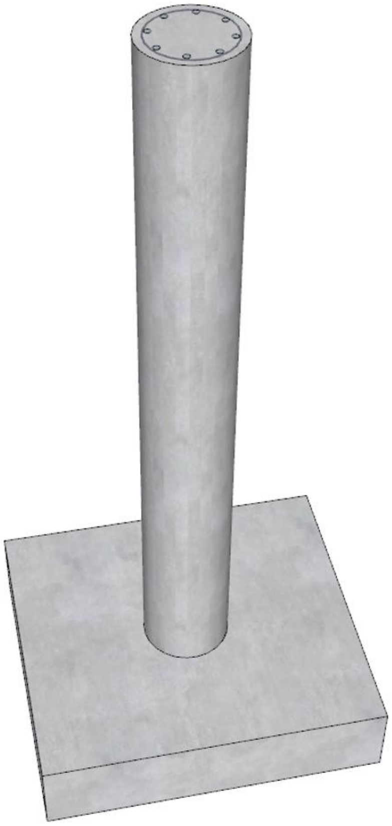
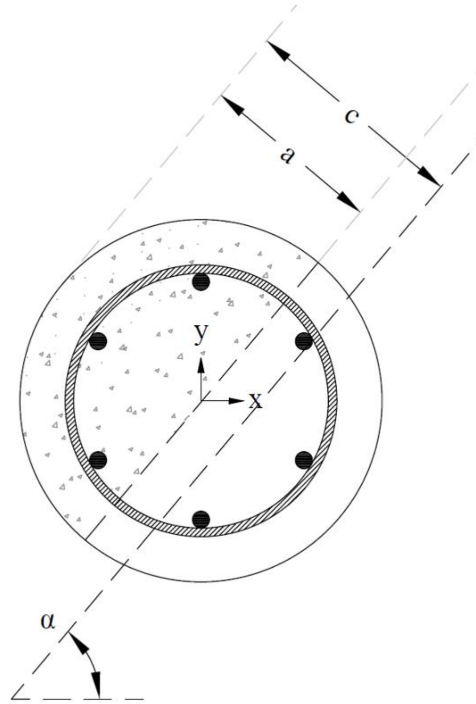
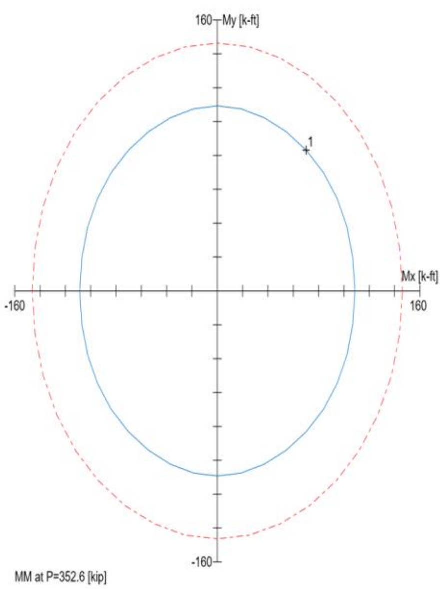
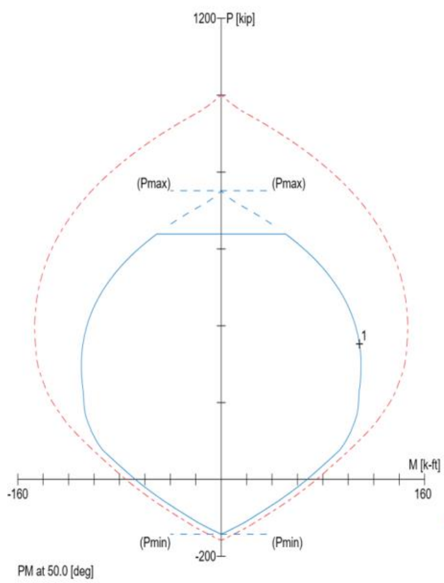
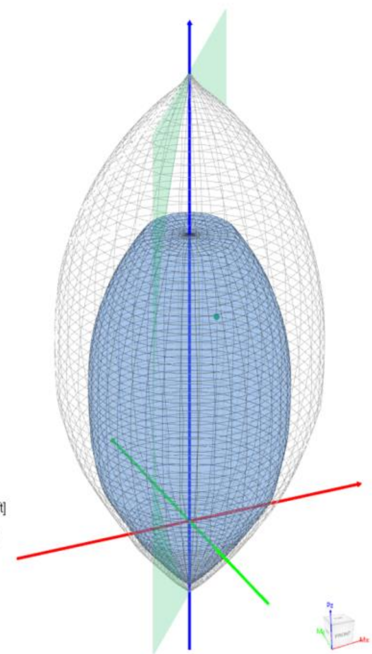
**Biaxial Bending Interaction Diagrams for Spiral Reinforced Circular Concrete Column Design (ACI 318-19)**





**Biaxial Bending Interaction Diagrams for Spiral Reinforced Circular Concrete Column Design (ACI 318-19)**

Biaxial bending of columns occurs when the loading causes bending simultaneously about both principal axes. The commonly encountered case of such loading occurs in corner columns. Corner and other columns exposed to known moments about each axis simultaneously should be designed for biaxial bending and axial load.

A uniaxial interaction diagram defines the load-moment strength along a single plane of a section under an axial load P and a uniaxial moment M. The biaxial bending resistance of an axially loaded column can be represented schematically as a surface formed by a series of uniaxial interaction curves drawn radially from the P axis. Data for these intermediate curves are obtained by varying the angle of the neutral axis (for assumed strain configurations) with respect to the major axes.

The difficulty associated with the determination of the strength of reinforced columns subjected to combined axial load and biaxial bending is primarily an arithmetic one. The bending resistance of an axially loaded column about a particular skewed axis is determined through iterations involving simple but lengthy calculations. These extensive calculations are compounded when optimization of the reinforcement or cross-section is sought.

This example demonstrates the determination of the design axial load capacity, $\phi P_n$, and the design $\phi M_{nx}$ and $\phi M_{ny}$ moments corresponding to the following case: The neutral axis depth of 10.05 in., at an angle of 50º counterclockwise from the x-axis of the cross section. The figure below shows the reinforced concrete circular column cross section in consideration. The calculated values of the column axial strength and biaxial bending strength are compared with the exact values from spColumn engineering software program from StructurePoint. The steps to develop the three-dimensional failure surface (interaction diagram) using spColumn will be shown in detail as well.

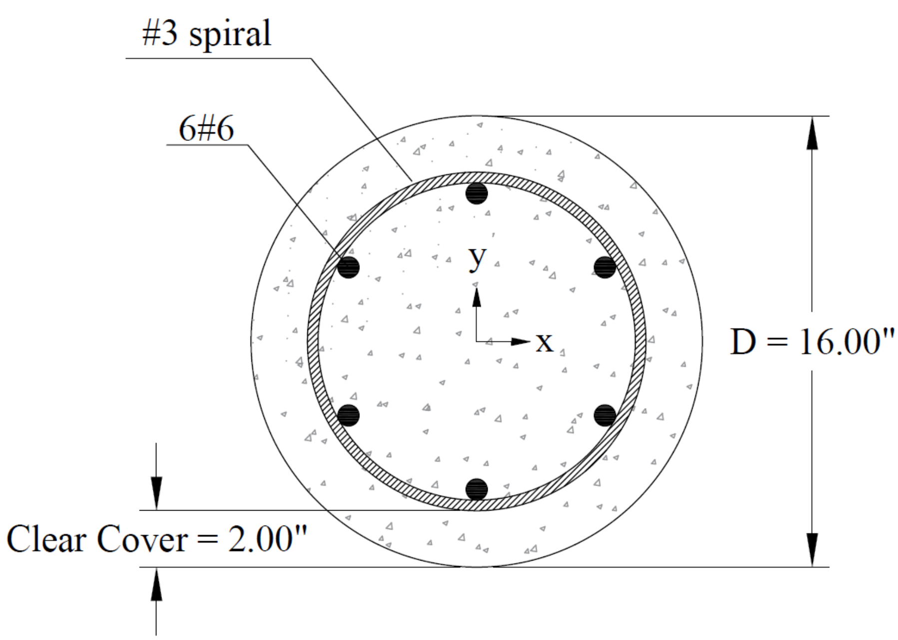



Figure 1 – Reinforced Concrete Column Cross-Section

# Contents

**Code**

Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)

**References**

- spColumn Engineering Software Program Manual v10.00, STRUCTUREPOINT, 2021
- "Biaxial Bending Interaction Diagrams for Square Reinforced Concrete Column Design (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Biaxial Bending Interaction Diagrams for Rectangular Reinforced Concrete Column Design (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Biaxial Bending Interaction Diagrams for C-Shaped Concrete Core Wall Design (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Manual Design Procedure for Columns and Walls with Biaxial Bending (ACI 318-11/14/19)" Design Example, STRUCTUREPOINT, 2022

**Design Data**

$f_c'$ = 5,000 psi

$f_y$ = 60,000 psi

Diameter = 16 in.

Clear Cover = 2.0 in.

Column dimensions and reinforcement locations are shown in following figure.

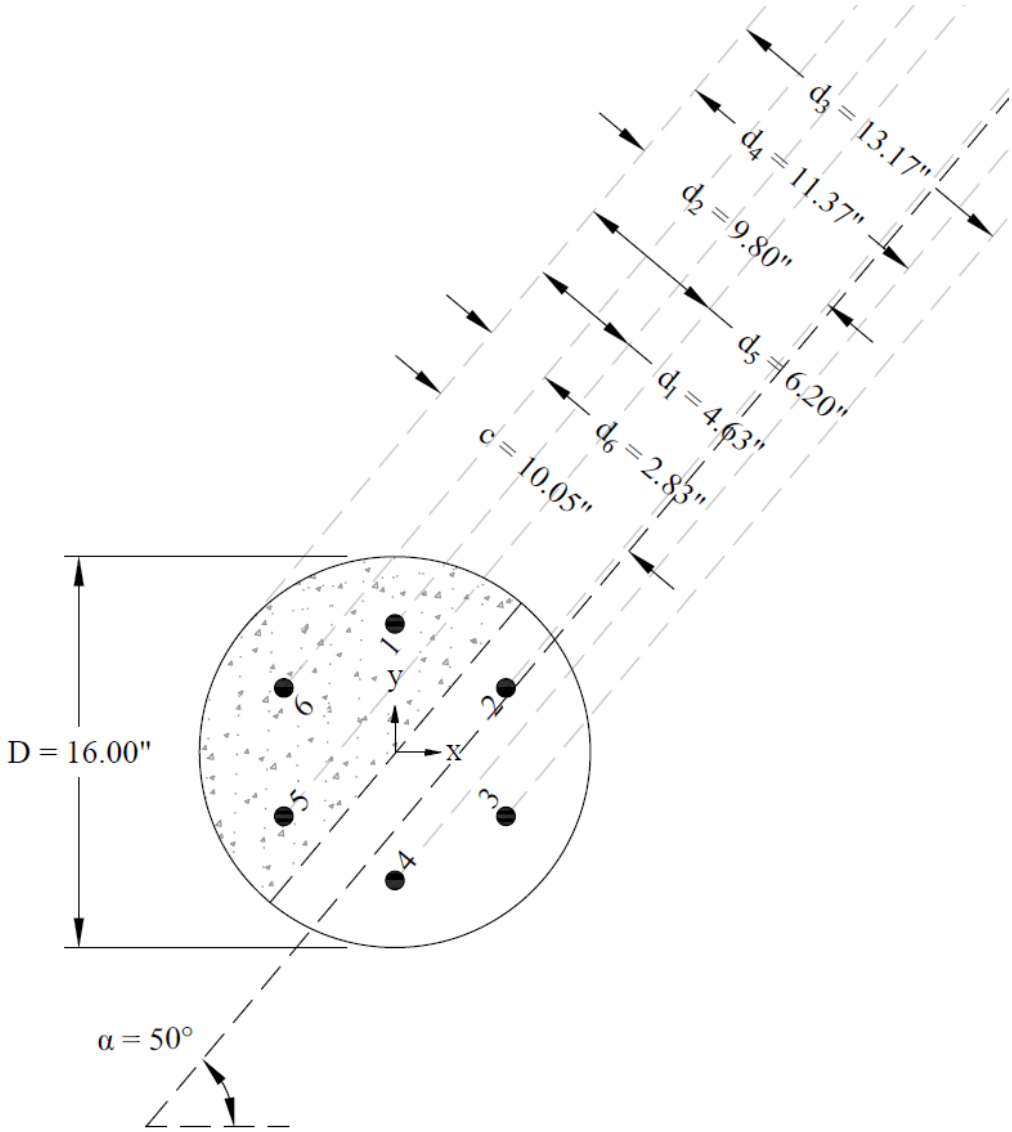



Figure 2 – Reinforced Concrete Column Cross-Section and Reinforcement Locations

**Solution**

In a reinforced concrete column, the determination of the nominal axial load capacity, $P_n$, and the nominal $M_{nx}$ and $M_{ny}$ moments involves a trial-and-error process for calculating the neutral axis depth and angle α. In this example, the neutral axis depth and angle are provided as an input (c = 10.05 in. and an angle of α = 50º) for illustration.

The steps to calculate biaxial flexural strength of a circular reinforced concrete column for nominal axial strength and biaxial bending moments are as follows:

1. Use the provided values for the angle of the neutral axis (α) and the neutral axis depth (c) to calculate the strain values in each reinforcement layer
2. Calculate the forces values in the concrete ($C_c$) and reinforcement layers ($F_{si}$)
3. Calculate $P_n$, $M_{nx}$ and $M_{ny}$ using the following equations

$$P_n = C_c + \sum F_s$$

$$M_{ny} = C_c \times \left(\frac{D}{2} - \bar{x}_c\right) + \sum_{i=1}^{n=6}\left(F_{si} \times \left(\frac{D}{2} - x_i\right)\right)$$

$$M_{nx} = C_c \times \left(\frac{D}{2} - \bar{y}_c\right) + \sum_{i=1}^{n=6}\left(F_{si} \times \left(\frac{D}{2} - y_i\right)\right)$$

The following figure demonstrates the procedure explained above:

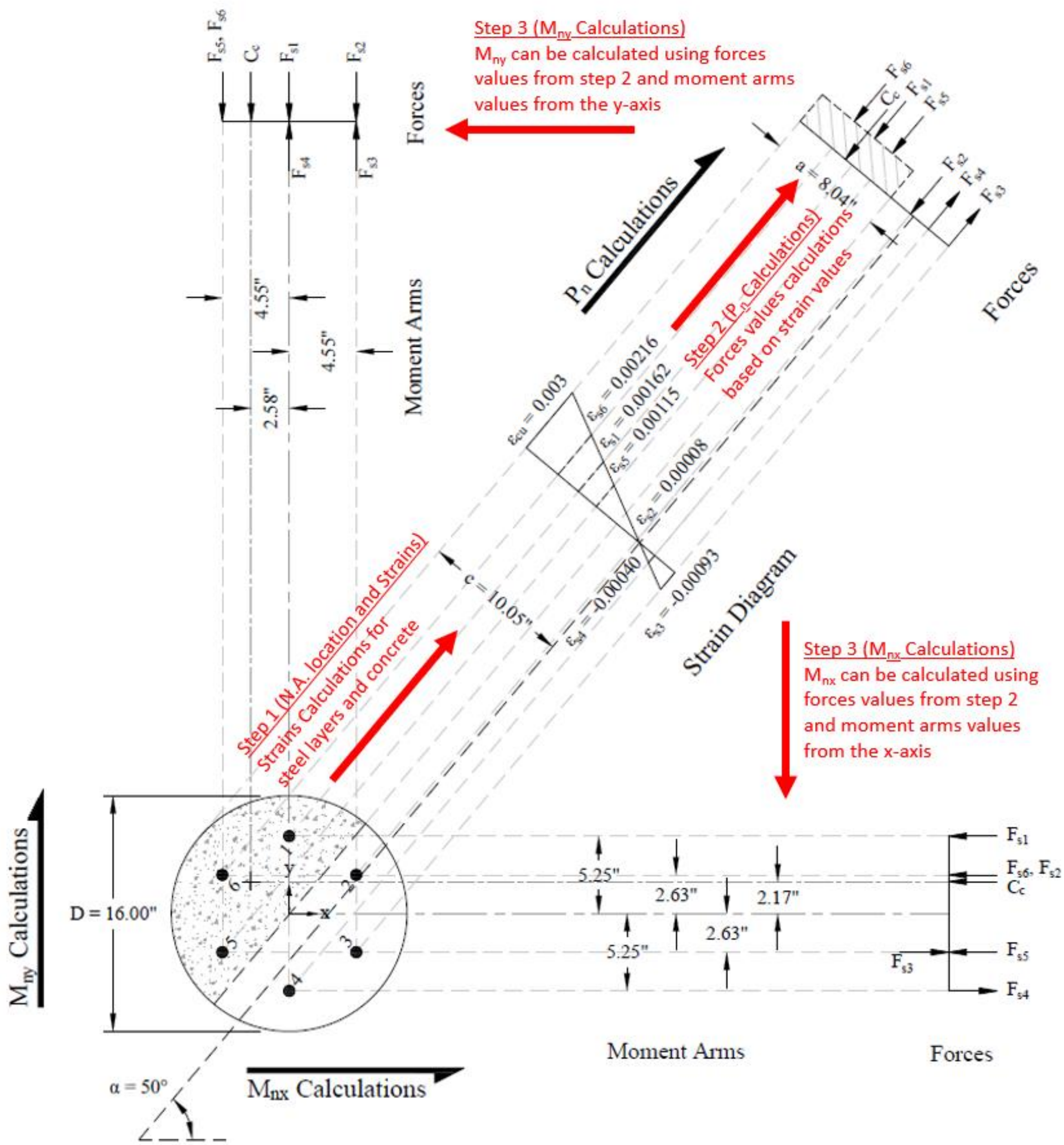



Figure 3 – Nominal Axial Load and Biaxial Flexural Strength Calculation Methods for a Reinforced Concrete Column

1. **Concrete Column Biaxial Strength Calculations**

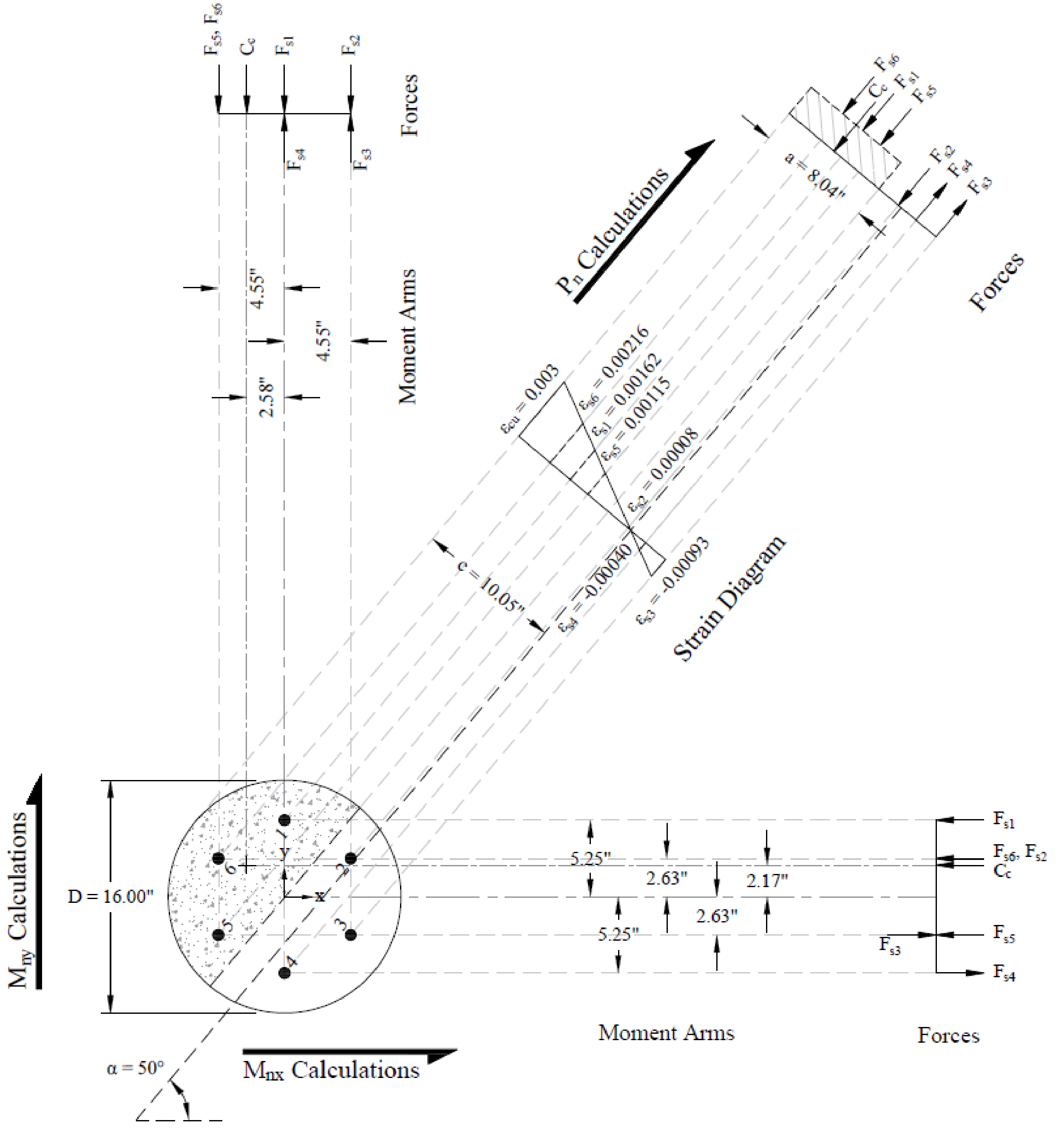



Figure 4 – Strains, Forces, and Moment Arms Diagram

## 1.1. Location of Neutral Axis and Concrete Compression Force

The trial-and-error process for calculating the neutral axis depth and angle α is not required in this example since these values are given (c = 10.05 in. and α = 50°). Where c is the distance from the fiber of maximum compressive strain to the neutral axis and α is the angle of the neutral axis.

***ACI 318-19 (22.2.2.4.2)***

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29{,}000} = 0.00207$$

$$\varepsilon_{s3} = (c - d_3) \times \frac{\varepsilon_{cu}}{c} = (10.05 - 13.17) \times \frac{0.003}{10.05} = -0.00093 \text{ (Tension)} < \varepsilon_y \rightarrow \text{reinforcement has not yielded}$$

$\therefore \phi = 0.75$ (for spiral) ***ACI 318-19 (Table 21.2.2)***

$a = \beta_1 \times c = 0.80 \times 10.05 = 8.04$ in. ***ACI 318-19 (22.2.2.4.1)***

$\varepsilon_{cu} = 0.003$ ***ACI 318-19 (22.2.2.1)***

Where:

*a* = Depth of equivalent rectangular stress block

$\beta_1 = 0.85 - \frac{0.05 \times (f'_c - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$ ***ACI 318-19 (Table 22.2.2.4.3)***

$C_c = 0.85 \times f'_c \times A_{comp} = 0.85 \times 5000 \times 101.18 = 430.03 \text{ kip (Compression)}$ ***ACI 318-19 (22.2.2.4.1)***

Where (see the following figure):

$$\theta = \cos^{-1}\left(\frac{\frac{D}{2} - a}{\frac{D}{2}}\right) = \cos^{-1}\left(\frac{\frac{16}{2} - 8.04}{\frac{16}{2}}\right) = 90.3°$$

$$A_{comp} = D^2 \times \frac{\theta - \sin(\theta) \times \cos(\theta)}{4}$$

$$A_{comp} = 16^2 \times \frac{\left(90.3° \times \frac{\pi}{180°}\right) - \sin(90.3°) \times \cos(90.3°)}{4} = 101.18 \text{ in.}^2$$

$$\bar{R} = \frac{D^3 \times \sin^3(\theta)}{12 \times A_{comp}} = \frac{16^3 \times \sin^3(90.3°)}{12 \times 101.18} = 3.37 \text{ in.}$$

$$\bar{x} = \bar{R} \times \sin(\alpha) = 3.37 \times \sin(50°) = -2.58 \text{ in.}$$

$$\bar{y} = \bar{R} \times \cos(\alpha) = 3.37 \times \cos(50^o) = 2.17 \text{ in.}$$

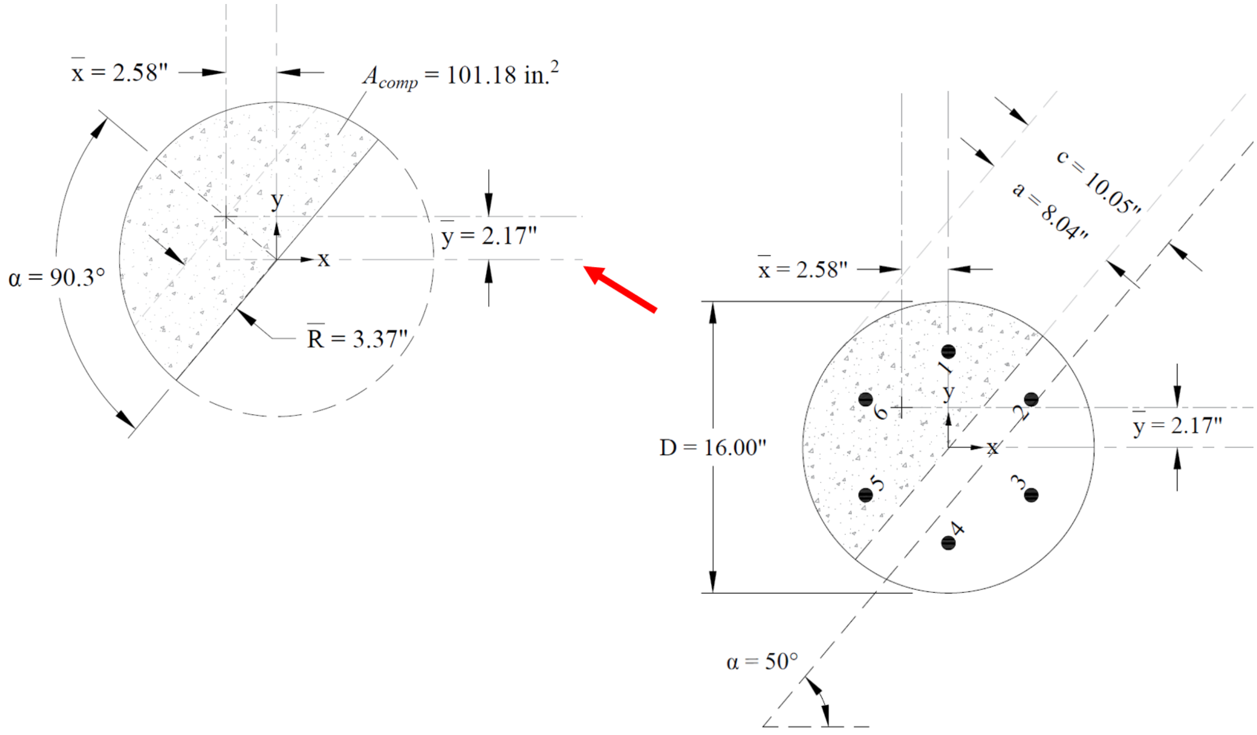



Figure 5 – Cracked Concrete Column Section Centroid Calculations

1.2. Strains and Forces Determination in Reinforcement Layers

The following shows the calculations of forces in the reinforcement layers with the extreme tension (at bar 3) and extreme compression (at bar 6) strains. The calculations for the rest of layers are shown the table at the end of this section.

For extreme tension reinforcement layer (at bar 3):

$$\varepsilon_{s3} = -0.00093 \text{ (Tension)} < \varepsilon_y \rightarrow \text{reinforcement has not yielded}$$

$$\therefore f_{s5} = \varepsilon_{s5} \times E_s = -0.00093 \times 29000000 = -26998 \text{ psi}$$

$$\text{F}_{s5} = f_{s5} \times A_{s5} = -26998 \times (1 \times 0.44) = -11.88 \text{ kip} \ \ \text{(Tension)}$$

For extreme compression reinforcement layer (at bar 6):

$$\varepsilon_{s6} = (c - d_6) \times \frac{\varepsilon_{cu}}{c} = (10.05 - 2.83) \times \frac{0.003}{10.05} = 0.00216 \text{ (Compression)} > \varepsilon_y \rightarrow \text{reinforcement has yielded}$$

$$\therefore f_{s1} = f_y = 60000 \text{ psi}$$

The area of the reinforcement in this layer is included in the area used to compute $C_c$ (a = 8.04 in. > $d_6$ = 2.83 in.). As a result, it is necessary to subtract 0.85$f_c$' from $f_{s6}$ before computing $F_{s6}$:

$$F_{s6} = f_{s6} \times A_{s6} = (60000 - 0.85 \times 5000) \times (1 \times 0.44) = 24.53 \text{ kip} \text{ (Compression)}$$

The same procedure shown above can be repeated to calculate the forces in the remaining reinforcement locations, results are summarized in the following table:

Table 1 - Strains, internal force resultants and Moments

| Location | d, in. | ε, in./in. | $f_s$, psi | $F_s$, kip | $C_c$, kip | Moment arm (x), in. | $M_y$, kip-ft | Moment arm (y), in. | $M_x$, kip-ft |
|---|---|---|---|---|---|---|---|---|---|
| Concrete | --- | 0.00300 | --- | --- | 430.04 | 2.58 | 92.60 | 2.17 | 77.70 |
| Bar 1 | 4.625 | 0.00162 | 46964 | 18.79* | --- | 5.25 | 8.22 | 0.00 | 0.00 |
| Bar 2 | 9.796 | 0.00008 | 2211 | 0.97* | --- | 2.63 | 0.21 | -4.55 | -0.37 |
| Bar 3 | 13.170 | -0.00093 | -26998 | -11.88 | --- | -2.63 | 2.60 | -4.55 | 4.50 |
| Bar 4 | 11.375 | -0.00040 | -11456 | -5.04 | --- | -5.25 | 2.21 | 0.00 | 0.00 |
| Bar 5 | 6.204 | 0.00115 | 33296 | 12.78* | --- | -2.63 | -2.80 | 4.55 | 4.84 |
| Bar 6 | 2.830 | 0.00216 | 60000 | 24.53* | --- | 2.63 | 5.37 | 4.55 | 9.29 |
| Axial Force and Biaxial Bending Moments Capacities | | | $P_n$, kip | 470.19 | | $M_{ny}$, kip-ft | 110.87 | $M_{nx}$, kip-ft | 93.51 |
| | | | ϕ$P_n$, kip | 352.64 | | ϕ$M_{ny}$, kip-ft | 83.15 | ϕ$M_{nx}$, kip-ft | 70.13 |

* The area of the reinforcement in this layer has been included in the area used to compute $C_c$. As a result, 0.85$f_c$' is subtracted from $f_s$ in the computation of $F_s$.

### 1.3. Calculation of $P_n$, $M_{nx}$ and $M_{ny}$

$$P_n = C_c + \sum F_s$$ (+) = Compression (-) = Tension

$$\phi P_n = \phi \times P_n = 0.75 \times P_n$$

$$M_{ny} = C_c \times \left(\frac{D}{2} - \bar{x}_c\right) + \sum_{i=1}^{n=6}\left(F_{si} \times \left(\frac{D}{2} - x_i\right)\right)$$ (+) = Counter Clockwise (-) = Clockwise

$$\phi M_{ny} = \phi \times M_{ny} = 0.75 \times M_{ny}$$

$$M_{nx} = C_c \times \left(\frac{D}{2} - \bar{y}_c\right) + \sum_{i=1}^{n=6}\left(F_{si} \times \left(\frac{D}{2} - y_i\right)\right)$$ (+) = Counter Clockwise (-) = Clockwise

$$\phi M_{nx} = \phi \times M_{nx} = 0.75 \times M_{nx}$$

## 2. Column Biaxial Bending Interaction Diagram – spColumn Software

spColumn is a StructurePoint software program that performs the analysis and design of reinforced concrete sections subjected to axial force combined with uniaxial or biaxial bending. Using the provisions of the Strength Design Method and Unified Design Provisions, slenderness considerations are used for moment magnification due to second order effect (P-Delta) for sway and non-sway frames.

For biaxial runs, the values of maximum compressive axial load capacity and maximum tensile load capacity are computed. These two values set the range within which the moment capacities are computed for a predetermined number of axial load values. For each level of axial load, the section is rotated in 10-degree increments from 0 degrees to 360 degrees and the $M_x$ and $M_y$ moment capacities are computed. Thus, for each level of axial load, an $M_x$-$M_y$ contour is developed. Repeating this for the entire range of axial loads, the three-dimensional failure surface is computed. A three-dimensional visualization of the resulting entire nominal and factored failure surface is provided to support enhanced understanding of the section capacity.

The **"biaxial"** feature allows the user to investigate the P-M interaction diagrams, the $M_x$-$M_y$ moment contour plots, as well as the 3D failure surface for even the most irregular column and shear wall sections quickly, simply, and accurately.

For this column section, we ran in investigation mode with **"biaxial"** option for "Run Axis" using the ACI 318-19.

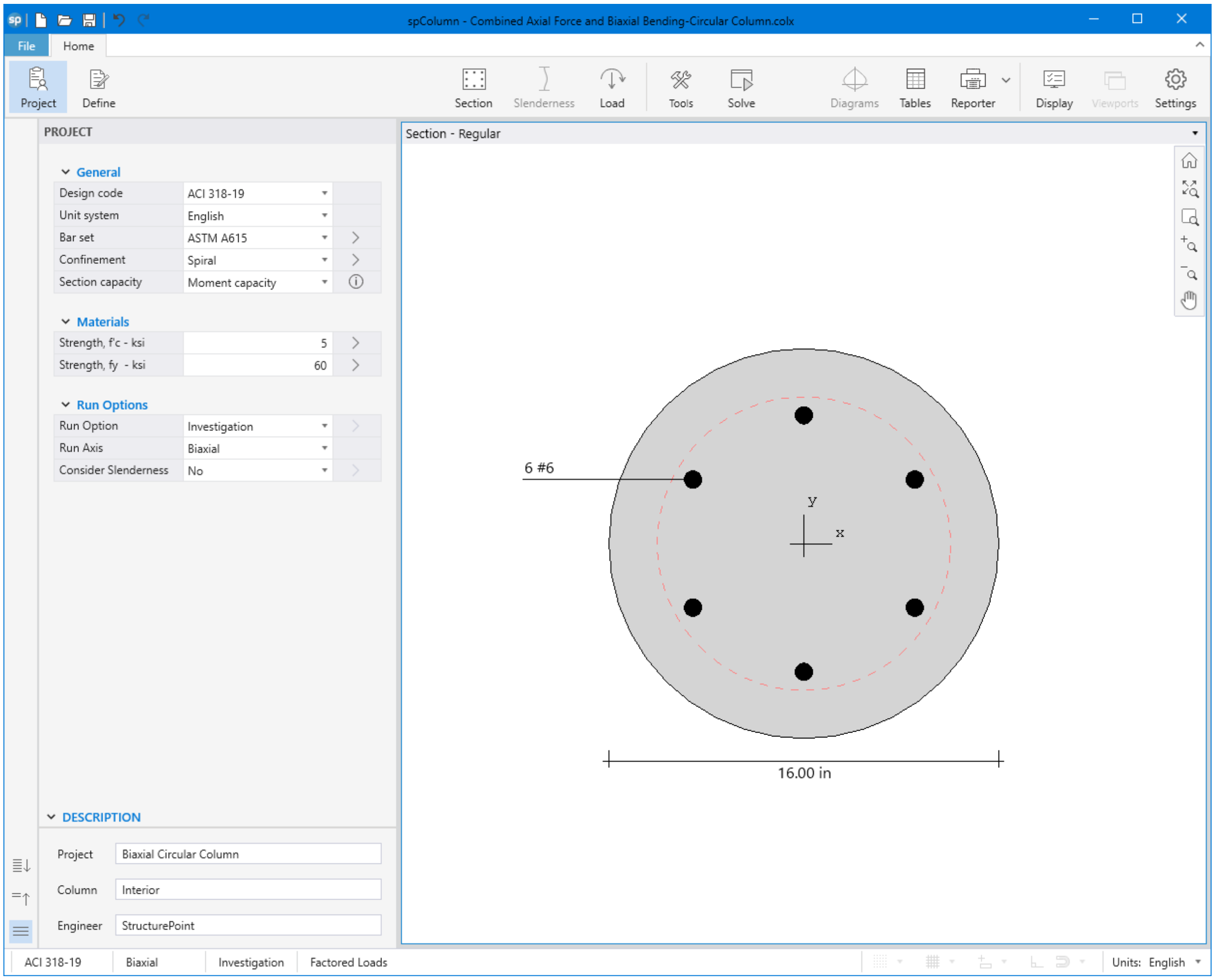



Figure 6 – spColumn Interface

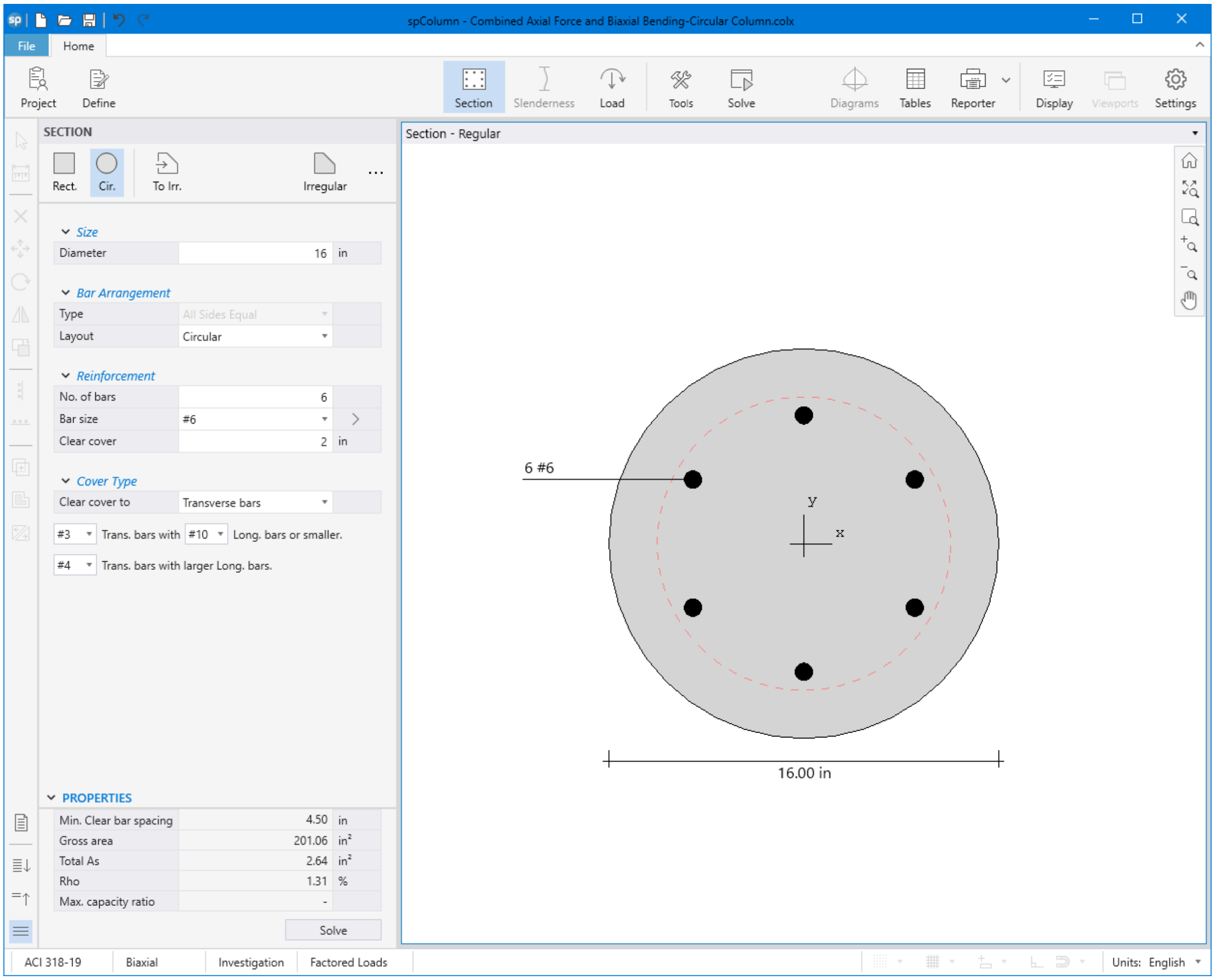



Figure 7 – spColumn Model Editor

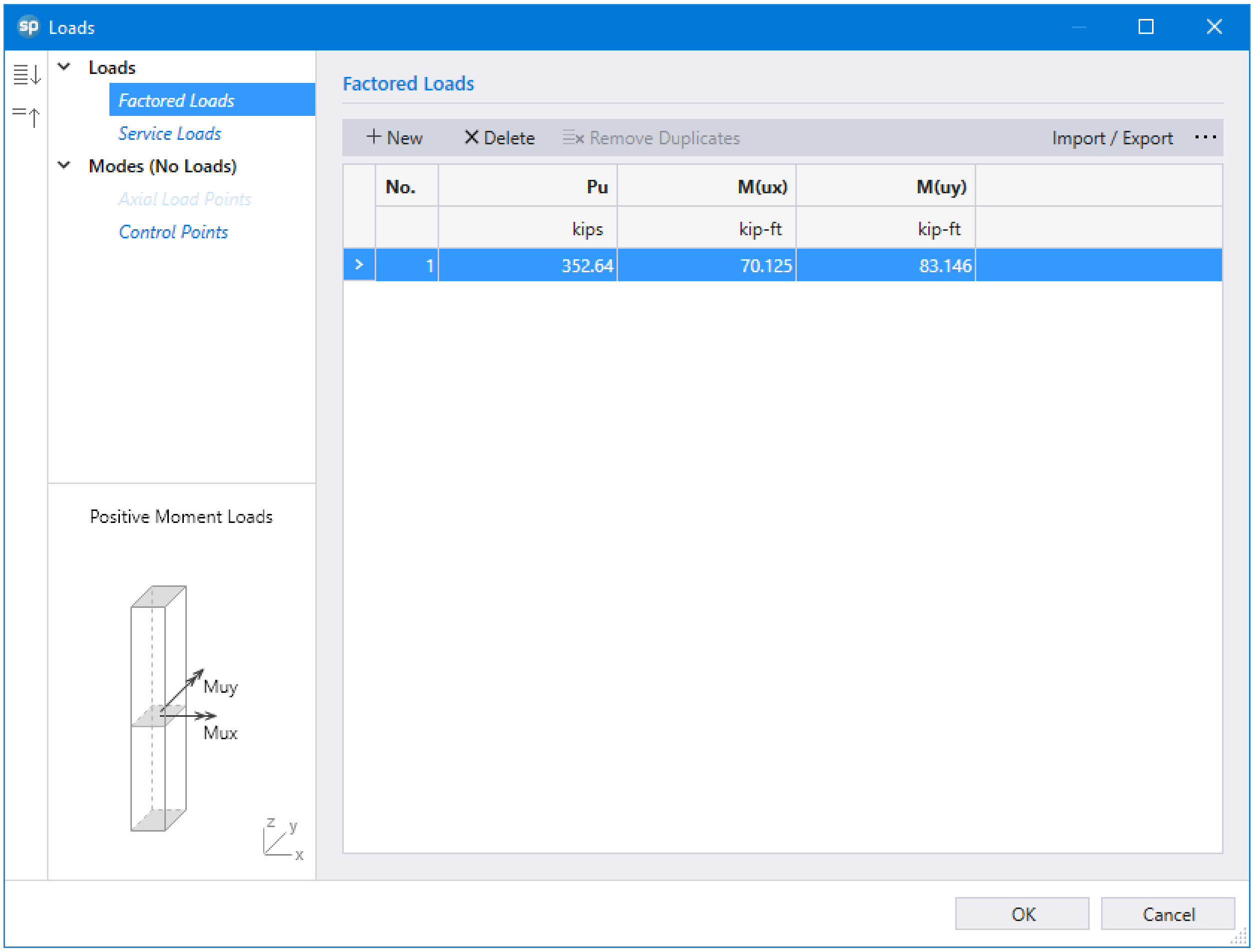



Figure 8 – Defining Loads / Modes (spColumn)

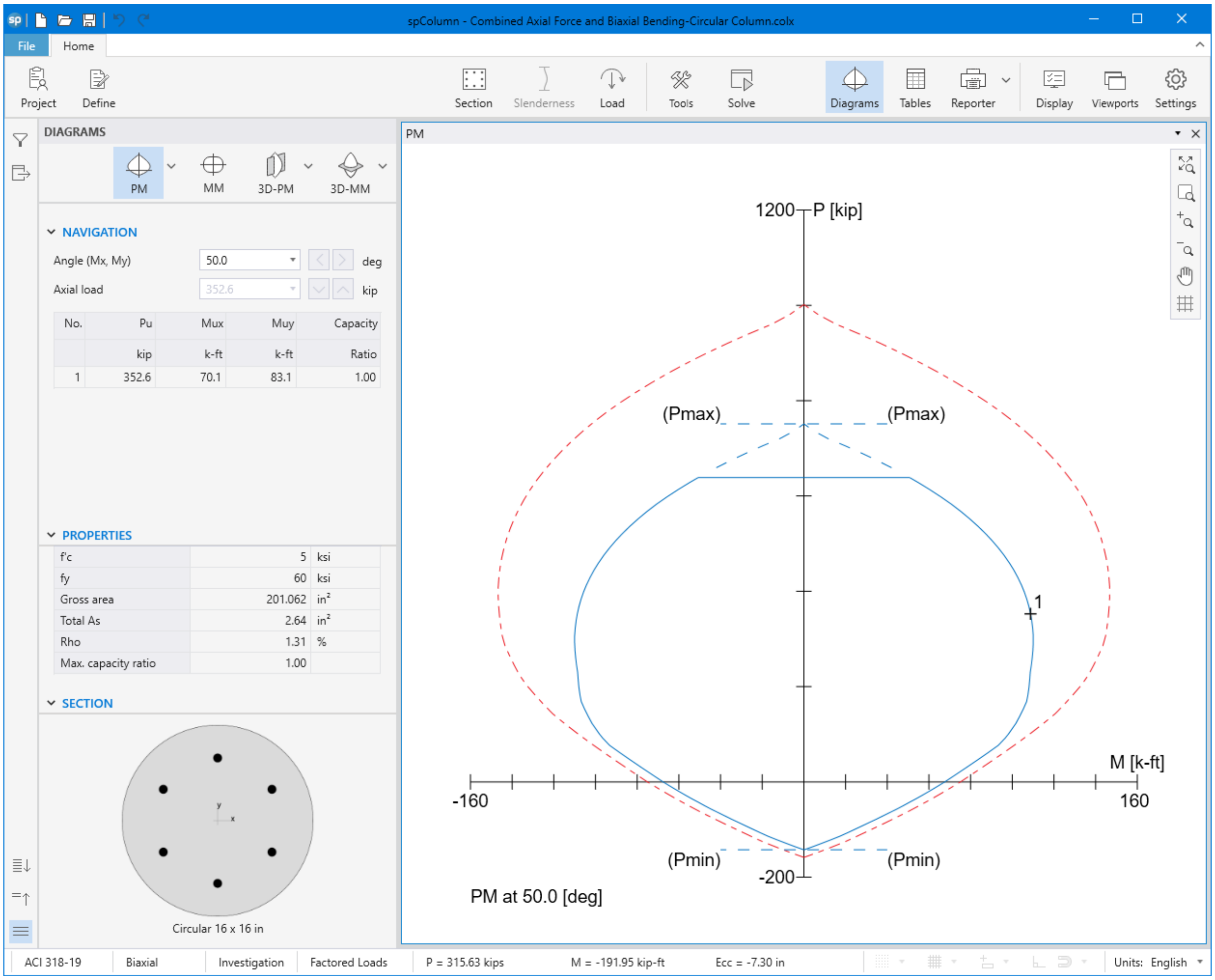
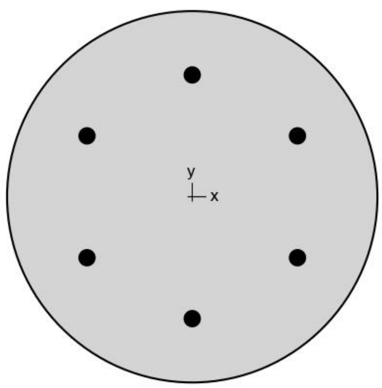



Figure 9 – Column Section Interaction Diagram at 50º (spColumn)

spColumn v10.00 (TM)
Computer program for the Strength Design of Reinforced Concrete Sections

## Contents



## List of Figures

# 1. General Information

| | |
|---|---|
| File Name | ...\Combined Axial Force and Biaxial Bending-C... |
| Project | Biaxial Circular Column |
| Column | Interior |
| Engineer | StructurePoint |
| Code | ACI 318-19 |
| Bar Set | ASTM A615 |
| Units | English |
| Run Option | Investigation |
| Run Axis | Biaxial |
| Slenderness | Not Considered |
| Column Type | Structural |
| Capacity Method | Moment capacity |

# 2. Material Properties

## *2.1. Concrete*

| | |
|---|---|
| Type | Standard |
| $f'_c$ | 5 ksi |
| $E_c$ | 4030.51 ksi |
| $f_c$ | 4.25 ksi |
| $\varepsilon_u$ | 0.003 in/in |
| $\beta_1$ | 0.8 |

## *2.2. Steel*

| | |
|---|---|
| Type | Standard |
| $f_y$ | 60 ksi |
| $E_s$ | 29000 ksi |
| $\varepsilon_{ty}$ | 0.00206897 in/in |

# 3. Section

## *3.1. Shape and Properties*

| | |
|---|---|
| Type | Circular |
| Diameter | 16 in |
| $A_g$ | 201.062 in² |
| $I_x$ | 3216.99 in⁴ |
| $I_y$ | 3216.99 in⁴ |
| $r_x$ | 4 in |
| $r_y$ | 4 in |
| $X_o$ | 0 in |
| $Y_o$ | 0 in |

### *3.2. Section Figure*

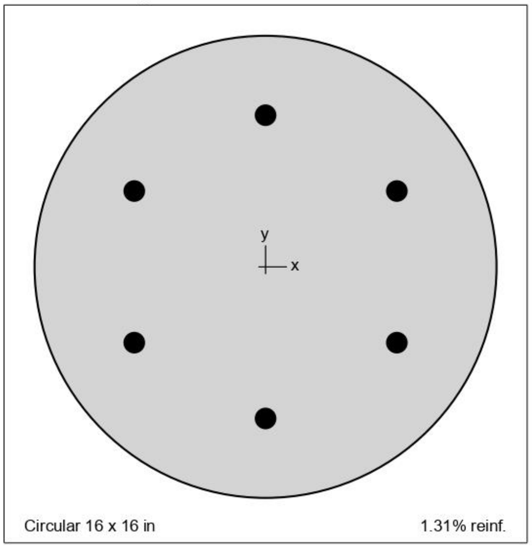



**Figure 1: Column section**

## 4. Reinforcement

### *4.1. Bar Set: ASTM A615*

| Bar | Diameter | Area | Bar | Diameter | Area | Bar | Diameter | Area |
|---|---|---|---|---|---|---|---|---|
| | in | in² | | in | in² | | in | in² |
| #3 | 0.38 | 0.11 | #4 | 0.50 | 0.20 | #5 | 0.63 | 0.31 |
| #6 | 0.75 | 0.44 | #7 | 0.88 | 0.60 | #8 | 1.00 | 0.79 |
| #9 | 1.13 | 1.00 | #10 | 1.27 | 1.27 | #11 | 1.41 | 1.56 |
| #14 | 1.69 | 2.25 | #18 | 2.26 | 4.00 | | | |

### *4.2. Confinement and Factors*

| | |
|---|---|
| Confinement type | Spiral |
| For #10 bars or less | #3 ties |
| For larger bars | #4 ties |
| | |
| **Capacity Reduction Factors** | |
| Axial compression, (a) | 0.85 |
| Tension controlled $\phi$, (b) | 0.9 |
| Compression controlled $\phi$, (c) | 0.75 |

### *4.3. Arrangement*

| | |
|---|---|
| Pattern | All sides equal |
| Bar layout | Circular |
| Cover to | Transverse bars |
| Clear cover | 2 in |
| Bars | 6 #6 |

| | |
|---|---|
| Total steel area, $A_s$ | 2.64 in² |
| Rho | 1.31 % |
| Minimum clear spacing | 4.50 in |

## 5. Control Points

| About | Point | P | X-Moment | Y-Moment | NA Depth | $d_t$ Depth | $\varepsilon_t$ | ϕ |
|---|---|---|---|---|---|---|---|---|
| | | kip | k-ft | k-ft | in | in | | |
| X | @ Max compression | 751.3 | 0.00 | 0.00 | 42.69 | 13.25 | -0.00207 | 0.75000 |
| X | @ Allowable comp. | 638.6 | 51.16 | 0.00 | 16.37 | 13.25 | -0.00057 | 0.75000 |
| X | @ $f_s$ = 0.0 | 511.8 | 89.10 | 0.00 | 13.25 | 13.25 | 0.00000 | 0.75000 |
| X | @ $f_s$ = 0.5 $f_y$ | 341.7 | 109.10 | 0.00 | 9.85 | 13.25 | 0.00103 | 0.75000 |
| X | @ Balanced point | 225.4 | 108.41 | 0.00 | 7.84 | 13.25 | 0.00207 | 0.75000 |
| X | @ Tension control | 83.2 | 94.96 | 0.00 | 4.93 | 13.25 | 0.00507 | 0.90000 |
| X | @ Pure bending | 0.0 | 67.16 | 0.00 | 3.66 | 13.25 | 0.00785 | 0.90000 |
| X | @ Max tension | -142.6 | 0.00 | 0.00 | 0.00 | 13.25 | 9.99999 | 0.90000 |
| Y | @ Max compression | 751.3 | 0.00 | 0.00 | 40.43 | 12.55 | -0.00207 | 0.75000 |
| Y | @ Allowable comp. | 638.6 | 0.00 | 50.18 | 16.46 | 12.55 | -0.00071 | 0.75000 |
| Y | @ $f_s$ = 0.0 | 478.0 | 0.00 | 95.93 | 12.55 | 12.55 | 0.00000 | 0.75000 |
| Y | @ $f_s$ = 0.5 $f_y$ | 314.9 | 0.00 | 110.20 | 9.33 | 12.55 | 0.00103 | 0.75000 |
| Y | @ Balanced point | 200.4 | 0.00 | 107.38 | 7.43 | 12.55 | 0.00207 | 0.75000 |
| Y | @ Tension control | 55.8 | 0.00 | 89.34 | 4.66 | 12.55 | 0.00507 | 0.90000 |
| Y | @ Pure bending | 0.0 | 0.00 | 67.64 | 3.57 | 12.55 | 0.00756 | 0.90000 |
| Y | @ Max tension | -142.6 | 0.00 | 0.00 | 0.00 | 12.55 | 9.99999 | 0.90000 |
| -X | @ Max compression | 751.3 | 0.00 | 0.00 | 42.69 | 13.25 | -0.00207 | 0.75000 |
| -X | @ Allowable comp. | 638.6 | -51.16 | 0.00 | 16.37 | 13.25 | -0.00057 | 0.75000 |
| -X | @ $f_s$ = 0.0 | 511.8 | -89.10 | 0.00 | 13.25 | 13.25 | 0.00000 | 0.75000 |
| -X | @ $f_s$ = 0.5 $f_y$ | 341.7 | -109.10 | 0.00 | 9.85 | 13.25 | 0.00103 | 0.75000 |
| -X | @ Balanced point | 225.4 | -108.41 | 0.00 | 7.84 | 13.25 | 0.00207 | 0.75000 |
| -X | @ Tension control | 83.2 | -94.96 | 0.00 | 4.93 | 13.25 | 0.00507 | 0.90000 |
| -X | @ Pure bending | 0.0 | -67.16 | 0.00 | 3.66 | 13.25 | 0.00785 | 0.90000 |
| -X | @ Max tension | -142.6 | 0.00 | 0.00 | 0.00 | 13.25 | 9.99999 | 0.90000 |
| -Y | @ Max compression | 751.3 | 0.00 | 0.00 | 40.43 | 12.55 | -0.00207 | 0.75000 |
| -Y | @ Allowable comp. | 638.6 | 0.00 | -50.18 | 16.46 | 12.55 | -0.00071 | 0.75000 |
| -Y | @ $f_s$ = 0.0 | 478.0 | 0.00 | -95.93 | 12.55 | 12.55 | 0.00000 | 0.75000 |
| -Y | @ $f_s$ = 0.5 $f_y$ | 314.9 | 0.00 | -110.20 | 9.33 | 12.55 | 0.00103 | 0.75000 |
| -Y | @ Balanced point | 200.4 | 0.00 | -107.38 | 7.43 | 12.55 | 0.00207 | 0.75000 |
| -Y | @ Tension control | 55.8 | 0.00 | -89.34 | 4.66 | 12.55 | 0.00507 | 0.90000 |
| -Y | @ Pure bending | 0.0 | 0.00 | -67.64 | 3.57 | 12.55 | 0.00756 | 0.90000 |
| -Y | @ Max tension | -142.6 | 0.00 | 0.00 | 0.00 | 12.55 | 9.99999 | 0.90000 |

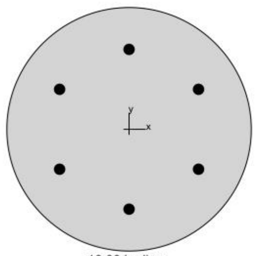
## 6. Factored Loads and Moments with Corresponding Capacity Ratios

NOTE: Calculations are based on "Moment Capacity" Method.

| No. | Demand | | | Capacity | | | Parameters at Capacity | | | Capacity |
|---|---|---|---|---|---|---|---|---|---|---|
| | $P_u$ | $M_{ux}$ | $M_{uy}$ | $\phi P_n$ | $\phi M_{nx}$ | $\phi M_{ny}$ | NA Depth | $\varepsilon_t$ | ϕ | Ratio |
| | kip | k-ft | k-ft | kip | k-ft | k-ft | in | | | |
| 1 | 352.64 | 70.13 | 83.15 | 352.64 | 70.13 | 83.15 | 10.05 | 0.00093 | 0.750 | 1.00 |

## 7. Diagrams

### *7.1. PM at θ=50 [deg]*




16.00 in diam.

**General Information**

| | |
|---|---|
| Project | Biaxial Circular Column |
| Column | Interior |
| Engineer | StructurePoint |
| Code | ACI 318-19 |
| Bar Set | ASTM A615 |
| Units | English |
| Run Option | Investigation |
| Run Axis | Biaxial |
| Slenderness | Not Considered |
| Column Type | Structural |
| Capacity Method | Moment capacity |

**Materials**

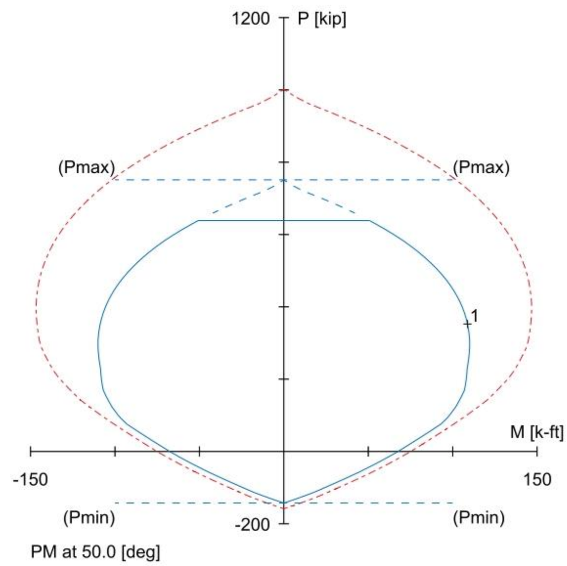
| | |
|---|---|
| $f'_c$ | 5 ksi |
| $E_c$ | 4030.51 ksi |
| | |
| $f_y$ | 60 ksi |
| $E_s$ | 29000 ksi |

**Section**

| | |
|---|---|
| Type | Circular |
| Diameter | 16 in |
| $A_g$ | 201.062 in² |
| $I_x$ | 3216.99 in⁴ |
| $I_y$ | 3216.99 in⁴ |

**Reinforcement**

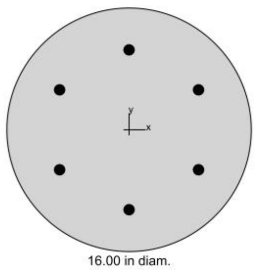
| | |
|---|---|
| Pattern | All sides equal |
| Bar layout | Circular |
| Cover to | Transverse bars |
| Clear cover | 2 in |
| Bars | 6 #6 |
| | |
| Confinement type | Spiral |
| | |
| Total steel area, $A_s$ | 2.64 in² |
| Rho | 1.31 % |
| Min. clear spacing | 4.50 in |




| No. | $P_u$ | $M_{ux}$ | $M_{uy}$ | $\phi P_n$ | $\phi M_{nx}$ | $\phi M_{ny}$ | Capacity |
|---|---|---|---|---|---|---|---|
| | kip | k-ft | k-ft | kip | k-ft | k-ft | Ratio |
| 1 | 352.6 | 70.1 | 83.1 | 352.64 | 70.13 | 83.15 | 1.00 |

Max. Capacity Ratio: 1.00

### *7.2. MM at P=353 [kip]*

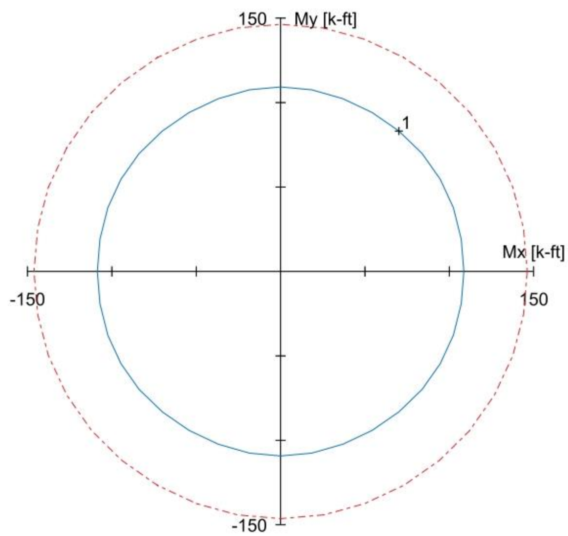



| General Information | |
|---|---|
| Project | Biaxial Circular Column |
| Column | Interior |
| Engineer | StructurePoint |
| Code | ACI 318-19 |
| Bar Set | ASTM A615 |
| Units | English |
| Run Option | Investigation |
| Run Axis | Biaxial |
| Slenderness | Not Considered |
| Column Type | Structural |
| Capacity Method | Moment capacity |

| Materials | |
|---|---|
| $f'_c$ | 5 ksi |
| $E_c$ | 4030.51 ksi |
| | |
| $f_y$ | 60 ksi |
| $E_s$ | 29000 ksi |

| Section | |
|---|---|
| Type | Circular |
| Diameter | 16 in |
| $A_g$ | 201.062 in² |
| $I_x$ | 3216.99 in⁴ |
| $I_y$ | 3216.99 in⁴ |

| Reinforcement | |
|---|---|
| Pattern | All sides equal |
| Bar layout | Circular |
| Cover to | Transverse bars |
| Clear cover | 2 in |
| Bars | 6 #6 |
| | |
| Confinement type | Spiral |
| | |
| Total steel area, $A_s$ | 2.64 in² |
| Rho | 1.31 % |
| Min. clear spacing | 4.50 in |




MM at P=353.0 [kip]

| No. | $P_u$ | $M_{ux}$ | $M_{uy}$ | $\phi P_n$ | $\phi M_{nx}$ | $\phi M_{ny}$ | Capacity |
|---|---|---|---|---|---|---|---|
| | kip | k-ft | k-ft | kip | k-ft | k-ft | Ratio |
| 1 | 352.6 | 70.1 | 83.1 | 352.64 | 70.13 | 83.15 | 1.00 |

Max. Capacity Ratio: 1.00

## 3. Summary and Comparison of Design Results

| Table 2 - Comparison of Results | | |
|---|---|---|
| Parameter | Hand | spColumn |
| c, in. | 10.05 | 10.05 |
| $\varepsilon_{s3}$, in./in. | 0.00093 | 0.00093 |
| $\phi P_n$, kip | 352.64 | 352.64 |
| $\phi M_{nx}$, kip-ft | 70.13 | 70.13 |
| $\phi M_{ny}$, kip-ft | 83.15 | 83.15 |

In all of the hand calculations used illustrated above, the results are in precise agreement with the automated exact results obtained from the spColumn program.

## 4. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In most building design calculations, such as the examples shown for flat plate or flat slab concrete floor systems, all building columns may be subjected to biaxial bending ($M_x$ and $M_y$) due to lateral effects and unbalanced moments from both directions of analysis. This requires an investigation of the column P-$M_x$-$M_y$ interaction diagram in two directions simultaneously (axial force interaction with biaxial bending).

This example shows the calculations needed to obtain one point on the three-dimensional failure surface (biaxial $M_x$-$M_y$ interaction diagram). Generating the three-dimensional failure surface (interaction diagram) for a column section subjected to a combined axial force and biaxial bending moments is tedious and challenging for engineers and the use of a computer aid can save time and eliminate errors. StucturePoint's spColumn program can, quickly, simply and accurately generate the three-dimensional failure surface (interaction diagram) for all commonly encountered column, beam or wall sections in addition to highly complex and irregular cross-sections.

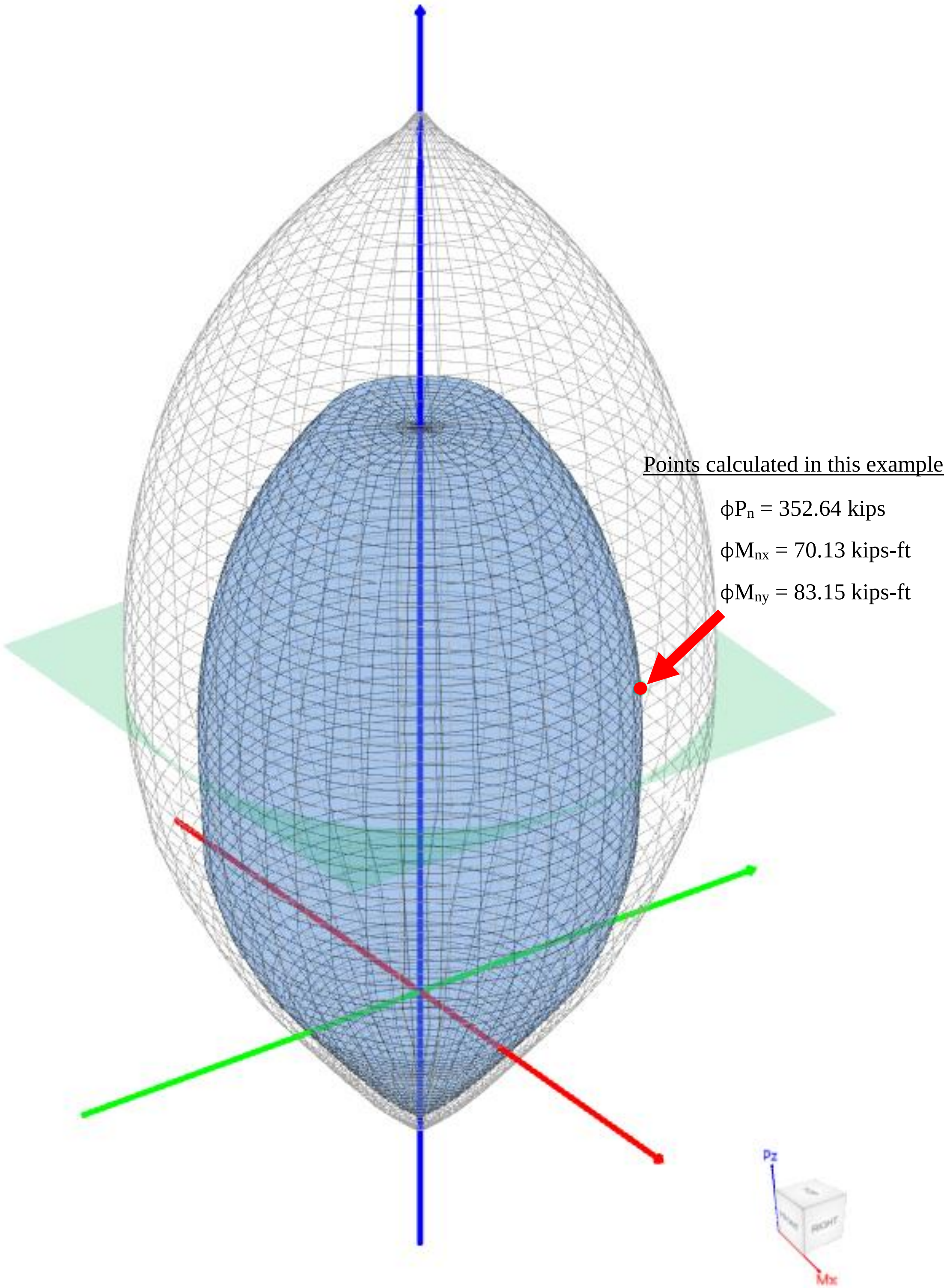



Figure 10 – Interaction Diagram in Two Directions (Biaxial) (spColumn)

The spColumn "Diagrams" module is a powerful tool especially for investigating interaction diagrams (failure surfaces) for columns and walls sections subjected to a combined axial force and biaxial bending moments. The module allows the user to view and analyze 2D interaction diagrams and contours along with 3D failure surfaces in a multi viewport environment. The following figure shows three views of:

1. P-M interaction diagram cut at angle of 50º
2. $M_x$-$M_y$ interaction diagram cut at axial load of 352.6 kip in compression
3. A 3D failure surface (interaction diagram) showing the points calculated in this example.

Figures 12 and 13 show 3D visualization of failure surface with a horizontal and vertical plane cut, respectively.

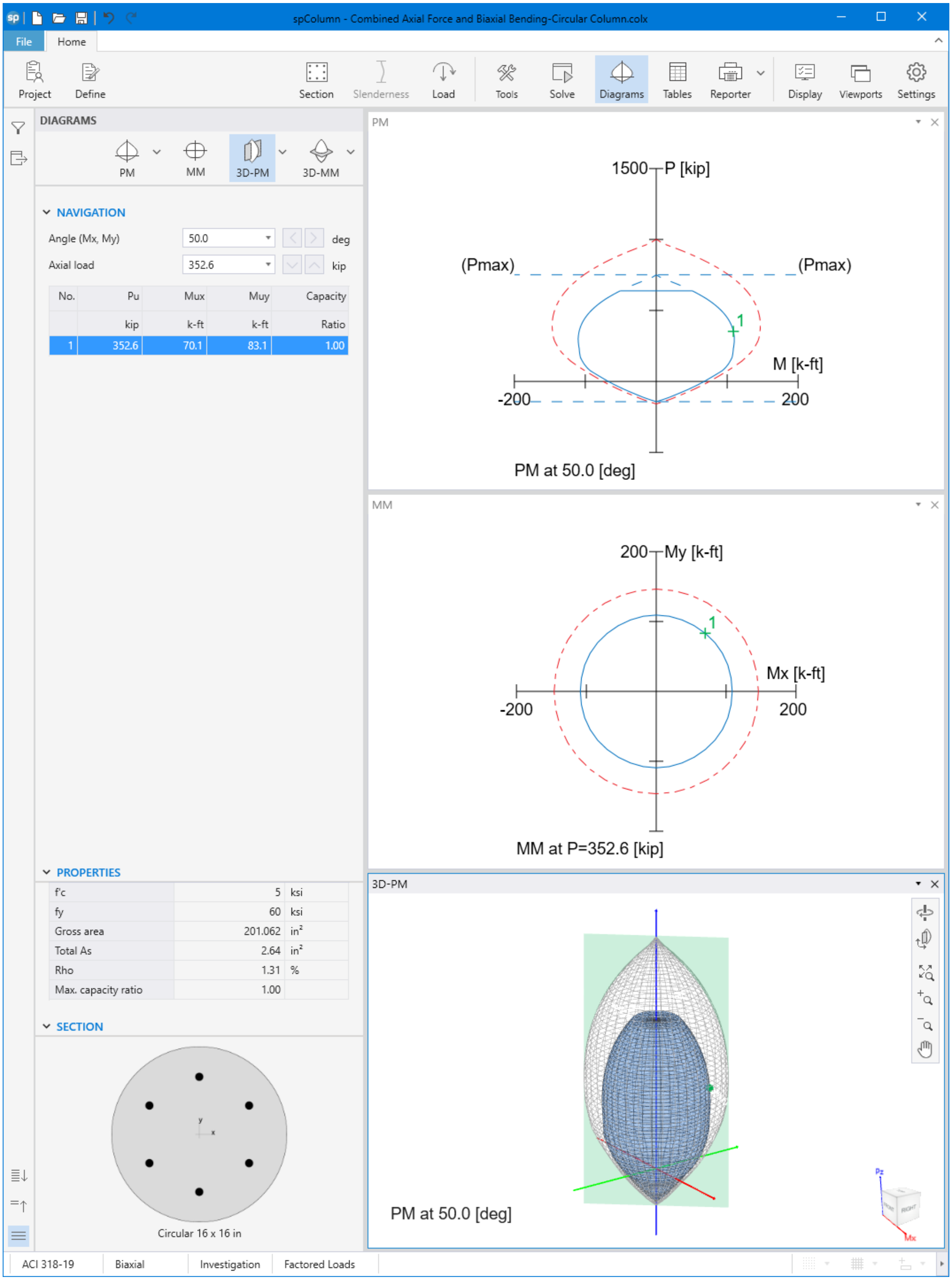



Figure 11 – Diagrams Module (spColumn)

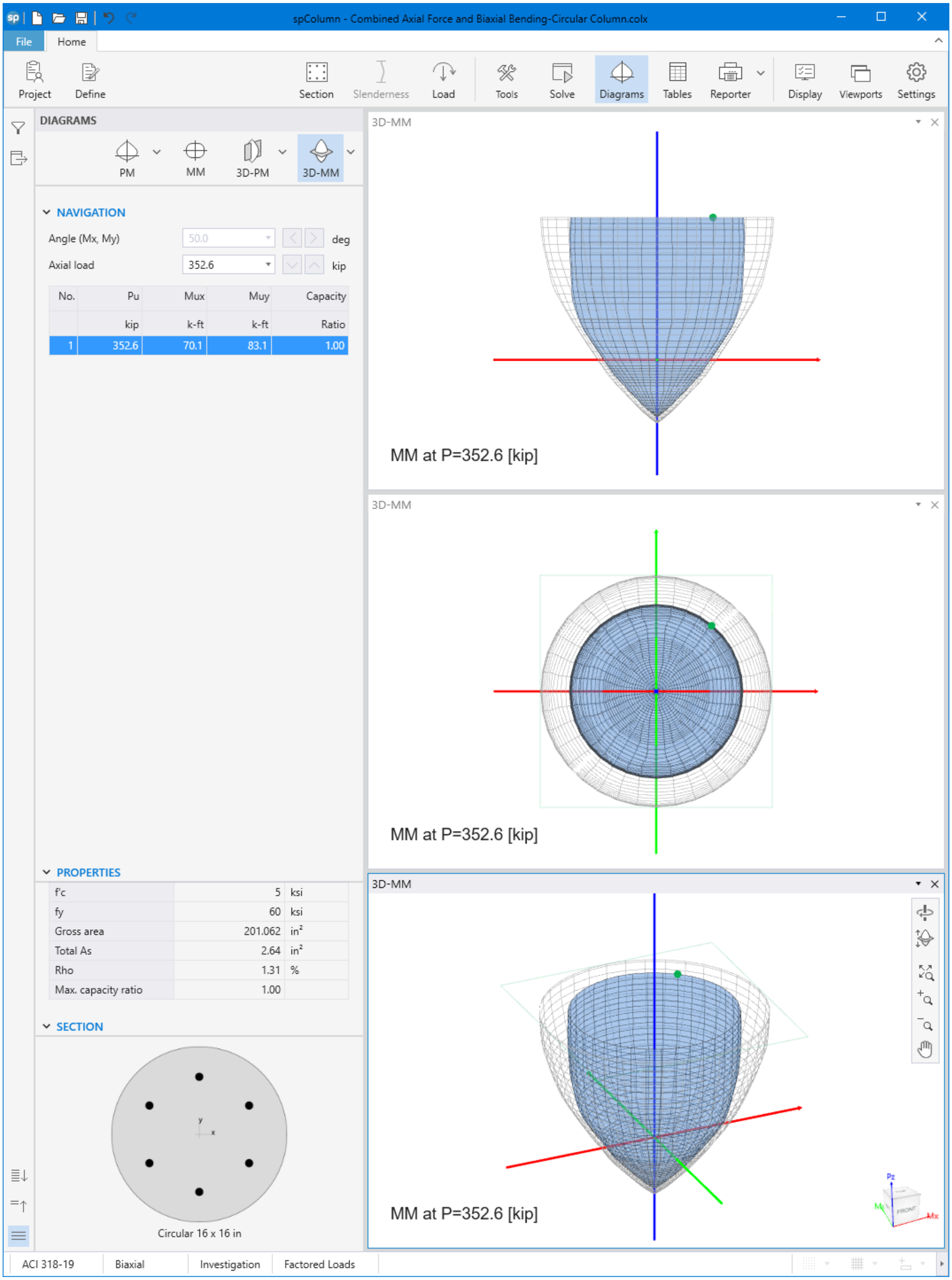



Figure 12 – 3D Visualization of Failure Surface with a Horizontal Plane Cut at P = 352.6 kip (spColumn)

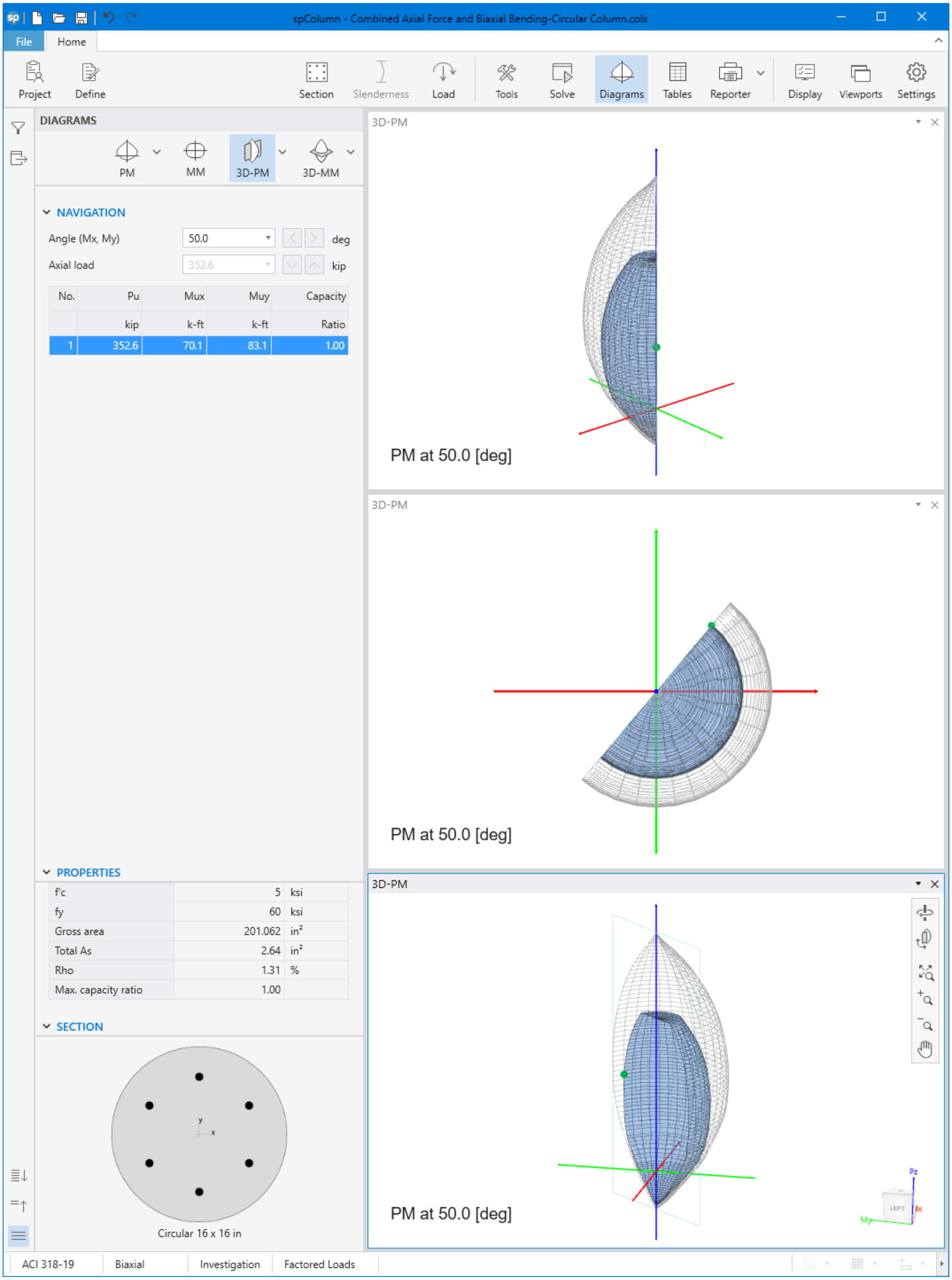



Figure 13 – 3D Visualization of Failure Surface with a Vertical Plane Cut at 50º (spColumn)