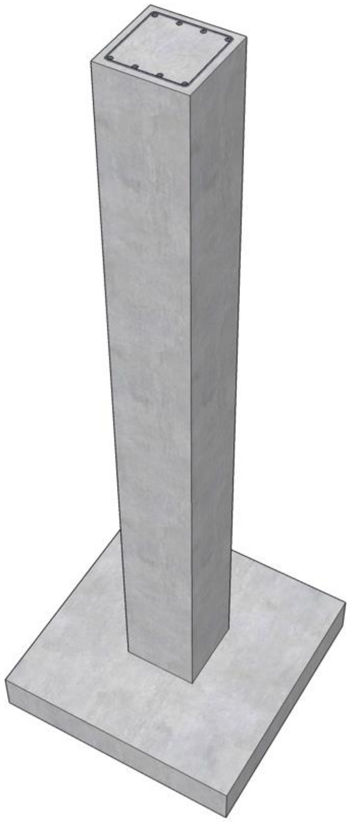
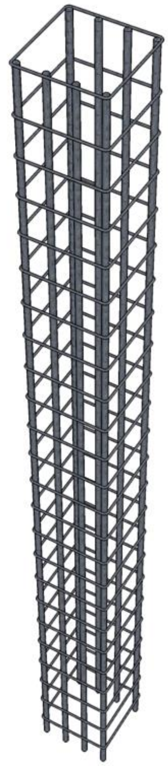
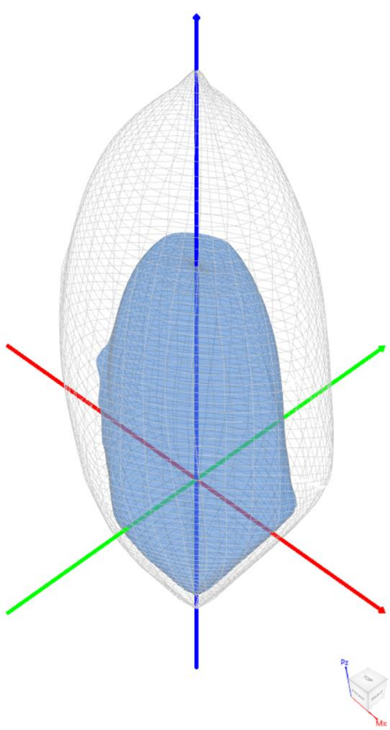
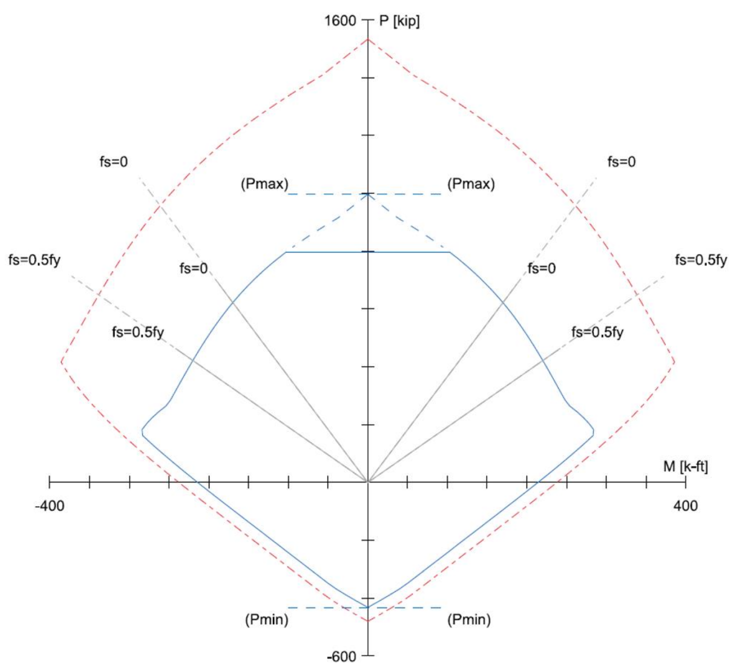
Interaction Diagram - Tied Reinforced Concrete Column Design Strength (ACI 318-19)









Interaction Diagram - Tied Reinforced Concrete Column Design Strength (ACI 318-19)

Develop an interaction diagram for the square tied concrete column shown in the figure below about the x-axis using ACI 318-19 provisions. Determine seven control points on the interaction diagram and compare the calculated values with the Reference and exact values from the complete interaction diagram generated by spColumn engineering software program from StructurePoint.

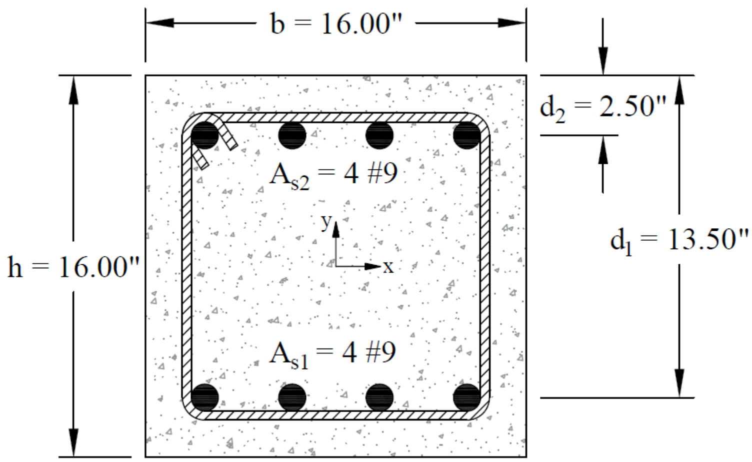


Figure 1 – Reinforced Concrete Column Cross-Section

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)

References

- Reinforced Concrete Mechanics and Design, 8th Edition, 2021, James Wight, Pearson, Example 11-1
- spColumn Engineering Software Program Manual v10.00, STRUCTUREPOINT, 2021
- "Interaction Diagram - Circular Spiral Reinforced Concrete Column (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Interaction Diagram - Tied Reinforced Concrete Column with High-Strength Reinforcing Bars (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Interaction Diagram - Barbell Concrete Shear Wall Unsymmetrical Boundary Elements (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Interaction Diagram – Building Elevator Reinforced Concrete Core Wall Design Strength (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022

Design Data

f_c' = 5000 psi

f_y = 60,000 psi

Cover = 2.5 in. to the center of the reinforcement

Column 16 in. x 16 in.

Top reinforcement = 4 #9

Bottom reinforcement = 4 #9

Solution

Use the traditional hand calculations approach to generate the interaction diagram for the concrete column section shown above by determining the following seven control points:

Point 1: Maximum compression

Point 2: Bar stress near tension face equal to zero, (f_s = 0)

Point 3: Bar stress near tension face equal to 0.5 f_y (f_s = 0.5 f_y)

Point 4: Bar stress near tension face equal to f_y (f_s = f_y)

Point 5: Bar strain near tension face equal to ε_y + 0.003

Point 6: Pure bending

Point 7: Maximum tension

Several terms are used to facilitate the following calculations:

P_o = nominal axial compressive strength, kip

ϕP_o = factored axial compressive strength, kip

$\phi P_{n,max}$ = maximum (allowable) factored axial compressive strength, kip

c = distance from the fiber of maximum compressive strain to the neutral axis, in.

a = depth of equivalent rectangular stress block, in.

C_c = compression force in equivalent rectangular stress block, kip

ε_s = strain value in reinforcement, in./in.

C_s = compression force in reinforcement, kip

T_s = tension force in reinforcement, kip

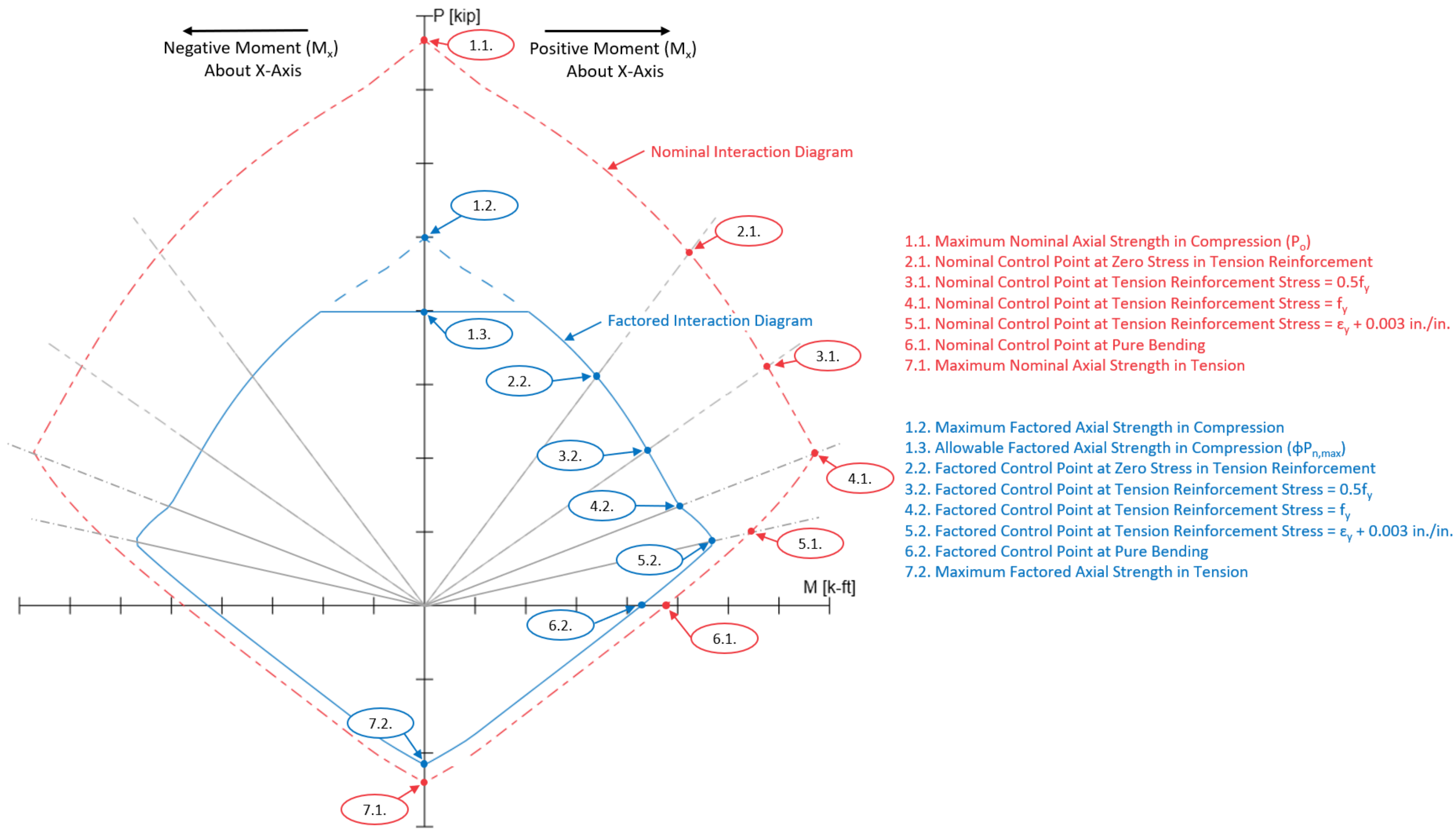


Figure 2 – Tied Column Section Interaction Diagram Control Points

1. Maximum Compression

1.1. Nominal axial compressive strength at zero eccentricity

$$P_o = 0.85 f_c'(A_g - A_{st}) + f_y A_{st}$$ ***ACI 318-19 (22.4.2.2)***

$$P_o = 0.85 \times 5000 \times (16 \times 16 - 8 \times 1.00) + 60000 \times 8 \times 1.00 = 1{,}534.0 \text{ kips}$$

1.2. Factored axial compressive strength at zero eccentricity

Since this column is a tied column with steel strain in compression:

$$\phi = 0.65$$ ***ACI 318-19 (Table 21.2.2)***

$$\phi P_o = 0.65 \times 1{,}534.0 = 997.1 \text{ kips}$$

Since the section is regular (symmetrical) about the x-axis, the moment capacity associated with the maximum axial compressive strength is equal to zero.

$$M_o = \phi M_o = 0.00 \text{ kip.ft}$$

1.3. Maximum (allowable) factored axial compressive strength

$$\phi P_{n,max} = 0.80 \times \phi P_o = 0.80 \times 997 = 797.7 \text{ kips}$$ ***ACI 318-19 (Table 22.4.2.1)***

2. Bar Stress Near Tension Face Equal to Zero, ($\varepsilon_s = f_s = 0$)

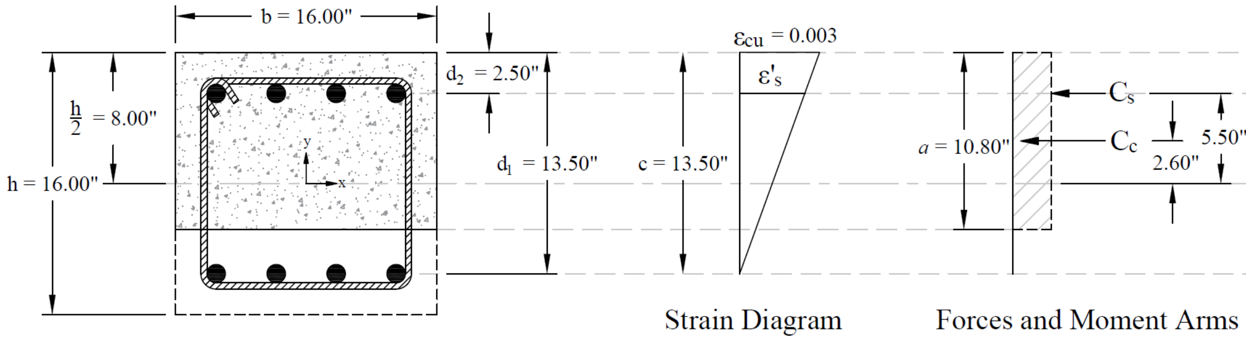


Figure 3 – Strains, Forces, and Moment Arms ($\varepsilon_t = f_s = 0$)

Strain ε_s is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices. ***ACI 318-19 (10.7.5.2.1 and 2)***

2.1. c, *a*, and strains in the reinforcement

$$c = d_1 = 13.5 \text{ in.}$$

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 13.5 = 10.80 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

a = Depth of equivalent rectangular stress block

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon_s = 0$$

$$\therefore \phi = 0.65$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

$$\varepsilon'_c = (c - d_2) \times \frac{\varepsilon_{cu}}{c} = (13.50 - 2.5) \times \frac{0.003}{13.50} = 0.00244 \text{ (Compression)} > \varepsilon_y = \frac{F_y}{E_s} = \frac{60}{29,000} = 0.00207$$

2.2. Forces in the concrete and steel

$$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 5{,}000 \times 10.80 \times 16 = 734.4 \text{ kip}$$ ***ACI 318-19 (22.2.2.4.1)***

$$f_s = 0 \text{ psi} \rightarrow \text{T}_s = f_s \times A_{s1} = 0 \text{ kip}$$

Since $\varepsilon_s' > \varepsilon_y \rightarrow$ compression reinforcement has yielded

$$\therefore f_s' = f_y = 60{,}000 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract $0.85 f_c'$ from f_s' before computing C_s:

$$\text{C}_s = (f_s' - 0.85 f_c') \times A_{s2} = (60{,}000 - 0.85 \times 5{,}000) \times 4 = 223 \text{ kip}$$

2.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 734.4 + 223 - 0 = 957.4 \text{ kip}$$

$$\phi P_n = 0.65 \times 957 = 622.3 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 734.4 \times \left(\frac{16}{2} - \frac{10.80}{2}\right) + 223 \times \left(\frac{16}{2} - 2.5\right) + 0 \times \left(13.50 - \frac{16}{2}\right) = 261.33 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 261 = 169.86 \text{ kip.ft}$$

3. Bar Stress Near Tension Face Equal to 0.5 f_y, (f_s = 0.5 f_y)

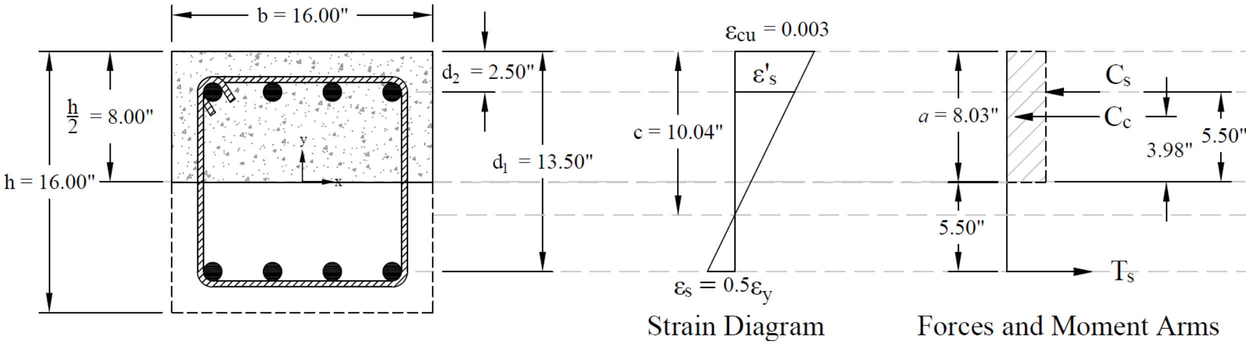


Figure 4 – Strains, Forces, and Moment Arms (f_s = 0.5 f_y)

3.1. c, *a*, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = \frac{\varepsilon_y}{2} = \frac{0.00207}{2} = 0.00103 < \varepsilon_y \rightarrow \text{tension reinforcement has not yielded}$$

$\therefore \phi = 0.65$ **_ACI 318-19 (Table 21.2.2)_**

$\varepsilon_{cu} = 0.003$ **_ACI 318-19 (22.2.2.1)_**

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00103 + 0.003} \times 0.003 = 10.04 \text{ in.}$$

Where *c* is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$a = \beta_1 \times c = 0.80 \times 10.04 = 8.03$ in. **_ACI 318-19 (22.2.2.4.1)_**

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ **_ACI 318-19 (Table 22.2.2.4.3)_**

$$\varepsilon_s' = (c - d_2) \times \frac{0.003}{c} = (10.04 - 2.5) \times \frac{0.003}{10.04} = 0.00225 \text{ (Compression)} > \varepsilon_y$$

3.2. Forces in the concrete and steel

$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 5,000 \times 8.03 \times 16 = 546.1$ kip **_ACI 318-19 (22.2.2.4.1)_**

$$f_s = \varepsilon_s \times E_s = 0.00103 \times 29,000,000 = 30,000 \text{ psi}$$

$$\text{T}_s = f_s \times A_{s1} = 30,000 \times 4 = 120 \text{ kip}$$

Since $\varepsilon_s' > \varepsilon_y \rightarrow$ compression reinforcement has yielded

$$\therefore f_s' = f_y = 60,000 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$$\text{C}_s = \left(f_s' - 0.85 f_c'\right) \times A_{s2} = \left(60,000 - 0.85 \times 5,000\right) \times 4 = 223 \text{ kip}$$

3.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 546.1 + 223 - 120 = 649.1 \text{ kip}$$

$$\phi P_n = 0.65 \times 649 = 421.9 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 546.1 \times \left(\frac{16}{2} - \frac{8.03}{2}\right) + 223 \times \left(\frac{16}{2} - 2.5\right) + 120 \times \left(13.50 - \frac{16}{2}\right) = 338.54 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 339 = 220.05 \text{ kip.ft}$$

4. Bar Stress Near Tension Face Equal to f_y, ($f_s = f_y$)

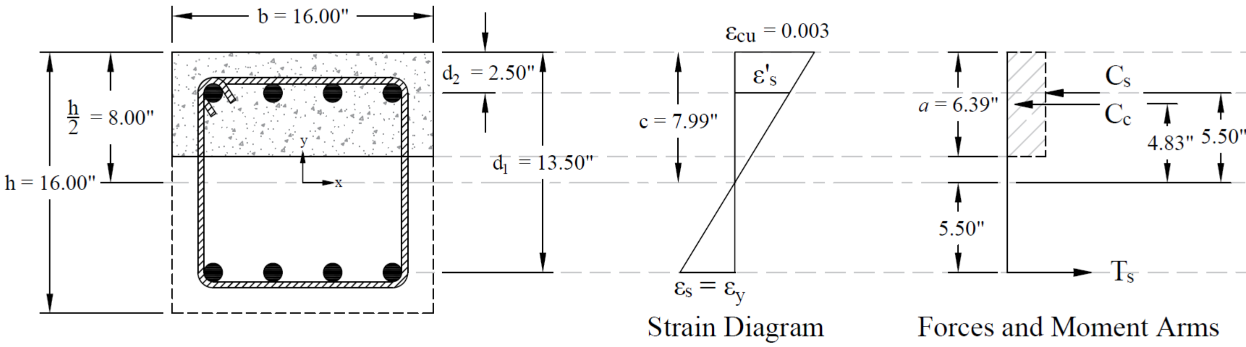


Figure 5 – Strains, Forces, and Moment Arms ($f_s = f_y$)

This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement. It also marks the start of the transition zone for ϕ for columns in which ϕ increases from 0.65 (or 0.75 for spiral columns) up to 0.90.

4.1. c, *a*, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = \varepsilon_y = 0.00207 \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.65$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00207 + 0.003} \times 0.003 = 7.99 \text{ in.}$$

Where *c* is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 7.99 = 6.39 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times \left(f_c' - 4000\right)}{1000} = 0.85 - \frac{0.05 \times \left(5000 - 4000\right)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon_s' = (c - d_2) \times \frac{0.003}{c} = (7.99 - 2.5) \times \frac{0.003}{7.99} = 0.00206 \text{ (Compression)} < \varepsilon_y$$

4.2. Forces in the concrete and steel

$$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 5{,}000 \times 6.39 \times 16 = 434.6 \text{ kip}$$ ***ACI 318-19 (22.2.2.4.1)***

$$f_s = f_y = 60{,}000 \text{ psi}$$

$$\text{T}_s = f_y \times A_{s1} = 60{,}000 \times 4 = 240 \text{ kip}$$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore\ f_s' = \varepsilon_s' \times E_s = 0.00206 \times 29{,}000{,}000 = 59{,}778 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (*ab*) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$$\text{C}_s = \left(f_s' - 0.85 f_c' \right) \times A_{s2} = \left(59{,}778 - 0.85 \times 5{,}000\right) \times 4 = 222.1 \text{ kip}$$

4.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 434.6 + 222.1 - 240 = 416.8 \text{ kip}$$

$$\phi P_n = 0.65 \times 417 = 270.9 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2} \right) + C_s \times \left(\frac{h}{2} - d_2 \right) + T_s \times \left(d_1 - \frac{h}{2} \right)$$

$$M_n = 434.6 \times \left(\frac{16}{2} - \frac{6.39}{2} \right) + 222.1 \times \left(\frac{16}{2} - 2.5 \right) + 240 \times \left(13.50 - \frac{16}{2} \right) = 385.81 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 386 = 250.77 \text{ kip.ft}$$

5. Bar Strain Near Tension Face Equal to ε_y + 0.003, (ε_s = 0.00507 in./in.)

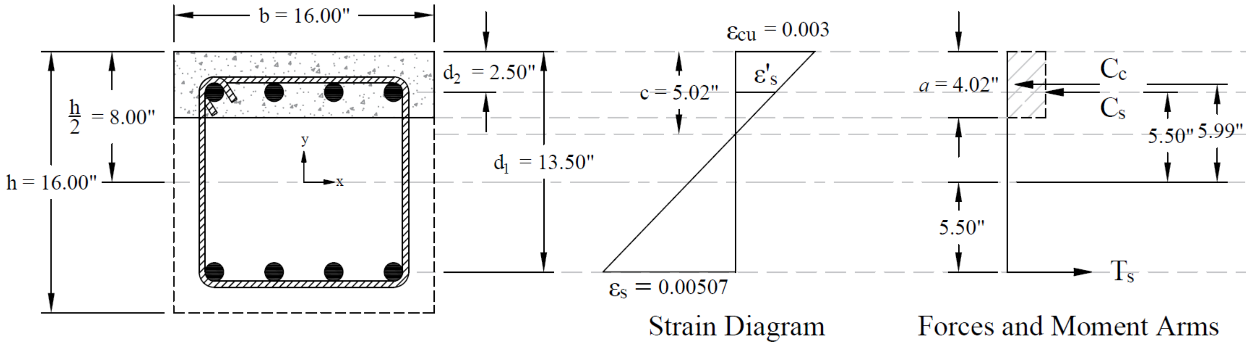


Figure 6 – Strains, Forces, and Moment Arms (ε_s = 0.00507 in./in.)

In ACI 318-19 provisions, this control point corresponds to the tension-controlled strain limit of ε_y + 0.003 (used to be 0.005 in ACI 318-14). It is the strain at the tensile limit of the transition zone for ϕ, used to define a tension-controlled section. Additional resources concerning code provision changes in ACI 318-19 can be found in "ACI 318-19 Code Revisions Impact on StructurePoint Software" technical article.

5.1. c, a, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = \varepsilon_y + 0.003 = 0.00207 + 0.003 = 0.00507 > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.9$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00507 + 0.003} \times 0.003 = 5.02 \text{ in.}$$

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 5.02 = 4.02 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon'_s = (c - d_2) \times \frac{0.003}{c} = (5.02 - 2.5) \times \frac{0.003}{5.02} = 0.00151 \text{ (Compression)} < \varepsilon_y$$

5.2. Forces in the concrete and steel

$$C_c = 0.85\times f_c' \times a \times b = 0.85\times 5,000\times 4.02\times 16 = 273.1 \text{ kip}$$ ***ACI 318-19 (22.2.2.4.1)***

$$f_s = f_y = 60,000 \text{ psi}$$

$$\text{T}_s = f_y \times A_{s1} = 60,000\times 4 = 240 \text{ kip}$$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore\ f_s' = \varepsilon_s' \times E_s = 0.00151\times 29,000,000 = 43,667 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$$\text{C}_s = (f_s' - 0.85 f_c')\times A_{s2} = (43,667 - 0.85\times 5,000)\times 4 = 157.7 \text{ kip}$$

5.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 273.1 + 157.7 - 240 = 190.7 \text{ kip}$$

$$\phi P_n = 0.90\times 191 = 171.6 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 273.1\times\left(\frac{16}{2} - \frac{4.02}{2}\right) + 157.7\times\left(\frac{16}{2} - 2.5\right) + 240\times\left(13.50 - \frac{16}{2}\right) = 318.61 \text{ kip.ft}$$

$$\phi M_n = 0.90\times 320 = 286.75 \text{ kip.ft}$$

6. Pure Bending

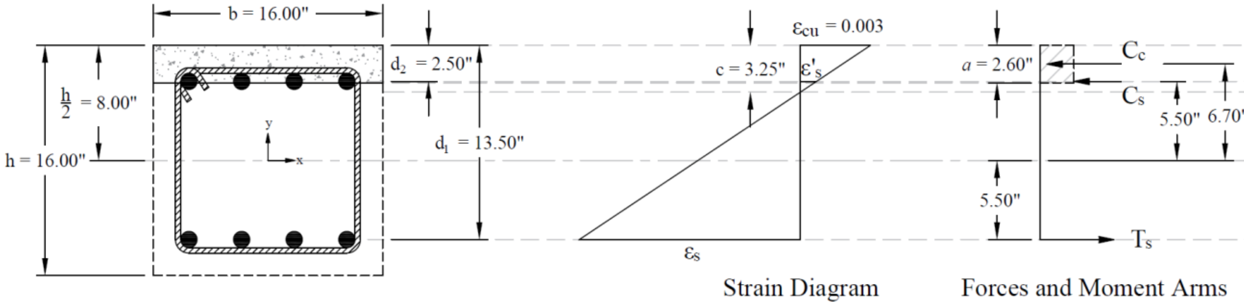


Figure 7 – Strains, Forces, and Moment Arms (Pure Moment)

This corresponds to the case where the nominal axial load capacity, P_n, is equal to zero. Iterative procedure is used to determine the nominal moment capacity as follows:

6.1. c, *a*, and strains in the reinforcement

Try $c = 3.25$ in.

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 3.25 = 2.60 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = (d_1 - c) \times \frac{0.003}{c} = (13.50 - 3.25) \times \frac{0.003}{3.25} = 0.00946 \text{ (Tension)} > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.9$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_s' = (c - d_2) \times \frac{0.003}{c} = (3.25 - 2.5) \times \frac{0.003}{3.25} = 0.00069 \text{ (Compression)} < \varepsilon_y$$

6.2. Forces in the concrete and steel

$$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 5,000 \times 2.6 \times 16 = 176.8 \text{ kip}$$ ***ACI 318-19 (22.2.2.4.1)***

$$f_s = f_y = 60,000 \text{ psi}$$

$$\text{T}_s = f_y \times A_{s1} = 60,000 \times 4 = 240 \text{ kip}$$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore\ f_s' = \varepsilon_s' \times E_s = 0.00069 \times 29,000,000 = 20,077 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (*ab*) used to compute C_c. As a result, it is necessary to subtract 0.85f_c' from f_s' before computing C_s:

$$\text{C}_s = \left(f_s' - 0.85 f_c' \right) \times A_{s2} = \left(20,077 - 0.85 \times 5,000\right) \times 4 = 63.3 \text{ kip}$$

6.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 176.8 + 63.3 - 240 \approx 0 \text{ kip} \rightarrow \phi P_n \approx 0 \text{ kip}$$

The assumption that c = 3.25 in. is correct

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 176.8 \times \left(\frac{16}{2} - \frac{2.60}{2}\right) + 63.3 \times \left(\frac{16}{2} - 2.5\right) + 240 \times \left(13.50 - \frac{16}{2}\right) = 237.73 \text{ kip.ft}$$

$$\phi M_n = 0.90 \times 238 = 213.96 \text{ kip.ft}$$

7. Maximum Tension

The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.

7.1. P_{nt} and ϕP_{nt}

$$P_{nt} = f_y \times (A_{s1} + A_{s2}) = 60{,}000 \times (4+4) = 480.0 \text{ kip}$$ ***ACI 318-19 (22.4.3.1)***

$$\phi = 0.9$$ ***ACI 318-19 (Table 21.2.2)***

$$\phi P_{nt} = 0.90 \times 480.0 = 432.0 \text{ kip}$$

7.2. M_n and ϕM_n

Since the section is symmetrical

$$M_n = \phi M_n = 0.00 \text{ kip.ft}$$

8. Column Interaction Diagram - spColumn Software

spColumn is a StructurePoint software program that performs the analysis and design of reinforced concrete sections subjected to axial force combined with uniaxial or biaxial bending. Using the provisions of the Strength Design Method and Unified Design Provisions, slenderness considerations are used for moment magnification due to second order effect (P-Delta) for sway and non-sway frames.

For this column section, investigation mode was used with no loads (the program will only report control points) and no slenderness considerations using ACI 318-19.

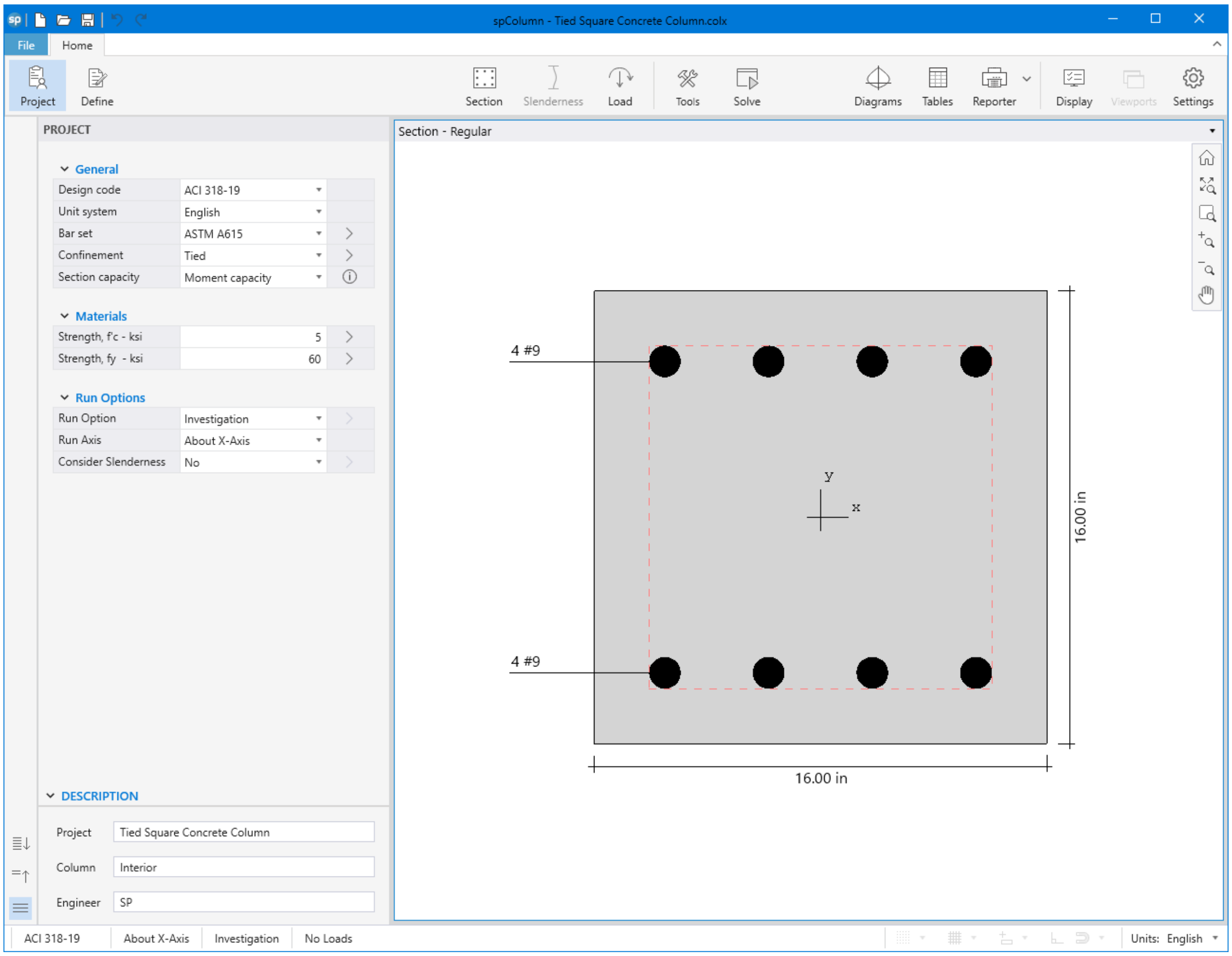


Figure 8 – spColumn Interface

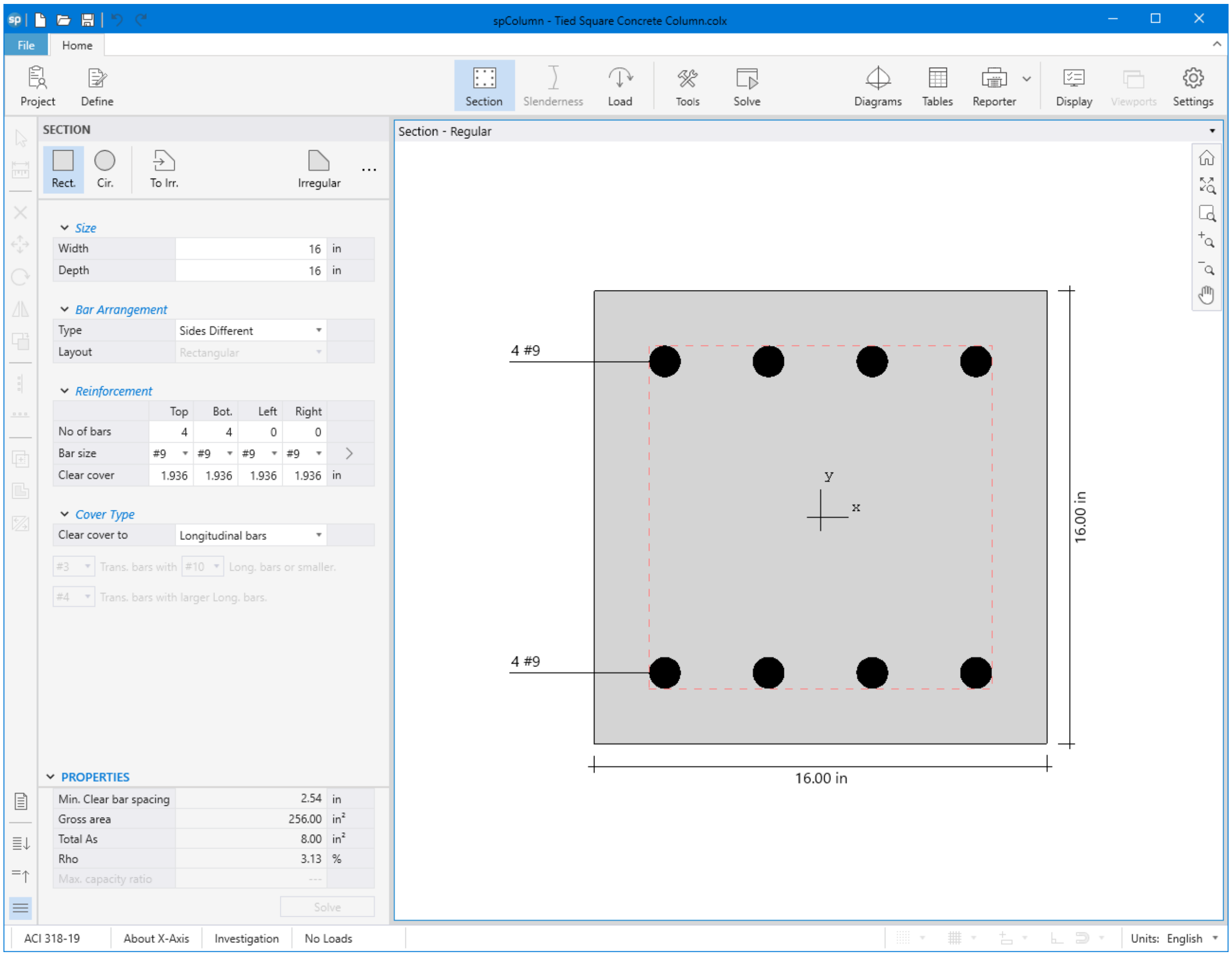


Figure 9 – spColumn Model Editor

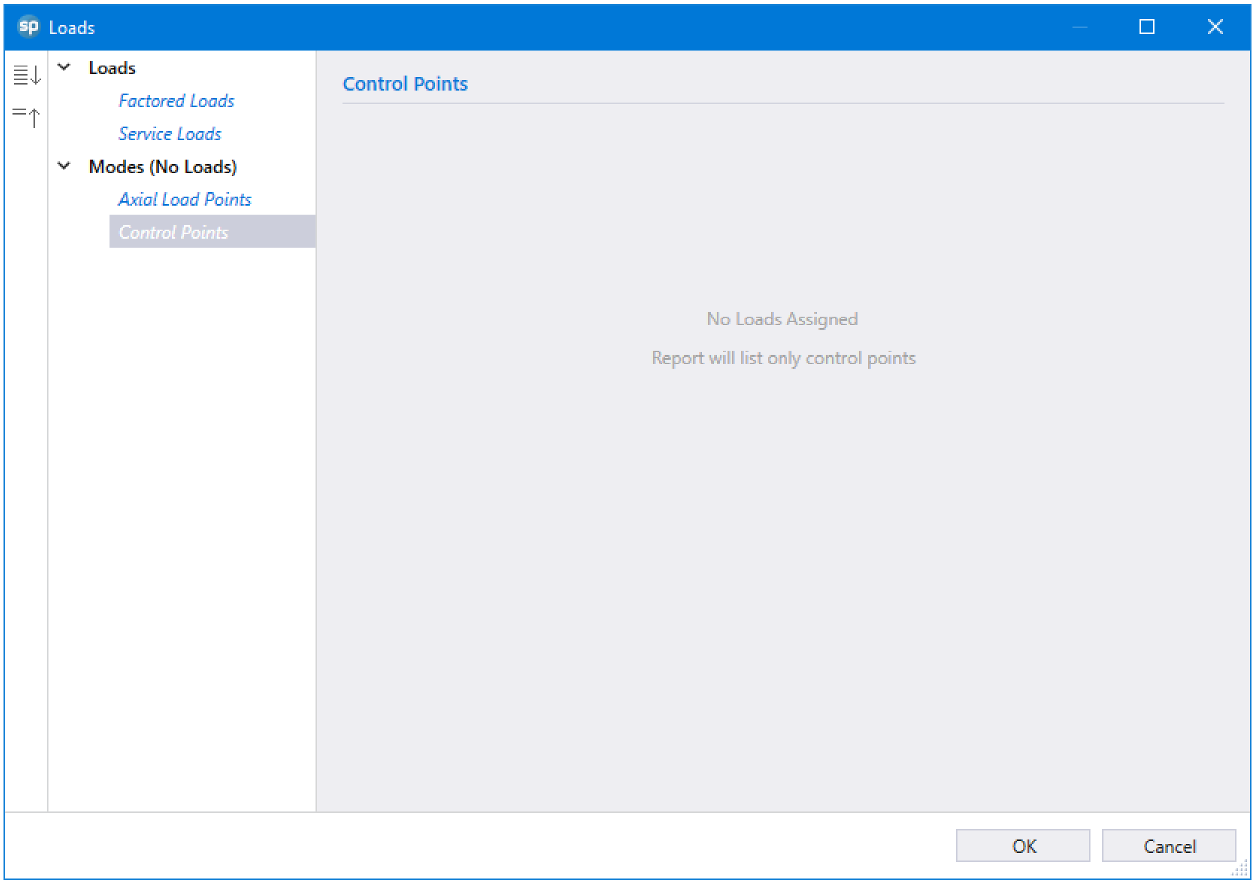


Figure 10 – Defining Loads / Modes (spColumn)

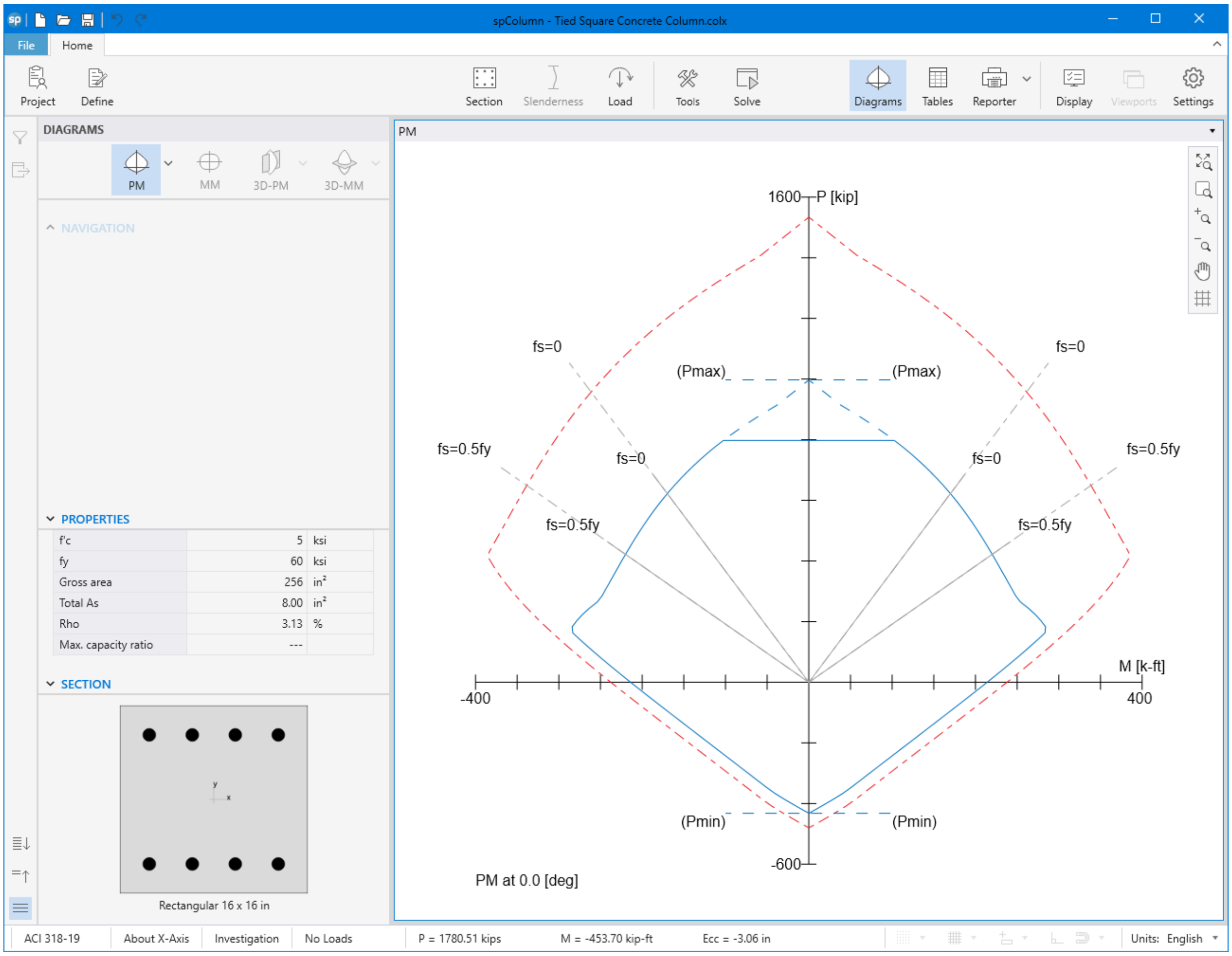
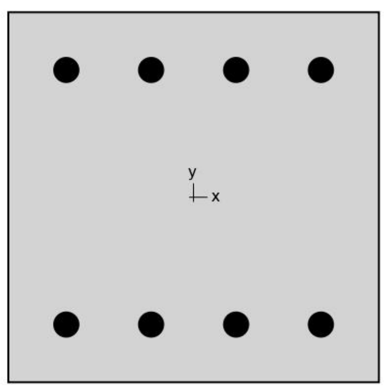


Figure 11 – Column P-M Interaction Diagram about the X-Axis (spColumn)

spColumn v10.00 (TM)
Computer program for the Strength Design of Reinforced Concrete Sections



Contents

List of Figures

1. General Information

File Name	E:\StructureP...\Tied Square Concrete Column.colx
Project	Tied Square Concrete Column
Column	Interior
Engineer	SP
Code	ACI 318-19
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	X - axis
Slenderness	Not Considered
Column Type	Structural
Capacity Method	Moment capacity

2. Material Properties

2.1. Concrete

Type	Standard	
f'_c	5	ksi
E_c	4030.51	ksi
f_c	4.25	ksi
ε_u	0.003	in/in
β_1	0.8	

2.2. Steel

Type	Standard	
f_y	60	ksi
E_s	29000	ksi
ε_{ty}	0.00206897	in/in

3. Section

3.1. Shape and Properties

Type	Rectangular	
Width	16	in
Depth	16	in
A_g	256	in^2
I_x	5461.33	in^4
I_y	5461.33	in^4
r_x	4.6188	in
r_y	4.6188	in
X_o	0	in
Y_o	0	in

3.2. Section Figure

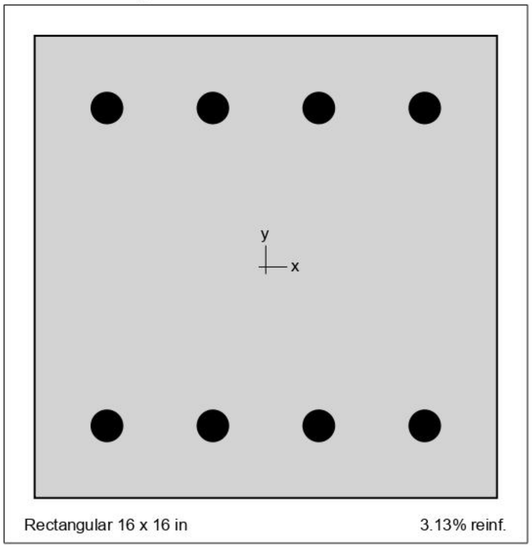


Figure 1: Column section

4. Reinforcement

4.1. Bar Set: ASTM A615

Bar	Diameter	Area	Bar	Diameter	Area	Bar	Diameter	Area
	in	in²		in	in²		in	in²
#3	0.38	0.11	#4	0.50	0.20	#5	0.63	0.31
#6	0.75	0.44	#7	0.88	0.60	#8	1.00	0.79
#9	1.13	1.00	#10	1.27	1.27	#11	1.41	1.56
#14	1.69	2.25	#18	2.26	4.00			

4.2. Confinement and Factors

Confinement type	Tied
For #10 bars or less	#3 ties
For larger bars	#4 ties
Capacity Reduction Factors	
Axial compression, (a)	0.8
Tension controlled ϕ, (b)	0.9
Compression controlled ϕ, (c)	0.65

4.3. Arrangement

Pattern	Sides different
Bar layout	Rectangular
Cover to	Longitudal bars
Clear cover	---
Bars	---

Total steel area, A_s	8.00 in²
Rho	3.13 %
Minimum clear spacing	2.54 in

4.4. Bars Provided

		Bars	Clear cover
			in
Top	4	#9	1.936
Bottom	4	#9	1.936
Left	0	#9	1.936
Right	0	#9	1.936

5. Control Points

About	Point	P	X-Moment	Y-Moment	NA Depth	d_t Depth	ε_t	ϕ
		kip	k-ft	k-ft	in	in		
X	@ Max compression	997.1	0.00	0.00	43.50	13.50	-0.00207	0.65000
X	@ Allowable comp.	797.7	102.64	0.00	17.35	13.50	-0.00067	0.65000
X	@ f_s = 0.0	622.3	169.86	0.00	13.50	13.50	0.00000	0.65000
X	@ f_s = 0.5 f_y	421.9	220.05	0.00	10.04	13.50	0.00103	0.65000
X	@ Balanced point	270.9	250.77	0.00	7.99	13.50	0.00207	0.65000
X	@ Tension control	171.6	286.75	0.00	5.02	13.50	0.00507	0.90000
X	@ Pure bending	0.0	213.91	0.00	3.25	13.50	0.00946	0.90000
X	@ Max tension	-432.0	0.00	0.00	0.00	13.50	9.99999	0.90000
-X	@ Max compression	997.1	0.00	0.00	43.50	13.50	-0.00207	0.65000
-X	@ Allowable comp.	797.7	-102.64	0.00	17.35	13.50	-0.00067	0.65000
-X	@ f_s = 0.0	622.3	-169.86	0.00	13.50	13.50	0.00000	0.65000
-X	@ f_s = 0.5 f_y	421.9	-220.05	0.00	10.04	13.50	0.00103	0.65000
-X	@ Balanced point	270.9	-250.77	0.00	7.99	13.50	0.00207	0.65000
-X	@ Tension control	171.6	-286.75	0.00	5.02	13.50	0.00507	0.90000
-X	@ Pure bending	0.0	-213.91	0.00	3.25	13.50	0.00946	0.90000
-X	@ Max tension	-432.0	0.00	0.00	0.00	13.50	9.99999	0.90000

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E:\StructurePoint\spColumn\Tied Square Concrete Column.colx

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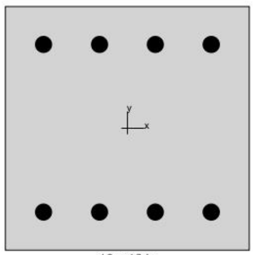
4:28 PM

6. Diagrams

6.1. PM at θ=0 [deg]



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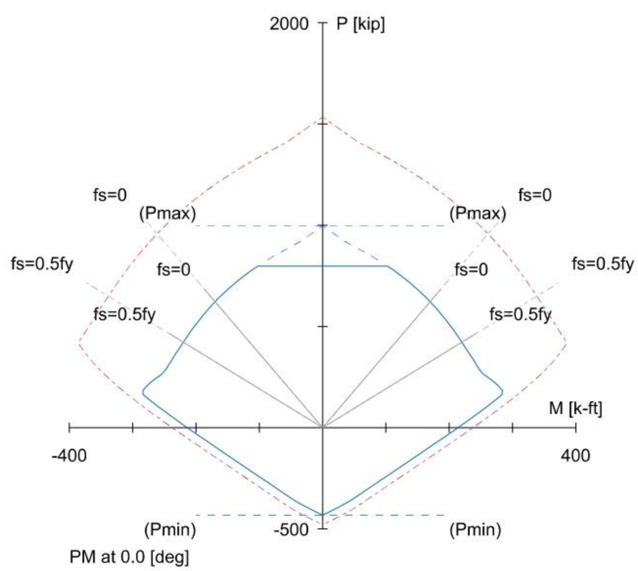
Section	
Type	Rectangular
Width	16 in
Depth	16 in
A_g	256 in^2
I_x	5461.33 in^4
I_y	5461.33 in^4
Reinforcement	
Pattern	Sides different
Bar layout	Rectangular
Cover to	Longitudinal bars
Clear cover	---
Bars	---
Confinement type	Tied
Total steel area, A_s	8.00 in^2
Rho	3.13 %
Min. clear spacing	2.54 in



9. Summary and Comparison of Design Results

Table 1 - Comparison of Results						
Support	ϕP_n, kip			ϕM_n, kip.ft		
	Reference	Hand	spColumn	Reference	Hand	spColumn
Max compression	997	997.1	997.1	0	0.00	0.00
Allowable compression	798	797.7	797.7	---	---	---
$f_s = 0.0$	622	622.3	622.3	170	169.86	169.86
$f_s = 0.5\ f_y$	422	421.9	421.9	220	220.05	220.05
Balanced point	271	270.9	270.9	251	250.77	250.77
Tension control	170*	171.6	171.6	286*	286.75	286.75
Pure bending	0	0.0	0.0	214	213.96	213.91
Max tension	432	432.0	432.0	0	0.00	0.00
* Reference rounded the strain value from 0.00507 to 0.0051.						

In all of the hand calculations and the reference used illustrated above, the results are in precise agreement with the automated exact results obtained from the spColumn program.

10. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In the calculation shown above a P-M interaction diagram was generated with moments about the X-Axis (Uniaxial bending). Since the reinforcement in the section is not symmetrical, a different P-M interaction diagram is needed for the other orthogonal direction about the Y-Axis (See the following Figure for the case where f_s = f_y).

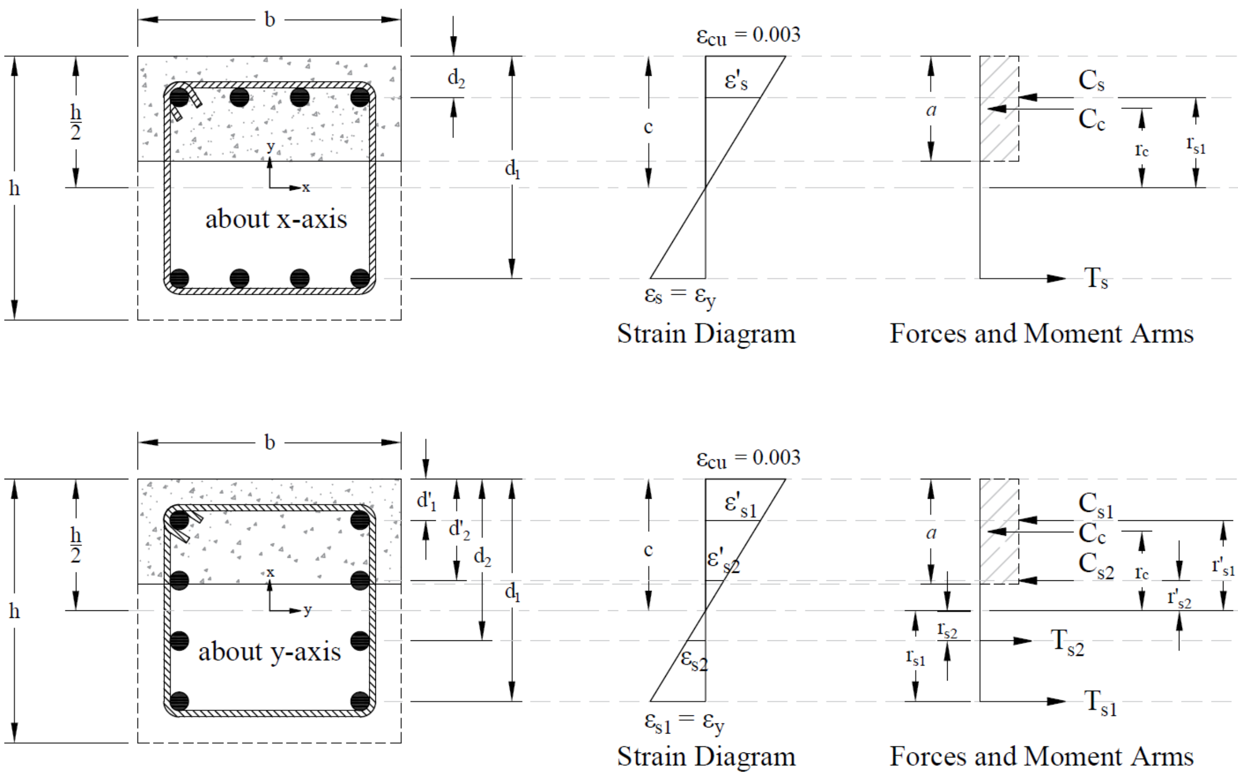


Figure 12 – Strains, Forces, and Moment Arms ($f_s = f_y$ Moments About x- and y-axis)

When running about the Y-Axis, we have 2 bars in 4 layers instead of 4 bars in just 2 layers (about X-Axis) resulting in a completely different interaction diagram as shown in the following Figure. Further differences in the interaction diagram in both directions can result if the column cross section geometry is irregular.

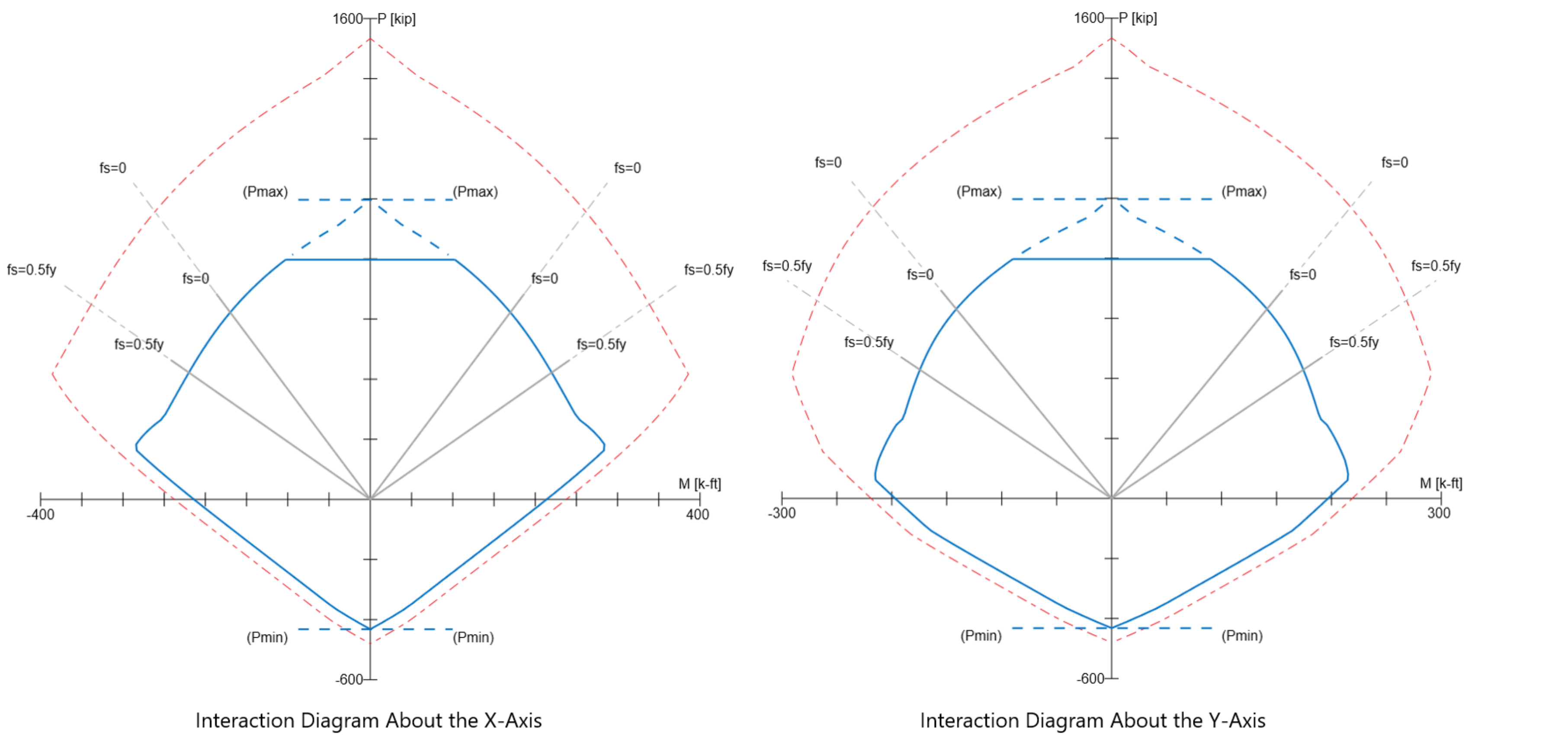


Figure 13 – Comparison of Column Interaction Diagrams about X-Axis and Y-Axis (spColumn)

In most building design calculations, such as the examples shown for flat plate or flat slab concrete floor systems, all building columns are subjected to M_x and M_y due to lateral forces and unbalanced moments from both directions of analysis. This requires an evaluation of the column P-M interaction diagram in two directions simultaneously (biaxial bending).

StucturePoint's spColumn program can also investigate column and wall sections in biaxial mode to produce the results shown in the following Figure for the column section in this example. In biaxial run mode, M_x and M_y diagrams at each axial force level can be viewed in 2D and 3D views.

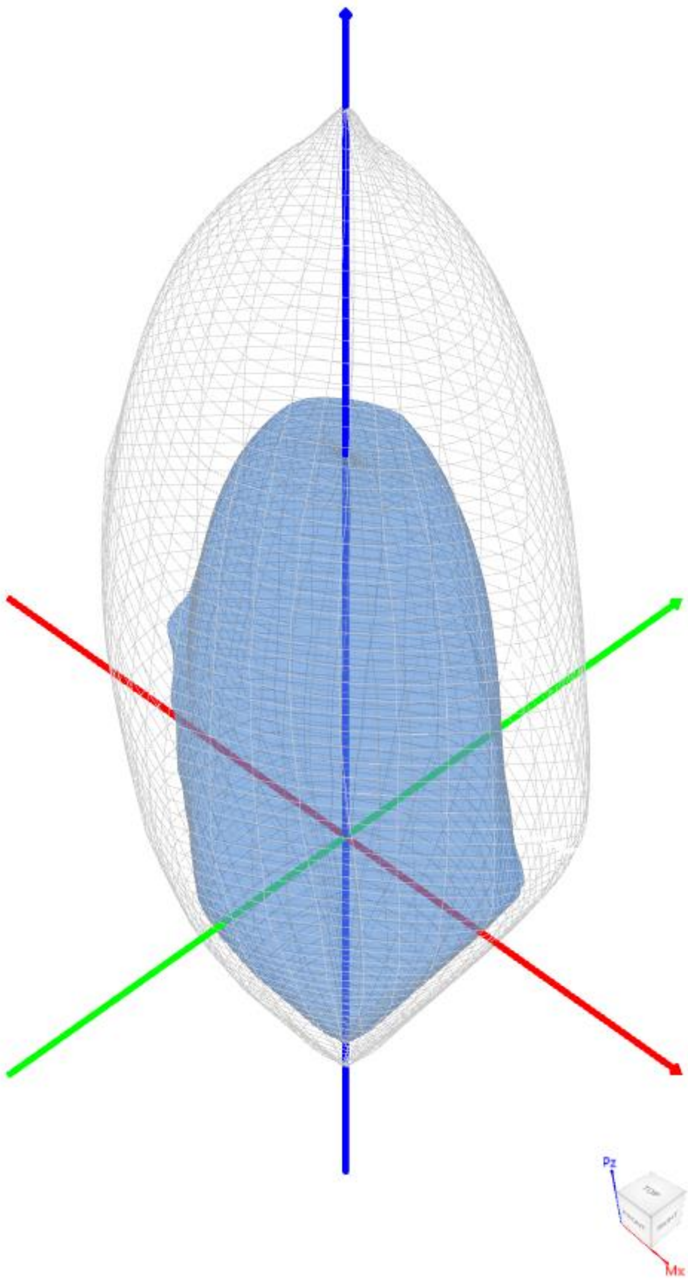


Figure 14 – Nominal & Design 3D Failure Surfaces (Biaxial) (spColumn)

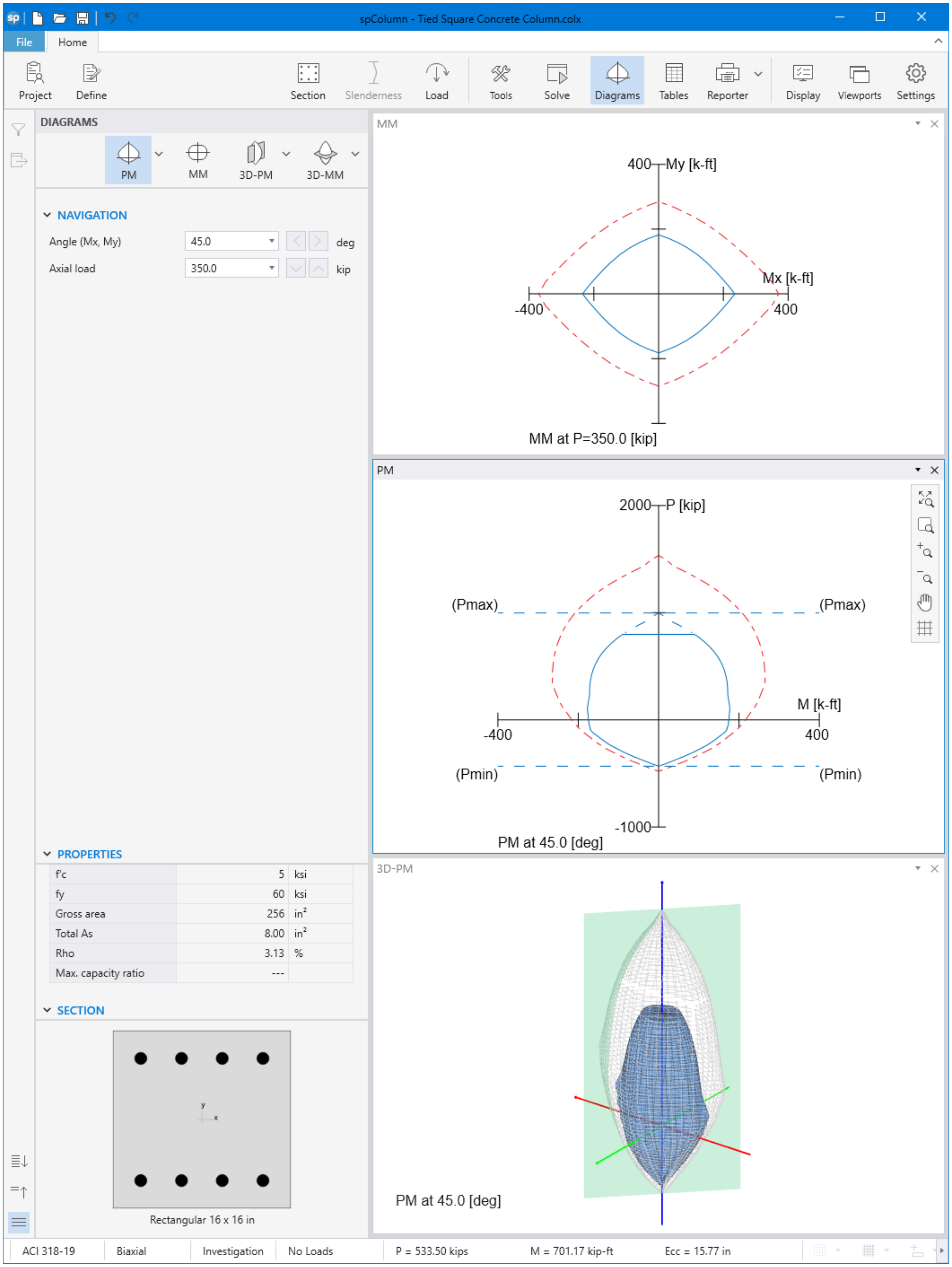


Figure 15 – Tied Column Interaction Diagram and 3D failure Surface Viewer (spColumn)

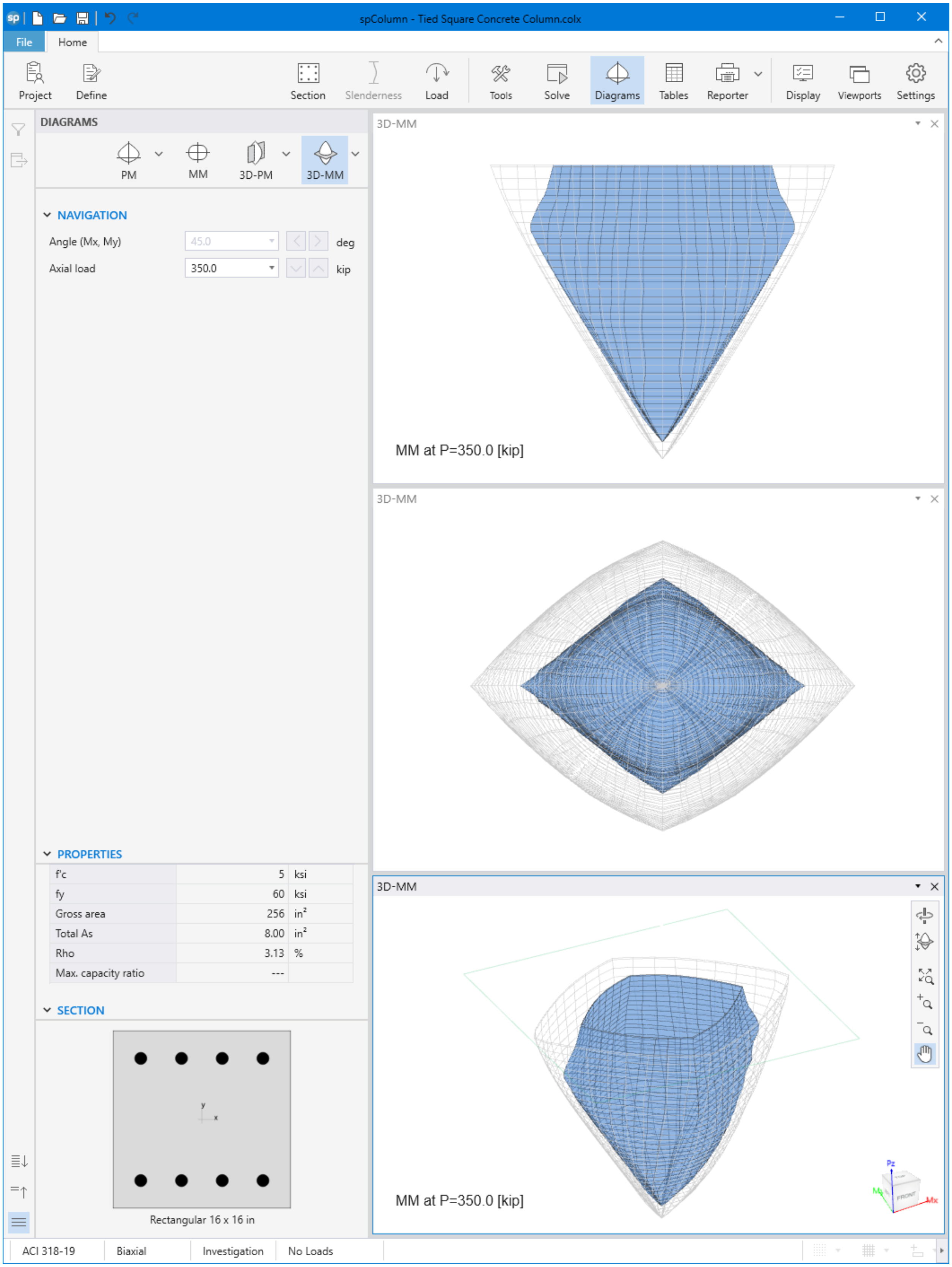


Figure 16 – Tied Column 3D Failure Surface with a Horizontal Plane Cut at P = 350 kip (spColumn)

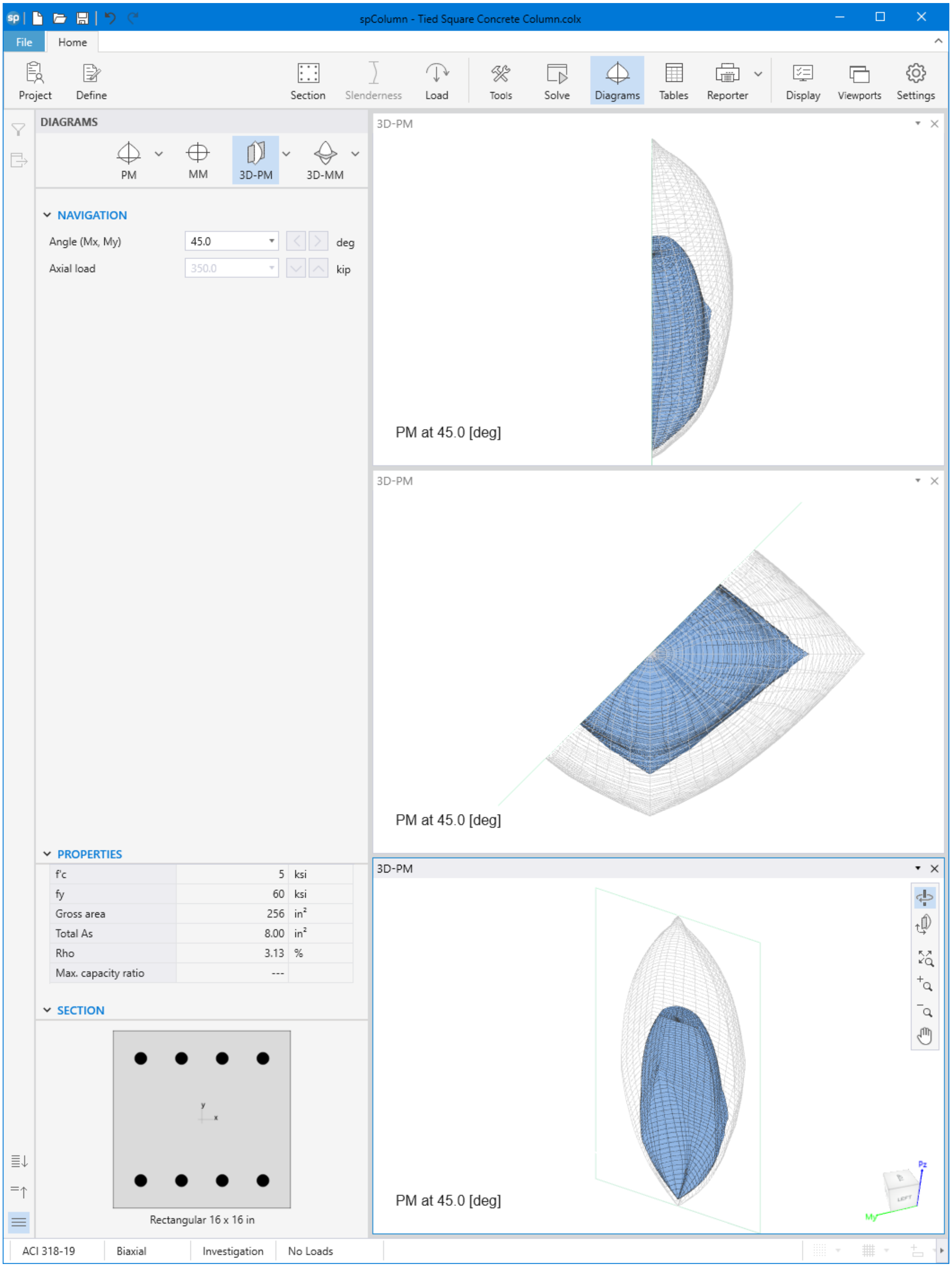


Figure 17 – Tied Column 3D Failure Surface with a Vertical Plane Cut at 45° (spColumn)