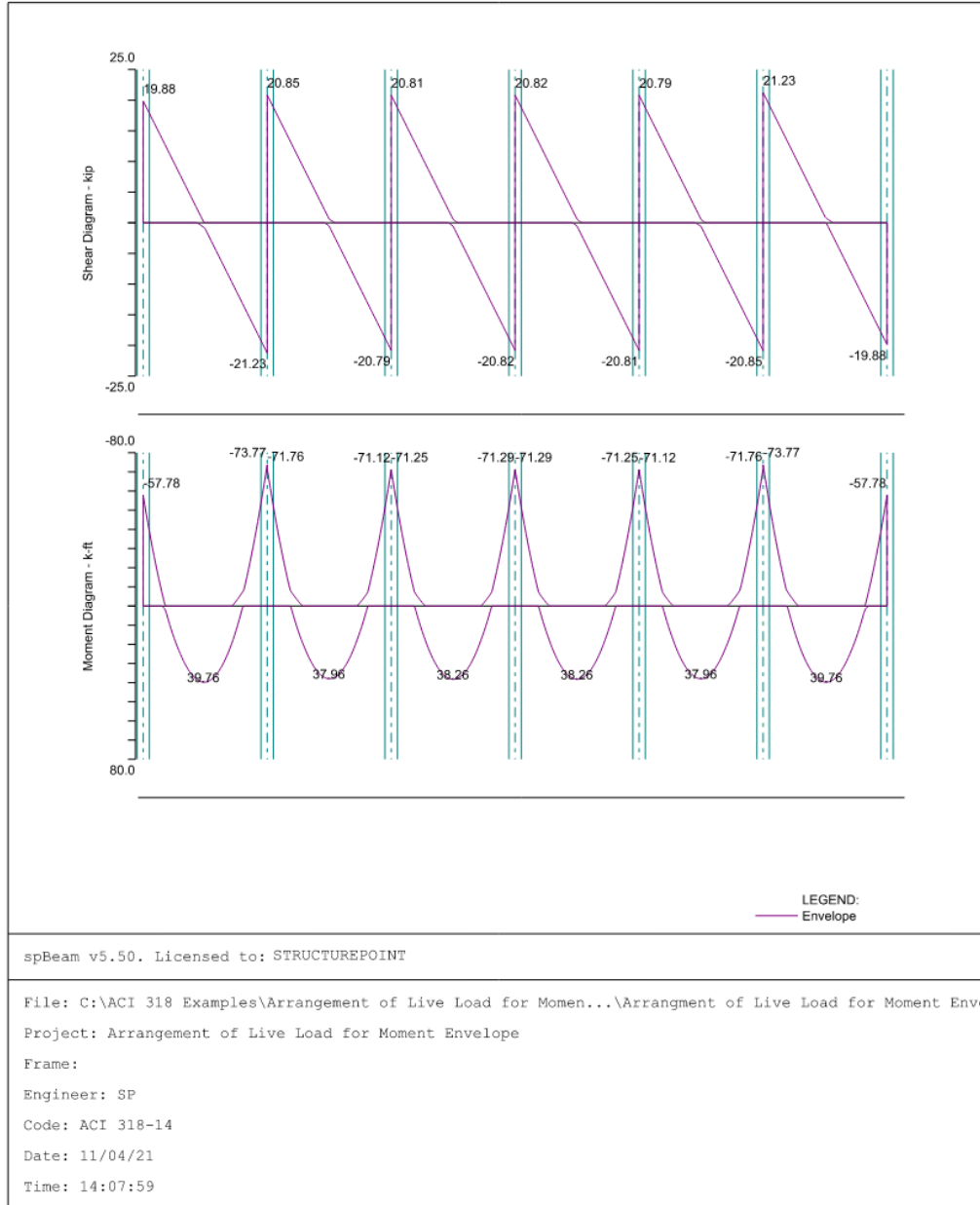
Case/Patt	Span	Wa lb/ft	La ft	Wb lb/ft	Lb ft
Dead	1	675.00	0.000	675.00	20.000
	2	675.00	0.000	675.00	20.000
	3	675.00	0.000	675.00	20.000
	4	675.00	0.000	675.00	20.000
	5	675.00	0.000	675.00	20.000
	6	675.00	0.000	675.00	20.000
Live	1	1350.00	0.000	1350.00	20.000
	2	1350.00	0.000	1350.00	20.000
	3	1350.00	0.000	1350.00	20.000
	4	1350.00	0.000	1350.00	20.000
	5	1350.00	0.000	1350.00	20.000
	6	1350.00	0.000	1350.00	20.000
Live/Odd	1	1350.00	0.000	1350.00	20.000
	3	1350.00	0.000	1350.00	20.000
	5	1350.00	0.000	1350.00	20.000
Live/Even	2	1350.00	0.000	1350.00	20.000
	4	1350.00	0.000	1350.00	20.000
	6	1350.00	0.000	1350.00	20.000
Live/S1	1	1350.00	0.000	1350.00	20.000
Live/S2	1	1350.00	0.000	1350.00	20.000
	2	1350.00	0.000	1350.00	20.000
Live/S3	2	1350.00	0.000	1350.00	20.000
	3	1350.00	0.000	1350.00	20.000
Live/S4	3	1350.00	0.000	1350.00	20.000
	4	1350.00	0.000	1350.00	20.000
Live/S5	4	1350.00	0.000	1350.00	20.000
	5	1350.00	0.000	1350.00	20.000
	6	1350.00	0.000	1350.00	20.000
Live/S6	5	1350.00	0.000	1350.00	20.000
	6	1350.00	0.000	1350.00	20.000
Live/S7	6	1350.00	0.000	1350.00	20.000

## 2. Diagrams

### 2.1. Loads



**2.2. Internal Forces**



#### 4. Comparison of Design Results

Span		AB		BC		CD		Line of symmetry
		M <sub>L</sub>	M <sub>R</sub>	M <sub>L</sub>	M <sub>R</sub>	M <sub>L</sub>	M <sub>R</sub>	
Negative Moment (kip-ft)	Hand	57.76	-73.83	72.03	-71.16	71.52	-71.62	
	Reference	57.67	-73.79	71.93	-71.12	71.44	-71.52	
	spBeam	57.78	-73.77	71.76	-71.12	71.25	-71.29	
Positive Moment (kip-ft)	Hand	39.72		37.74		38.24		
	Reference	39.85		37.99		38.31		
	spBeam	39.76		37.96		38.26		

Comparison of results has been limited to the envelope values for the first three spans AB, BC, and CD since this is the goal of this case study. The results of reference and the hand calculations illustrated above are in precise agreement with the automated exact results obtained from the [spBeam](#) program.