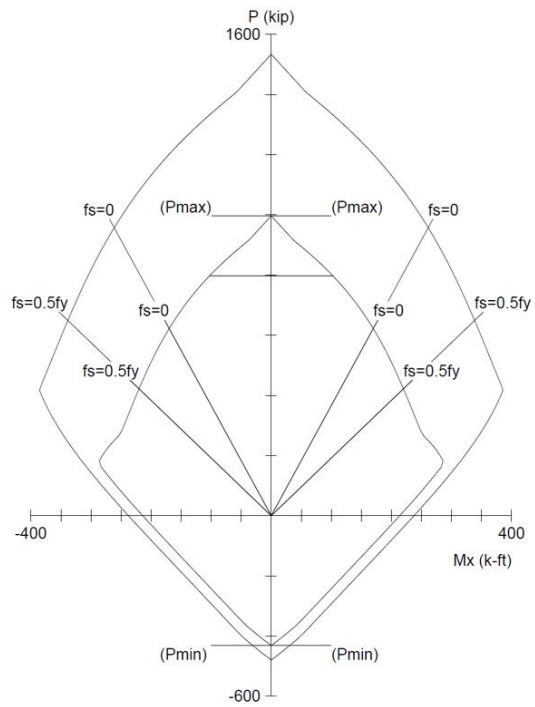
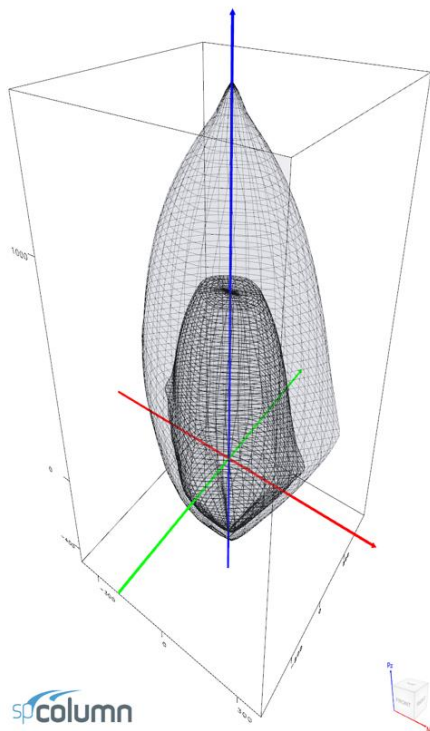
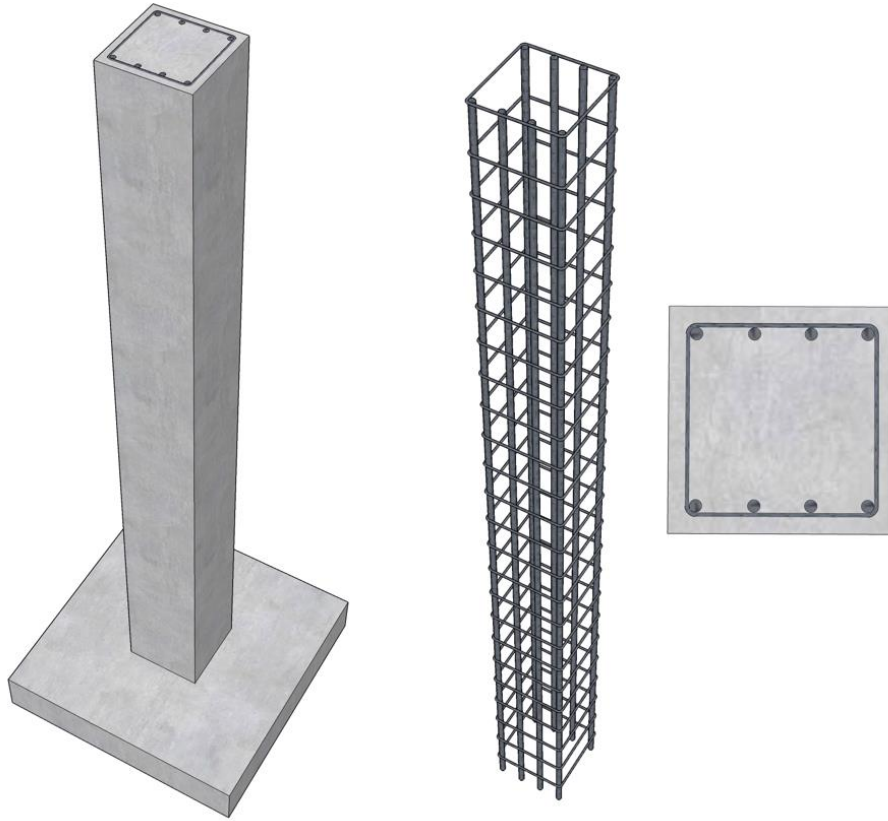
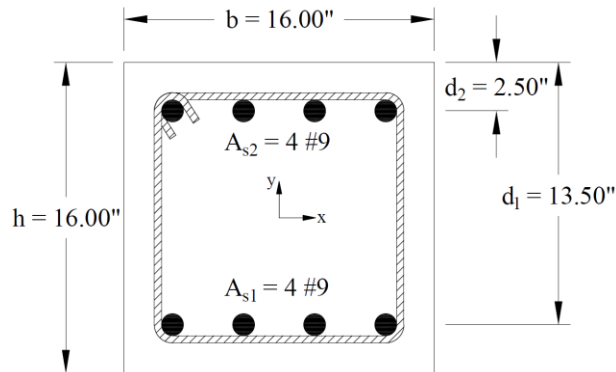


**Interaction Diagram - Tied Reinforced Concrete Column**



### Interaction Diagram - Tied Reinforced Concrete Column

Develop an interaction diagram for the square tied concrete column shown in the figure below about the x-axis. Determine seven control points on the interaction diagram and compare the calculated values in the Reference and with exact values from the complete interaction diagram generated by [spColumn](#) engineering software program from [StructurePoint](#).



**Figure 1 – Reinforced Concrete Column Cross-Section**

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### Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

### Reference

Reinforced Concrete Mechanics and Design, 6<sup>th</sup> Edition, 2011, James Wight and James MacGregor, Pearson

### Design Data

$$f_c' = 5000 \text{ psi}$$

$$f_y = 60,000 \text{ psi}$$

Cover = 2.5 in. to the center of the reinforcement

Column 16 in. x 16 in.

Top reinforcement = 4 #9

Bottom reinforcement = 4 #9

### Solution

Use the traditional hand calculations approach to generate the interaction diagram for the concrete column section shown above by determining the following seven control points:

Point 1: Pure compression

Point 2: Bar stress near tension face of member equal to zero, ( $f_s = 0$ )

Point 3: Bar stress near tension face of member equal to  $0.5 f_y$  ( $f_s = -0.5 f_y$ )

Point 4: Bar stress near tension face of member equal to  $f_y$  ( $f_s = -f_y$ )

Point 5: Bar strain near tension face of member equal to 0.005

Point 6: Pure bending

Point 7: Pure tension

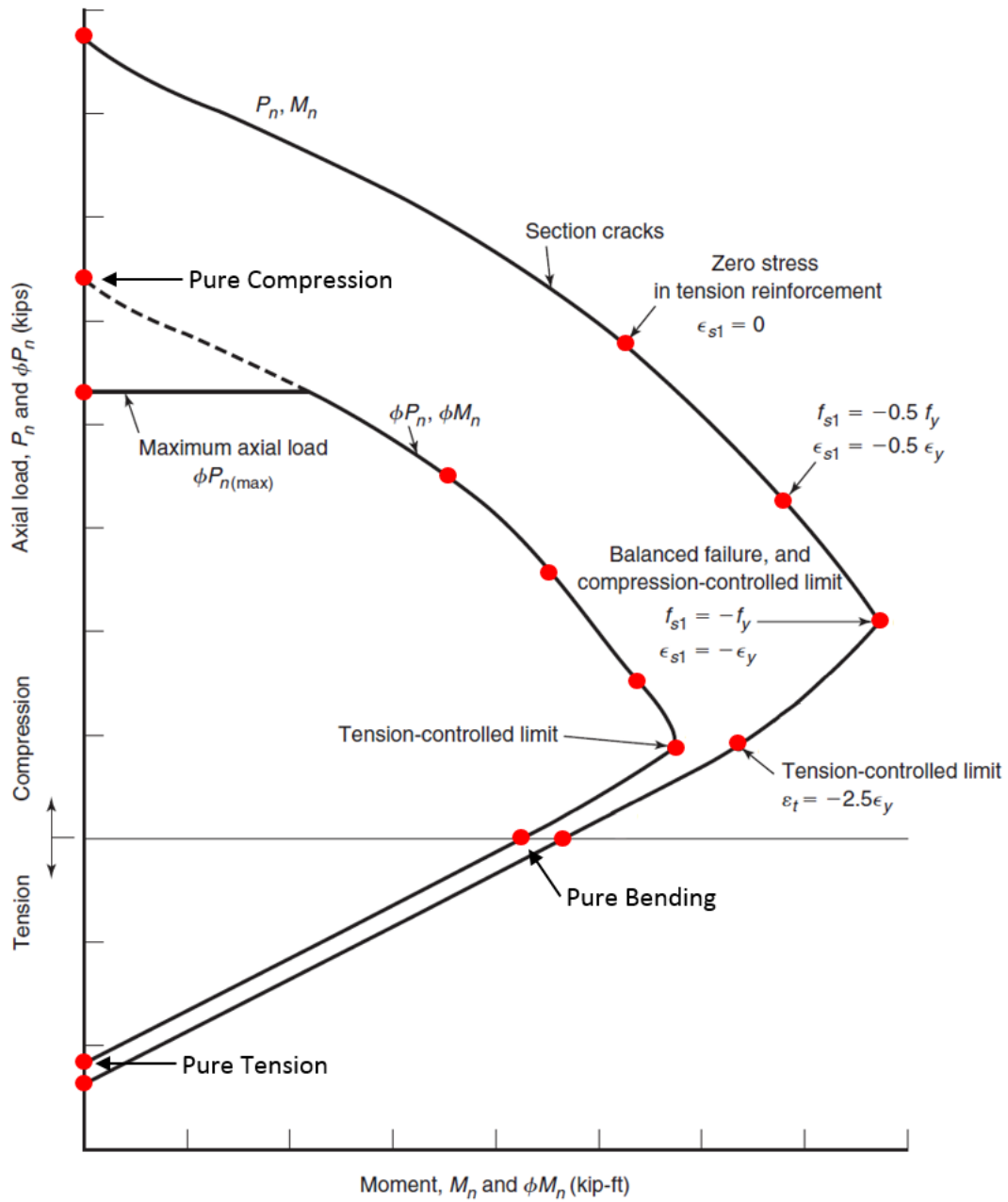


Figure 2 – Control Points

## 1. Pure Compression

### 1.1. Nominal axial compressive strength at zero eccentricity

$$P_o = 0.85f'_c(A_g - A_{st}) + f_y A_{st} \quad \text{ACI 318-14 (22.4.2.2)}$$

$$P_o = 0.85 \times 5000 \times (16 \times 16 - 8 \times 1.00) + 60000 \times 8 \times 1.00 = 1,530 \text{ kips}$$

### 1.2. Factored axial compressive strength at zero eccentricity

Since this column is a tied column with steel strain in compression:

$$\phi = 0.65 \quad \text{ACI 318-14 (Table 21.2.2)}$$

$$\phi P_o = 0.65 \times 1,530 = 997 \text{ kips}$$

### 1.3. Maximum (allowable) factored axial compressive strength

$$\phi P_{n,max} = 0.80 \times \phi P_o = 0.80 \times 997 = 798 \text{ kips} \quad \text{ACI 318-14 (Table 22.4.2.1)}$$

**2. Bar Stress Near Tension Face of Member Equal to Zero, ( $\epsilon_s = f_s = 0$ )**

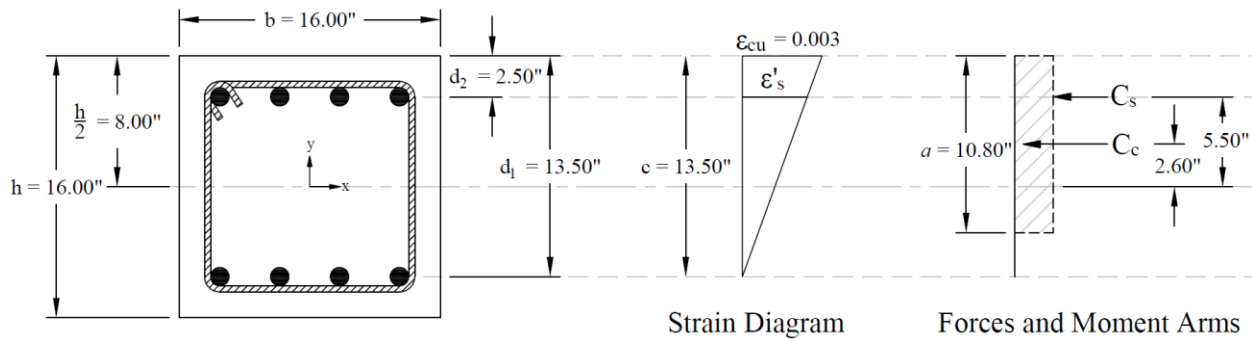


Figure 3 – Strains, Forces, and Moment Arms ( $\epsilon_t = f_s = 0$ )

Strain  $\epsilon_s$  is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices. ACI 318-14 (10.7.5.2.1 and 2)

**2.1. c, a, and strains in the reinforcement**

$$c = d_1 = 13.5 \text{ in.}$$

Where  $c$  is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-14 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 13.5 = 10.80 \text{ in.}$$

ACI 318-14 (22.2.2.4.1)

Where:

$a$  = Depth of equivalent rectangular stress block

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c \times 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$

ACI 318-14 (Table 22.2.2.4.3)

$$\epsilon_s = 0$$

$$\therefore \phi = 0.65$$

ACI 318-14 (Table 21.2.2)

$$\epsilon_{cu} = 0.003$$

ACI 318-14 (22.2.2.1)

$$\epsilon'_s = (c - d_2) \times \frac{\epsilon_{cu}}{c} = (13.50 - 2.5) \times \frac{0.003}{13.50} = 0.00244 \text{ (Compression)} > \epsilon_y = \frac{F_y}{E_s} = \frac{60}{29,000} = 0.00207$$

**2.2. Forces in the concrete and steel**

$$C_c = 0.85 \times f'_c \times a \times b = 0.85 \times 5,000 \times 10.80 \times 16 = 734.4 \text{ kip}$$

ACI 318-14 (22.2.2.4.1)

$$f_s = 0 \text{ psi} \rightarrow T_s = f_s \times A_{s1} = 0 \text{ kip}$$

Since  $\varepsilon_s' > \varepsilon_y$   $\rightarrow$  compression reinforcement has yielded

$$\therefore f_s' = f_y = 60,000 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area ( $ab$ ) used to compute  $C_c$ . As a result, it is necessary to subtract  $0.85f_c'$  from  $f_s'$  before computing  $C_s$ :

$$C_s = (f_s' - 0.85f_c') \times A_{s2} = (60,000 - 0.85 \times 5,000) \times 4 = 223 \text{ kip}$$

### 2.3. $\phi P_n$ and $\phi M_n$

$$P_n = C_c + C_s - T_s = 734.4 + 223 - 0 = 957 \text{ kip}$$

$$\phi P_n = 0.65 \times 957 = 622 \text{ kip}$$

$$M_n = C_c \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_s \times \left( \frac{h}{2} - d_2 \right) + T_s \times \left( d_1 - \frac{h}{2} \right)$$

$$M_n = 734.4 \times \left( \frac{16}{2} - \frac{10.80}{2} \right) + 223 \times \left( \frac{16}{2} - 2.5 \right) + 0 \times \left( 13.50 - \frac{16}{2} \right) = 261 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 261 = 170 \text{ kip.ft}$$



**3. Bar Stress Near Tension Face of Member Equal to  $0.5 f_y$ , ( $f_s = -0.5 f_y$ )**

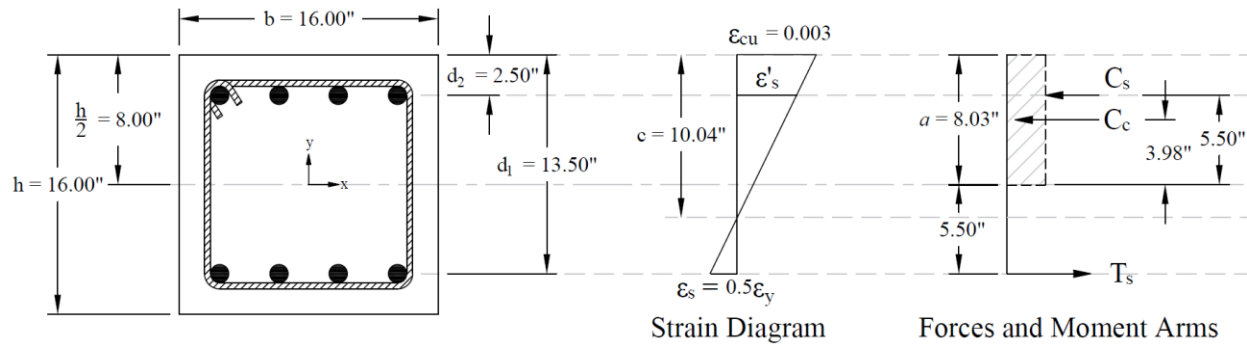


Figure 4 – Strains, Forces, and Moment Arms ( $f_s = -0.5 f_y$ )

3.1. c, a, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = \frac{\varepsilon_y}{2} = \frac{0.00207}{2} = 0.00103 < \varepsilon_y \rightarrow \text{tension reinforcement has not yielded}$$

$$\therefore \phi = 0.65$$

ACI 318-14 (Table 21.2.2)

$$\varepsilon_{cu} = 0.003$$

ACI 318-14 (22.2.2.1)

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00103 + 0.003} \times 0.003 = 10.04 \text{ in.}$$

Where  $c$  is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-14 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 10.04 = 8.03 \text{ in.}$$

ACI 318-14 (22.2.2.4.1)

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c \times 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$

ACI 318-14 (Table 22.2.2.4.3)

$$\varepsilon'_s = (c - d_2) \times \frac{0.003}{c} = (10.04 - 2.5) \times \frac{0.003}{10.04} = 0.00225 \text{ (Compression)} > \varepsilon_y$$

3.2. Forces in the concrete and steel

$$C_c = 0.85 \times f'_c \times a \times b = 0.85 \times 5,000 \times 8.03 \times 16 = 546.1 \text{ kip}$$

ACI 318-14 (22.2.2.4.1)

$$f_s = \varepsilon_s \times E_s = 0.00103 \times 29,000,000 = 30,000 \text{ psi}$$

$$T_s = f_s \times A_{s1} = 30,000 \times 4 = 120 \text{ kip}$$

Since  $\varepsilon'_s > \varepsilon_y \rightarrow$  compression reinforcement has yielded

$$\therefore f'_s = f_y = 60,000 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area ( $ab$ ) used to compute  $C_c$ . As a result, it is necessary to subtract  $0.85f'_c$  from  $f'_s$  before computing  $C_s$ :

$$C_s = (f'_s - 0.85f'_c) \times A_{s2} = (60,000 - 0.85 \times 5,000) \times 4 = 223 \text{ kip}$$

### 3.3. $\phi P_n$ and $\phi M_n$

$$P_n = C_c + C_s - T_s = 546.1 + 223 - 120 = 649 \text{ kip}$$

$$\phi P_n = 0.65 \times 649 = 422 \text{ kip}$$

$$M_n = C_c \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_s \times \left( \frac{h}{2} - d_2 \right) + T_s \times \left( d_1 - \frac{h}{2} \right)$$

$$M_n = 546.1 \times \left( \frac{16}{2} - \frac{8.03}{2} \right) + 223 \times \left( \frac{16}{2} - 2.5 \right) + 120 \times \left( 13.50 - \frac{16}{2} \right) = 339 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 339 = 220 \text{ kip.ft}$$

**4. Bar Stress Near Tension Face of Member Equal to  $f_y$ , ( $f_s = -f_y$ )**

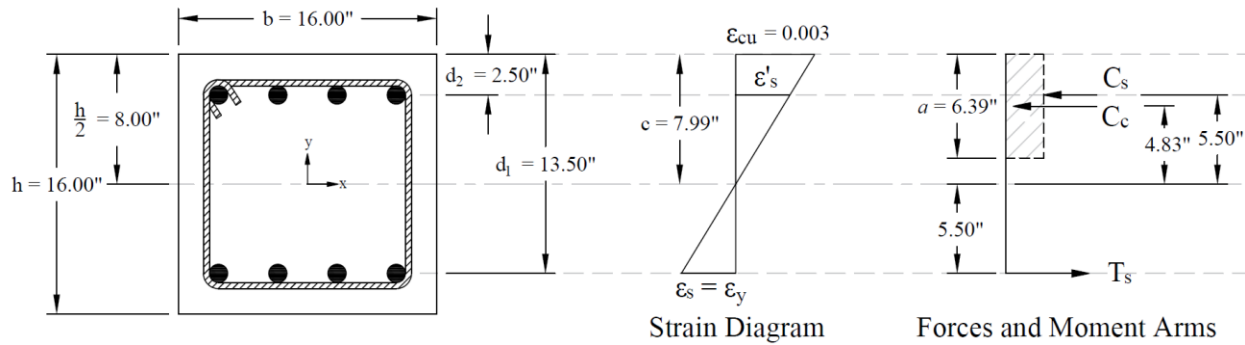


Figure 5 – Strains, Forces, and Moment Arms ( $f_s = -f_y$ )

This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement. It also marks the start of the transition zone for  $\phi$  for columns in which  $\phi$  increases from 0.65 (or 0.75 for spiral columns) up to 0.90.

4.1. c, a, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = \varepsilon_y = 0.00207 \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.65$$

ACI 318-14 (Table 21.2.2)

$$\varepsilon_{cu} = 0.003$$

ACI 318-14 (22.2.2.1)

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00207 + 0.003} \times 0.003 = 7.99 \text{ in.}$$

Where  $c$  is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-14 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 7.99 = 6.39 \text{ in.}$$

ACI 318-14 (22.2.2.4.1)

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c \times 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 \times 4000)}{1000} = 0.80$$

ACI 318-14 (Table 22.2.2.4.3)

$$\varepsilon'_s = (c - d_2) \times \frac{0.003}{c} = (7.99 - 2.5) \times \frac{0.003}{7.99} = 0.00206 \text{ (Compression)} < \varepsilon_y$$

4.2. Forces in the concrete and steel

$$C_c = 0.85 \times f'_c \times a \times b = 0.85 \times 5,000 \times 6.39 \times 16 = 434.6 \text{ kip}$$

ACI 318-14 (22.2.2.4.1)

$$f_s = f_y = 60,000 \text{ psi}$$

$$T_s = f_y \times A_{s1} = 60,000 \times 4 = 240 \text{ kip}$$

Since  $\epsilon'_s < \epsilon_y \rightarrow$  compression reinforcement has not yielded

$$\therefore f'_s = \epsilon'_s \times E_s = 0.00206 \times 29,000,000 = 59,778 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area ( $ab$ ) used to compute  $C_c$ . As a result, it is necessary to subtract  $0.85f'_c$  from  $f'_s$  before computing  $C_s$ :

$$C_s = (f'_s - 0.85f'_c) \times A_{s2} = (59,778 - 0.85 \times 5,000) \times 4 = 222.1 \text{ kip}$$

4.3.  $\phi P_n$  and  $\phi M_n$

$$P_n = C_c + C_s - T_s = 434.6 + 222.1 - 240 = 417 \text{ kip}$$

$$\phi P_n = 0.65 \times 417 = 271 \text{ kip}$$

$$M_n = C_c \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_s \times \left( \frac{h}{2} - d_2 \right) + T_s \times \left( d_1 - \frac{h}{2} \right)$$

$$M_n = 434.6 \times \left( \frac{16}{2} - \frac{6.39}{2} \right) + 222.1 \times \left( \frac{16}{2} - 2.5 \right) + 240 \times \left( 13.50 - \frac{16}{2} \right) = 386 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 386 = 251 \text{ kip.ft}$$

**5. Bar Strain Near Tension Face of Member Equal to 0.005 in./in., ( $\epsilon_s = -0.005$  in./in.)**

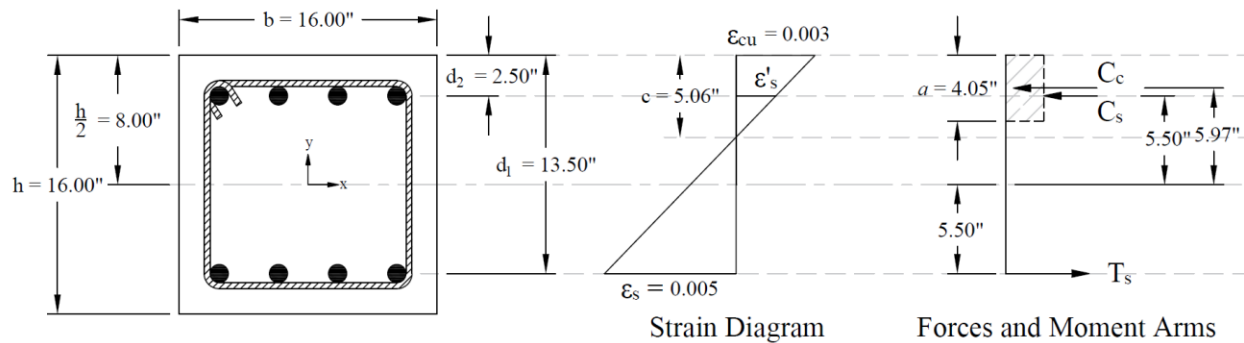


Figure 6 – Strains, Forces, and Moment Arms ( $\epsilon_s = -0.005$  in./in.)

This corresponds to the tension-controlled strain limit of 0.005. It is the strain at the tensile limit of the transition zone for  $\phi$ , used to define a tension-controlled section.

5.1. c, a, and strains in the reinforcement

$$\epsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\epsilon_s = 0.005 > \epsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.9$$

ACI 318-14 (Table 21.2.2)

$$\epsilon_{cu} = 0.003$$

ACI 318-14 (22.2.2.1)

$$c = \frac{d_1}{\epsilon_s + \epsilon_{cu}} \times \epsilon_{cu} = \frac{13.50}{0.005 + 0.003} \times 0.003 = 5.06 \text{ in.}$$

Where  $c$  is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-14 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 5.06 = 4.05 \text{ in.}$$

ACI 318-14 (22.2.2.4.1)

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c \times 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$

ACI 318-14 (Table 22.2.2.4.3)

$$\epsilon'_s = (c - d_2) \times \frac{0.003}{c} = (5.06 - 2.5) \times \frac{0.003}{5.06} = 0.00152 \text{ (Compression)} < \epsilon_y$$

5.2. Forces in the concrete and steel

$$C_c = 0.85 \times f'_c \times a \times b = 0.85 \times 5,000 \times 4.05 \times 16 = 275.4 \text{ kip}$$

ACI 318-14 (22.2.2.4.1)

$$f_s = f_y = 60,000 \text{ psi}$$

$$T_s = f_y \times A_{s1} = 60,000 \times 4 = 240 \text{ kip}$$

Since  $\epsilon'_s < \epsilon_y \rightarrow$  compression reinforcement has not yielded

$$\therefore f'_s = \epsilon'_s \times E_s = 0.00152 \times 29,000,000 = 44,037 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area ( $ab$ ) used to compute  $C_c$ . As a result, it is necessary to subtract  $0.85f'_c$  from  $f'_s$  before computing  $C_s$ :

$$C_s = (f'_s - 0.85f'_c) \times A_{s2} = (44,037 - 0.85 \times 5,000) \times 4 = 159.1 \text{ kip}$$

### 5.3. $\phi P_n$ and $\phi M_n$

$$P_n = C_c + C_s - T_s = 275.4 + 159.1 - 240 = 195 \text{ kip}$$

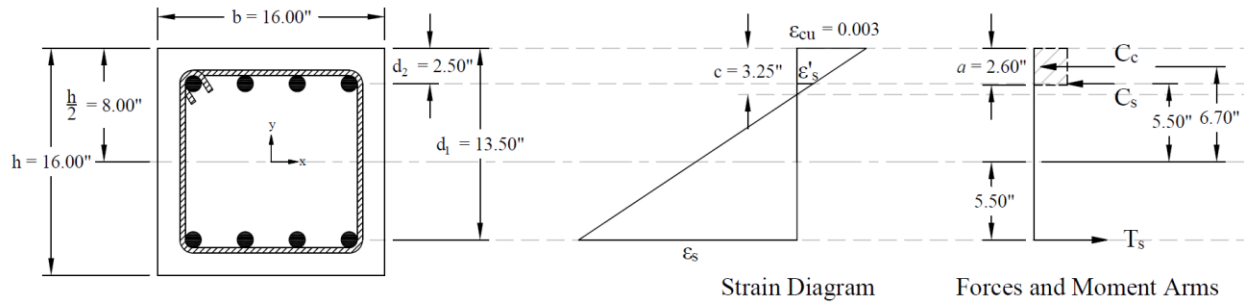
$$\phi P_n = 0.90 \times 195 = 175 \text{ kip}$$

$$M_n = C_c \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_s \times \left( \frac{h}{2} - d_2 \right) + T_s \times \left( d_1 - \frac{h}{2} \right)$$

$$M_n = 275.4 \times \left( \frac{16}{2} - \frac{4.05}{2} \right) + 159.1 \times \left( \frac{16}{2} - 2.5 \right) + 240 \times \left( 13.50 - \frac{16}{2} \right) = 320 \text{ kip.ft}$$

$$\phi M_n = 0.90 \times 320 = 288 \text{ kip.ft}$$

## 6. Pure Bending



**Figure 7 – Strains, Forces, and Moment Arms (Pure Moment)**

This corresponds to the case where the nominal axial load capacity,  $P_n$ , is equal to zero. Iterative procedure is used to determine the nominal moment capacity as follows:

### 6.1. c, a, and strains in the reinforcement

Try  $c = 3.25$  in.

Where  $c$  is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-14 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 3.25 = 2.60 \text{ in.}$$

ACI 318-14 (22.2.2.4.1)

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c \times 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 \times 4000)}{1000} = 0.80$$

ACI 318-14 (Table 22.2.2.4.3)

$$\varepsilon_{cu} = 0.003$$

ACI 318-14 (22.2.2.1)

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = (d_1 - c) \times \frac{0.003}{c} = (13.50 - 3.25) \times \frac{0.003}{3.25} = 0.00946 \text{ (Tension)} > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.9$$

ACI 318-14 (Table 21.2.2)

$$\varepsilon'_s = (c - d_2) \times \frac{0.003}{c} = (3.25 - 2.5) \times \frac{0.003}{3.25} = 0.00069 \text{ (Compression)} < \varepsilon_y$$

### 6.2. Forces in the concrete and steel

$$C_c = 0.85 \times f'_c \times a \times b = 0.85 \times 5,000 \times 2.6 \times 16 = 176.8 \text{ kip}$$

ACI 318-14 (22.2.2.4.1)

$$f_s = f_y = 60,000 \text{ psi}$$

$$T_s = f_y \times A_{s1} = 60,000 \times 4 = 240 \text{ kip}$$

Since  $\varepsilon'_s < \varepsilon_y \rightarrow$  compression reinforcement has not yielded

$$\therefore f'_s = \varepsilon'_s \times E_s = 0.00069 \times 29,000,000 = 20,077 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area ( $ab$ ) used to compute  $C_c$ . As a result, it is necessary to subtract  $0.85f'_c$  from  $f'_s$  before computing  $C_s$ :

$$C_s = (f'_s - 0.85f'_c) \times A_{s2} = (20,077 - 0.85 \times 5,000) \times 4 = 63.3 \text{ kip}$$

### 6.3. $\phi P_n$ and $\phi M_n$

$$P_n = C_c + C_s - T_s = 176.8 + 63.3 - 240 \approx 0 \text{ kip} \rightarrow \phi P_n \approx 0 \text{ kip}$$

The assumption that  $c = 3.25$  in. is correct

$$M_n = C_c \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_s \times \left( \frac{h}{2} - d_2 \right) + T_s \times \left( d_1 - \frac{h}{2} \right)$$

$$M_n = 176.8 \times \left( \frac{16}{2} - \frac{2.60}{2} \right) + 63.3 \times \left( \frac{16}{2} - 2.5 \right) + 240 \times \left( 13.50 - \frac{16}{2} \right) = 238 \text{ kip.ft}$$

$$\phi M_n = 0.90 \times 238 = 214 \text{ kip.ft}$$



## 7. Pure Tension

The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.

### 7.1. $P_m$ and $\phi P_m$

$$P_m = f_y \times (A_{s1} + A_{s2}) = 60,000 \times (4 + 4) = 480 \text{ kip} \quad \text{ACI 318-14 (22.4.3.1)}$$

$$\phi = 0.9 \quad \text{ACI 318-14 (Table 21.2.2)}$$

$$\phi P_m = 0.90 \times 480 = 432 \text{ kip}$$

### 7.2. $M_n$ and $\phi M_n$

Since the section is symmetrical

$$M_n = \phi M_n = 0 \text{ kip.ft}$$

## 8. Column Interaction Diagram - spColumn Software

spColumn program performs the analysis of the reinforced concrete section conforming to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility. For this column section, we ran in investigation mode with control points using the 318-14. In lieu of using program shortcuts, spSection (Figure 9) was used to place the reinforcement and define the cover to illustrate handling of irregular shapes and unusual bar arrangement.

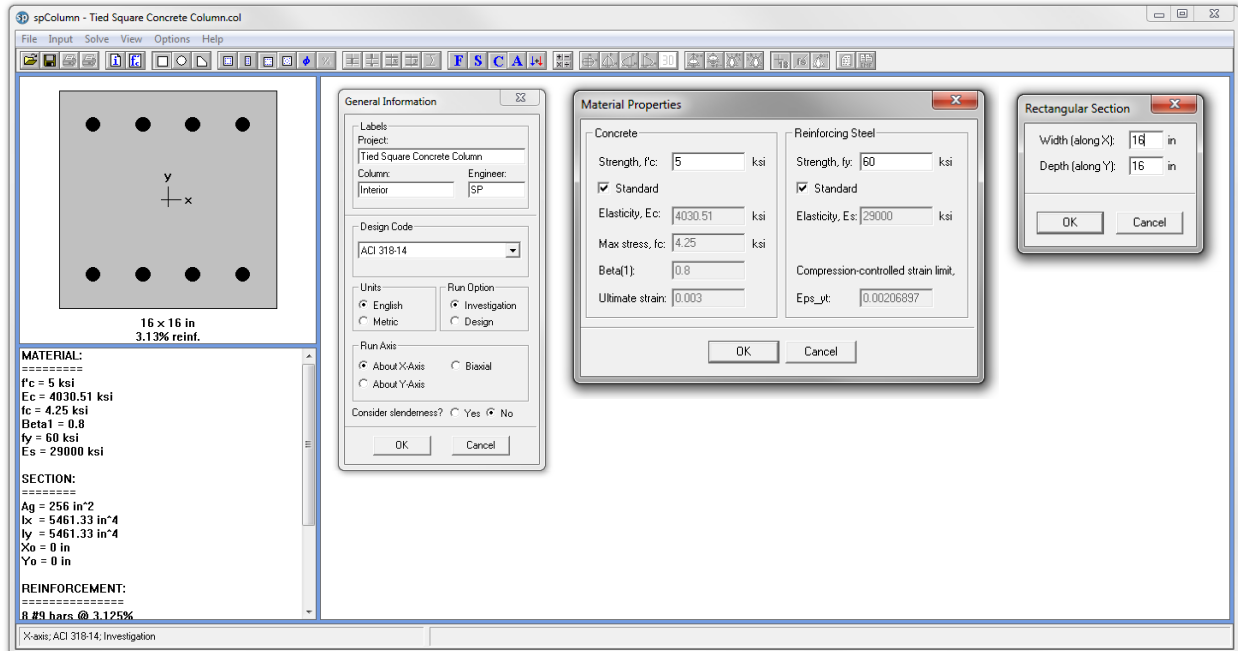


Figure 8 – Generating spColumn Model

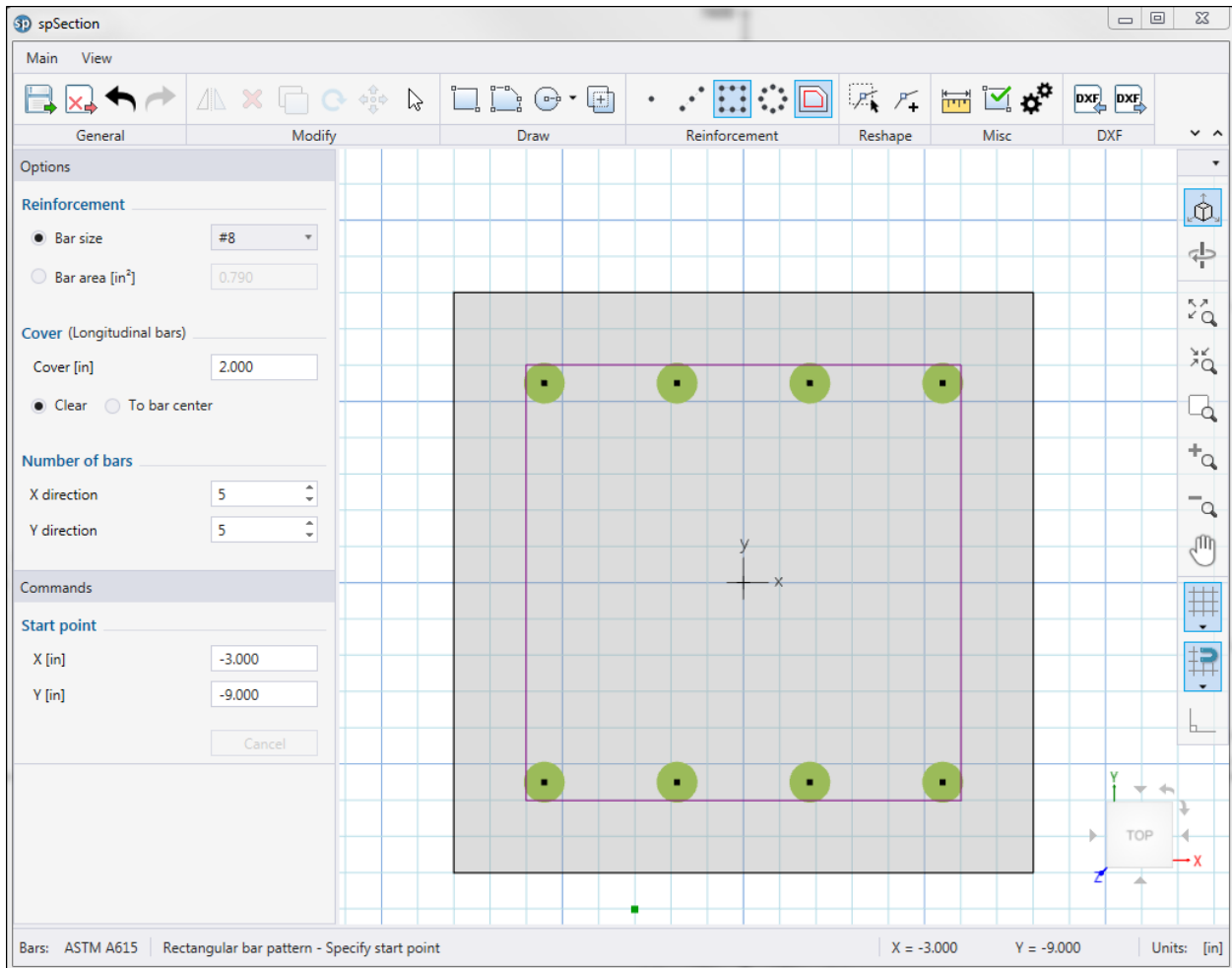


Figure 9 – spColumn Model Editor (spSection)

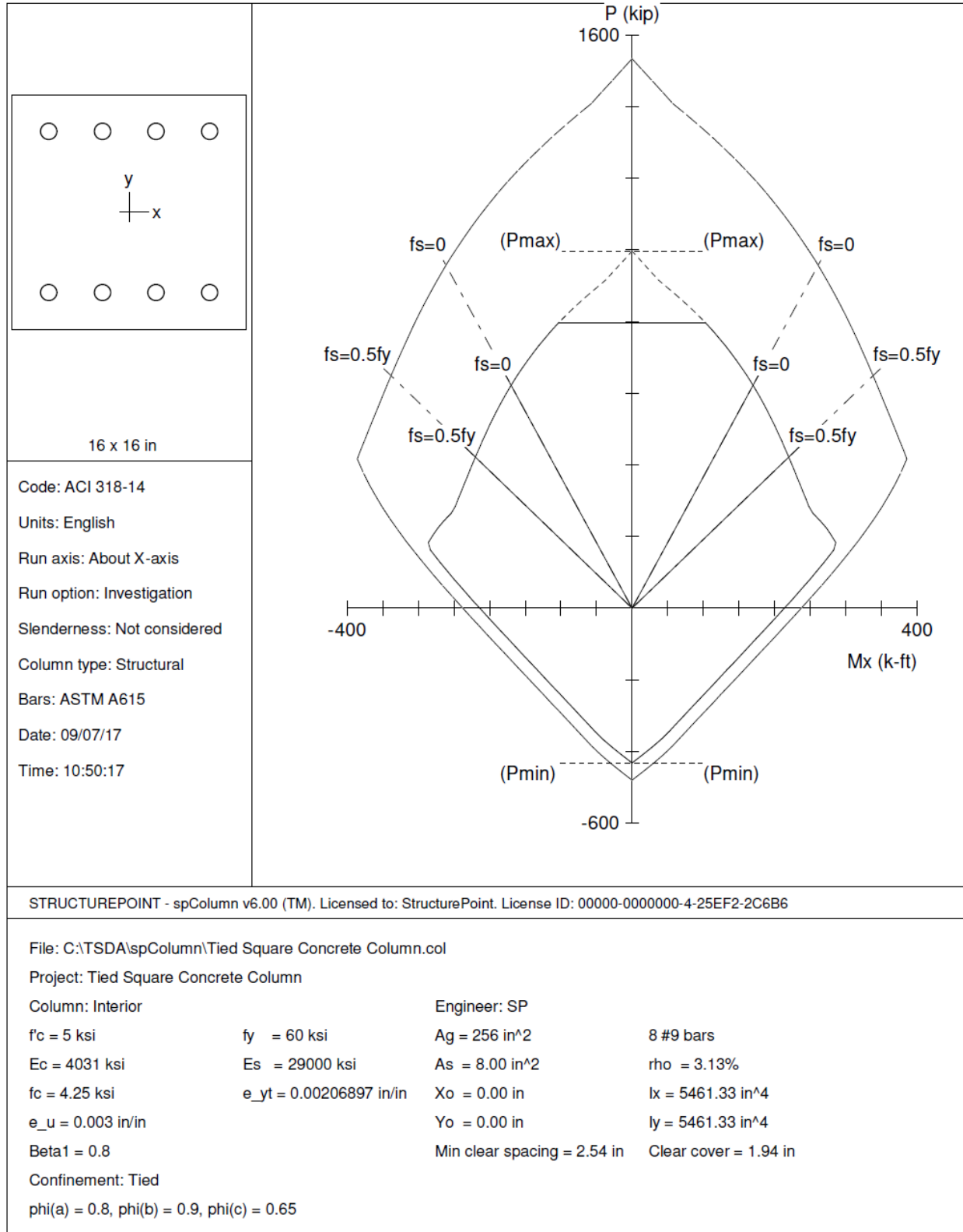


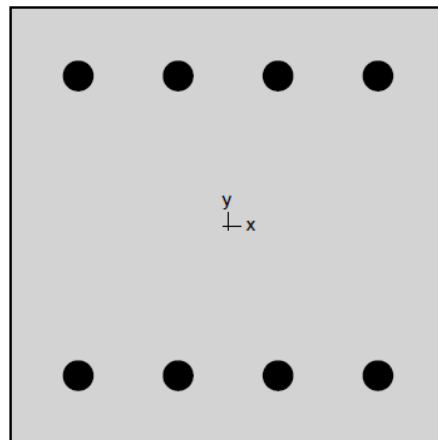
Figure 10 – Column Section Interaction Diagram about the X-Axis (spColumn)



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Computer program for the Strength Design of Reinforced Concrete Sections  
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## 1. General Information

File Name	C:\TSDA\spColumn\Tied Square Concrete Column.col
Project	Tied Square Concrete Column
Column	Interior
Engineer	SP
Code	ACI 318-14
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	X - axis
Slenderness	Not Considered
Column Type	Structural

## 2. Material Properties

### 2.1. Concrete

Type	Standard
$f_c$	5 ksi
$E_c$	4030.51 ksi
$f_c$	4.25 ksi
$\epsilon_u$	0.003 in/in
$\beta_1$	0.8

### 2.2. Steel

Type	Standard
$f_y$	60 ksi
$E_s$	29000 ksi
$\epsilon_{yt}$	0.00206897 in/in

## 3. Section

### 3.1. Shape and Properties

Type	Rectangular
Width	16 in
Depth	16 in
$A_g$	256 in <sup>2</sup>
$I_x$	5461.33 in <sup>4</sup>
$I_y$	5461.33 in <sup>4</sup>
$r_x$	4.6188 in
$r_y$	4.6188 in
$X_o$	0 in
$Y_o$	0 in

### 3.2. Section Figure

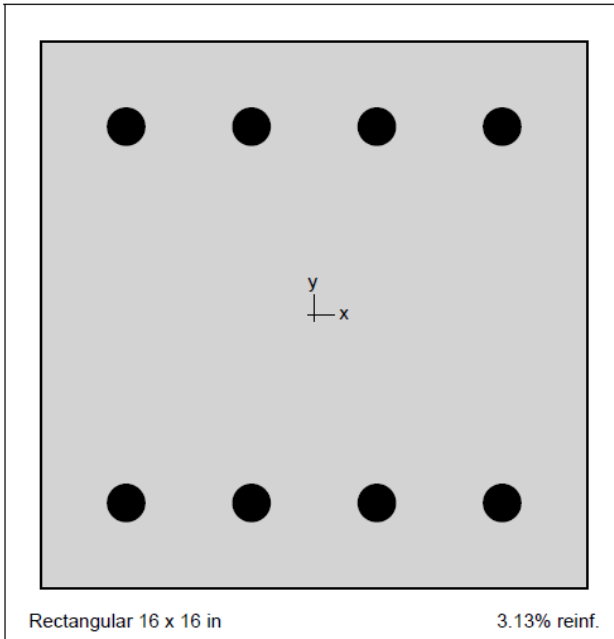


Figure 1: Column section

## 4. Reinforcement

### 4.1. Bar Set: ASTM A615

Bar	Diameter in	Area in <sup>2</sup>	Bar	Diameter in	Area in <sup>2</sup>	Bar	Diameter in	Area in <sup>2</sup>
#3	0.38	0.11	#4	0.50	0.20	#5	0.63	0.31
#6	0.75	0.44	#7	0.88	0.60	#8	1.00	0.79
#9	1.13	1.00	#10	1.27	1.27	#11	1.41	1.56
#14	1.69	2.25	#18	2.26	4.00			

### 4.2. Confinement and Factors

Confinement type	Tied
For #10 bars or less	#3 ties
For larger bars	#4 ties
<b>Capacity Reduction Factors</b>	
Axial compression, (a)	0.8
Tension controlled failure, (b)	0.9
Compression controlled failure, (c)	0.65

### 4.3. Arrangement

Pattern	Sides different
Bar layout	---
Cover to	Longitudinal bars
Clear cover	---
Bars	---



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Total steel area, $A_s$	8.00 in <sup>2</sup>
rho	3.13 %
Minimum clear spacing	2.54 in

#### 4.4. Bars Provided

	Bars	Cover in
Top	4 #9	1.936
Bottom	4 #9	1.936
Left	0 #9	1.936
Right	0 #9	1.936

#### 5. Control Points

About Point	P kip	X-Moment k-ft	Y-Moment k-ft	NA Depth in	$d_t$ Depth in	$\epsilon_t$	$\Phi$
X @ Max compression	997.1	0.00	0.00	43.50	13.50	-0.00207	0.65000
X @ Allowable comp.	797.7	102.64	0.00	17.35	13.50	-0.00067	0.65000
X @ $f_s = 0.0$	622.3	169.86	0.00	13.50	13.50	0.00000	0.65000
X @ $f_s = 0.5 f_y$	421.9	220.05	0.00	10.04	13.50	0.00103	0.65000
X @ Balanced point	270.9	250.77	0.00	7.99	13.50	0.00207	0.65000
X @ Tension control	175.1	288.06	0.00	5.06	13.50	0.00500	0.90000
X @ Pure bending	0.0	213.91	0.00	3.25	13.50	0.00946	0.90000
X @ Max tension	-432.0	0.00	0.00	0.00	13.50	9.99999	0.90000
-X @ Max compression	997.1	0.00	0.00	43.50	13.50	-0.00207	0.65000
-X @ Allowable comp.	797.7	-102.64	0.00	17.35	13.50	-0.00067	0.65000
-X @ $f_s = 0.0$	622.3	-169.86	0.00	13.50	13.50	0.00000	0.65000
-X @ $f_s = 0.5 f_y$	421.9	-220.05	0.00	10.04	13.50	0.00103	0.65000
-X @ Balanced point	270.9	-250.77	0.00	7.99	13.50	0.00207	0.65000
-X @ Tension control	175.1	-288.06	0.00	5.06	13.50	0.00500	0.90000
-X @ Pure bending	0.0	-213.91	0.00	3.25	13.50	0.00946	0.90000
-X @ Max tension	-432.0	0.00	0.00	0.00	13.50	9.99999	0.90000

## 9. Summary and Comparison of Design Results

Table 1 - Comparison of Results						
Support	$\phi P_n$ , kip			$\phi M_n$ , kip.ft		
	Hand	Reference*	spColumn	Hand	Reference*	spColumn
Max compression	997	997	997	0	0	0
Allowable compression	798	798	798	---	---	---
$f_s = 0.0$	622	622	622	170	170	170
$f_s = 0.5 f_y$	422	422	422	220	220	220
Balanced point	271	271	271	251	251	251
Tension control	175	175	175	288	288	288
Pure bending	0	0	0	214	214	214
Max tension	432	432	432	0	0	0

\* Reinforced Concrete Mechanic and Design, 6<sup>th</sup> Edition, James Wight & MacGregor – Example 11-1

In all of the hand calculations and the reference used illustrated above, the results are in precise agreement with the automated exact results obtained from the [spColumn](#) program.

## 10. Conclusions & Observations

The analysis of the reinforced concrete section performed by [spColumn](#) conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In the calculation shown above a P-M interaction diagram was generated with moments about the X-Axis (Uniaxial bending). Since the reinforcement in the section is not symmetrical, a different P-M interaction diagram is needed for the other orthogonal direction about the Y-Axis (See the following Figure for the case where  $f_s = f_y$ ).

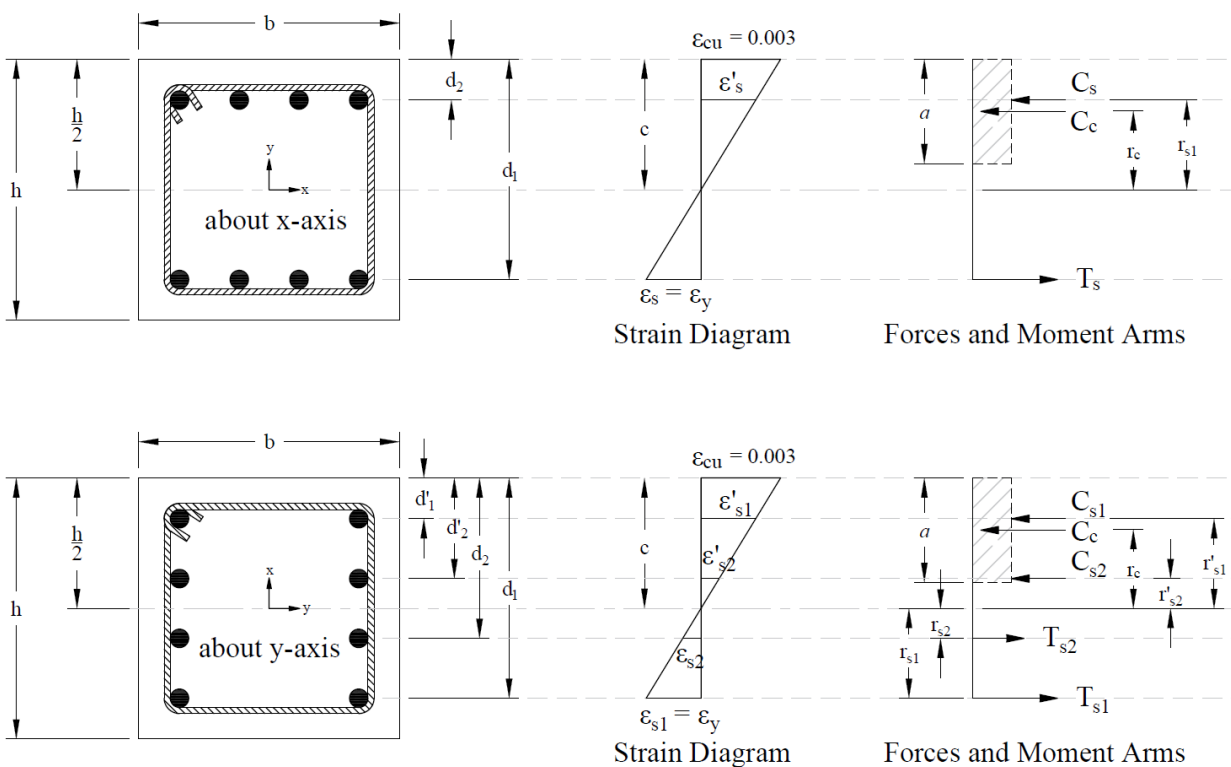
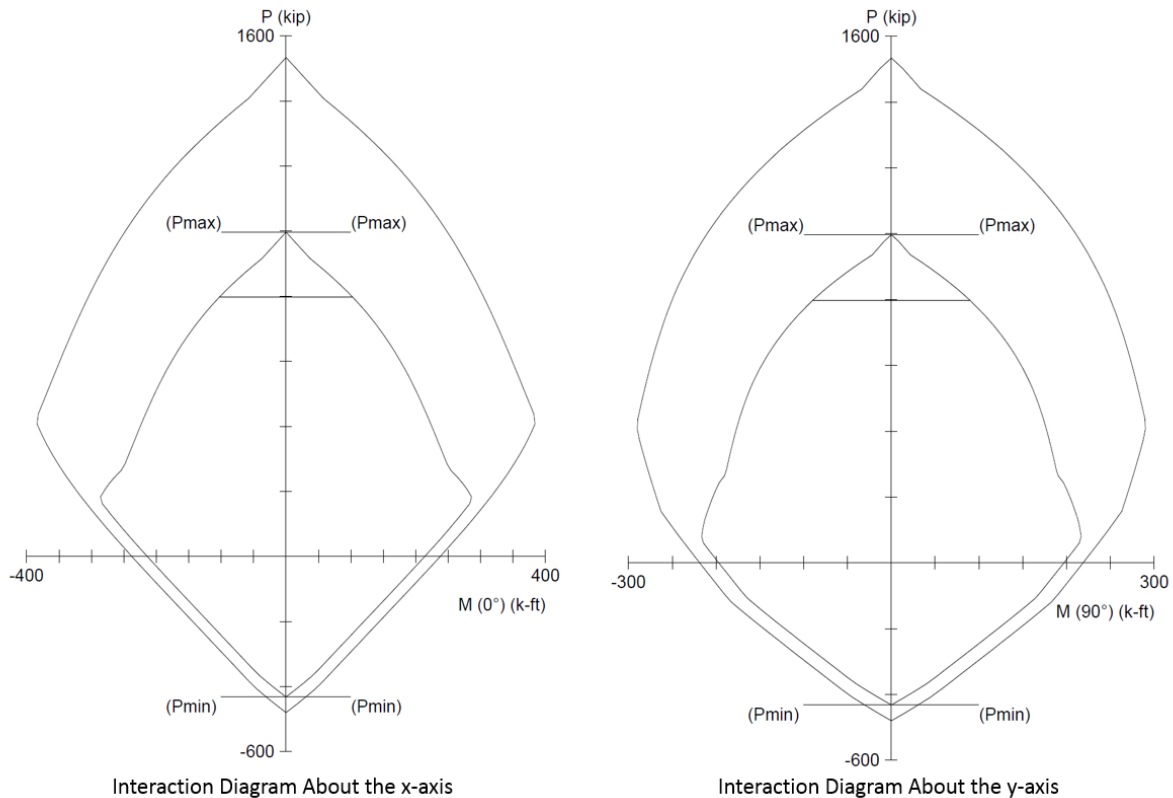


Figure 11 – Strains, Forces, and Moment Arms ( $f_s = -f_y$  Moments About x- and y-axis)

When running about the Y-Axis, we have 2 bars in 4 layers instead of 4 bars in just 2 layers (about X-Axis) resulting in a completely different interaction diagram as shown in the following Figure.



**Figure 12 – Comparison of Column Interaction Diagrams about X-Axis and Y-Axis (spColumn)**

Further differences in the interaction diagram in both directions can result if the column cross section geometry is irregular.

In most building design calculations, such as the examples shown for [flat plate](#) or [flat slab](#) concrete floor systems, all building columns are subjected to  $M_x$  and  $M_y$  due to lateral forces and unbalanced moments from both directions of analysis. This requires an evaluation of the column P-M interaction diagram in two directions simultaneously (biaxial bending).

StructurePoint's [spColumn](#) program can also evaluate column sections in biaxial mode to produce the results shown in the following Figure for the column section in this example.

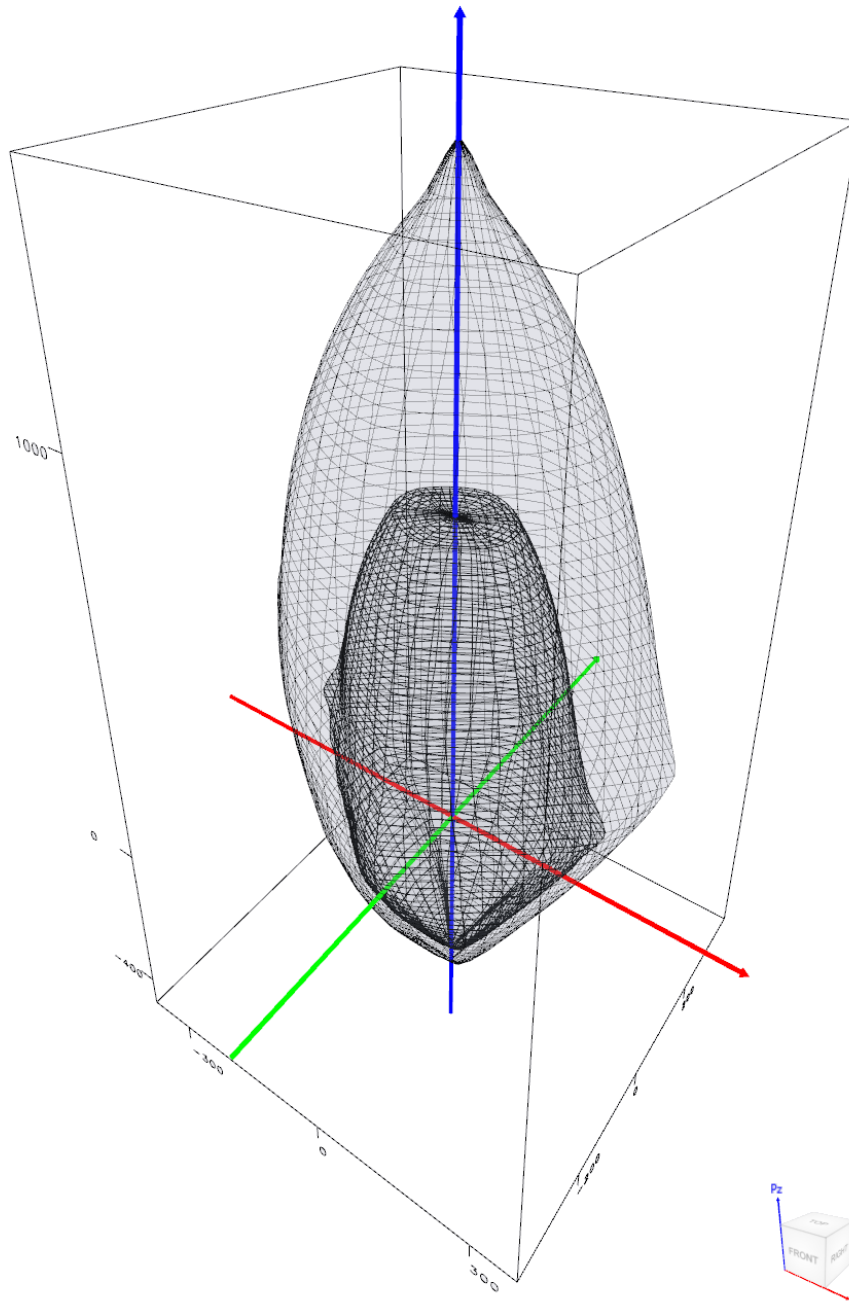


Figure 13 – Nominal & Design Interaction Diagram in Two Directions (Biaxial) (spColumn)