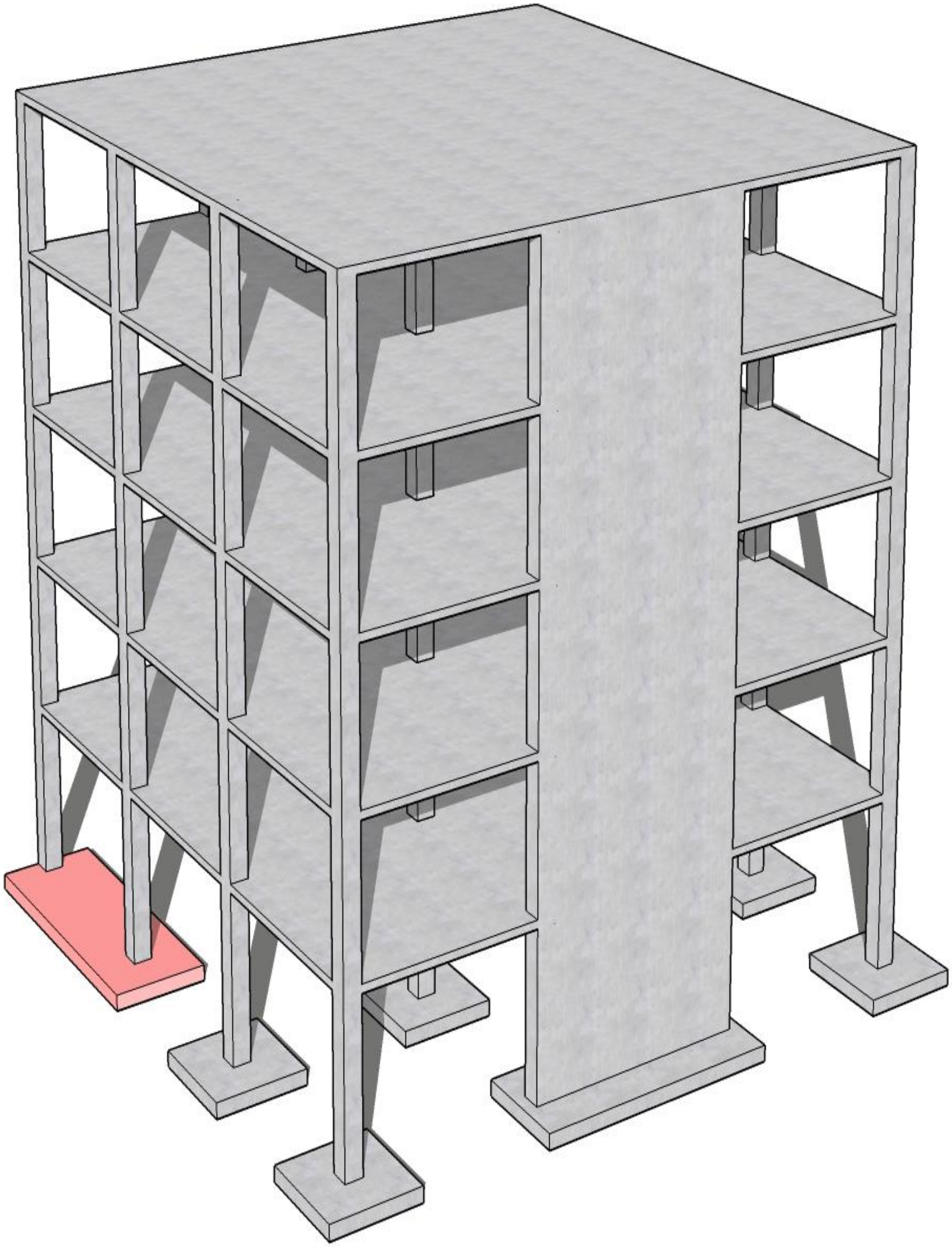
Reinforced Concrete Column Combined Footing Analysis and Design



Reinforced Concrete Column Combined Footing Analysis and Design

A combined footing was selected to support a 24 in. x 16 in. exterior column near a property line and a 24 in. x 24 in. Interior column. Each column carries the service dead and live loads shown in the following figure. The footing dimensions (25 ft 4 in. x 8 ft) were selected such that the centroid of the area in contact with soil coincides with the resultant of the column loads supported by the footing.

Check if the selected combined footing preliminary thickness of 36 in. is sufficient to resist two-way punching shear around the interior and exterior columns supported by the footing. Compare the calculated results with the values presented in the Reference and model results from spMats engineering software program from StructurePoint.

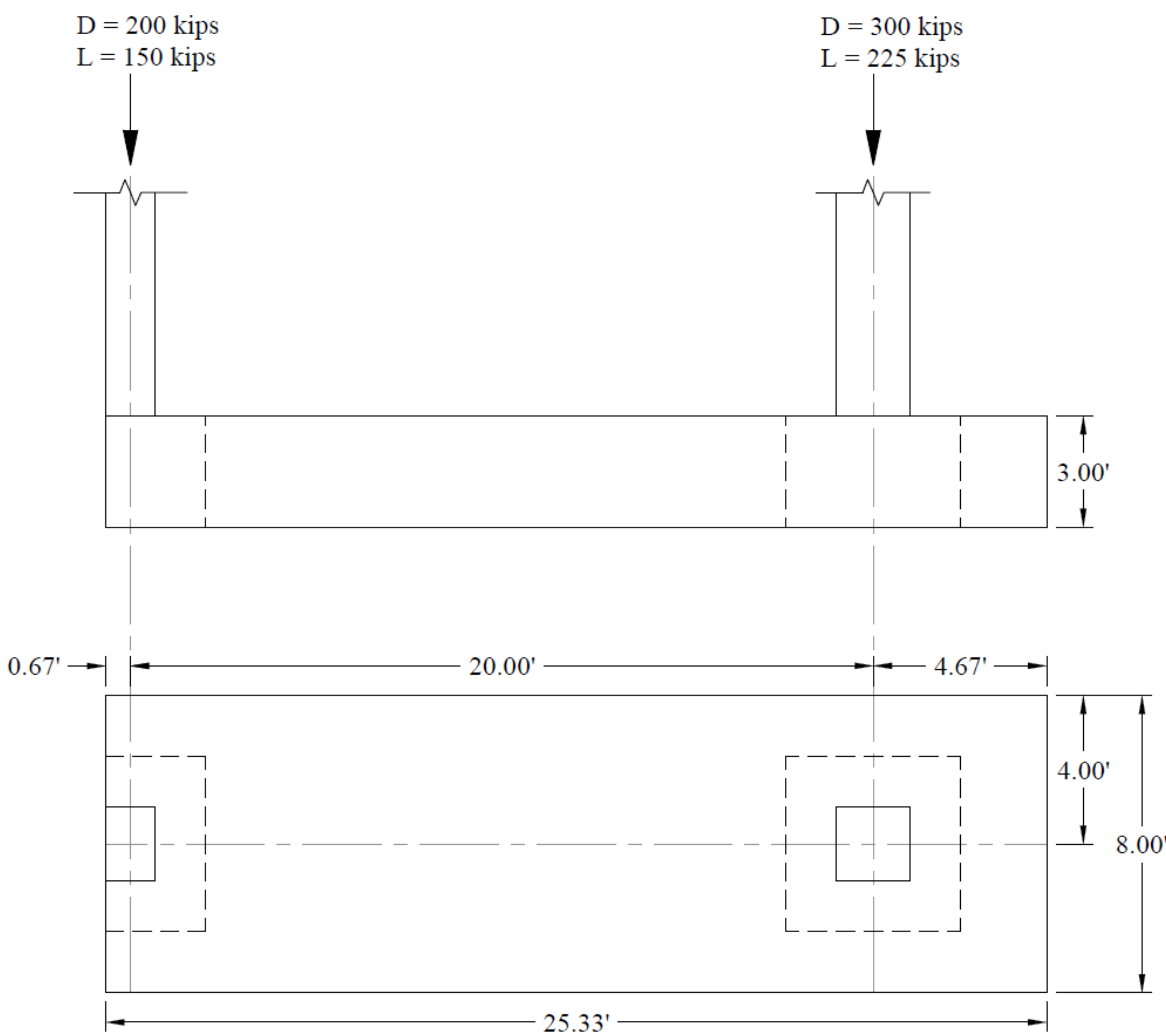


Figure 1 – Reinforced Concrete Combined Footing Geometry

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference

Reinforced Concrete Mechanics and Design, 7th Edition, 2016, James Wight, Pearson, Example 15-5

spMats Engineering Software Program Manual v8.12, StucturePoint LLC., 2016

Design Data

f_c' = 3,000 psi normal weight concrete

f_y = 60,000 psi

Preliminary footing thickness = 36 in.

Dead load, D = 200 kips for exterior column and 300 kips for interior column

Live load, L = 150 kips for exterior column and 225 kips for interior column

Soil density, γ_s = 120 pcf

Concrete density, γ_c = 150 pcf for normal weight concrete

Allowable soil pressure, q_a = 5000 psf

Footing length = 25 ft 4 in.

Footing width = 8 ft

Preliminary footing depth = 36 in. with effective depth, d = 32.5 in.

1. Preliminary Member Sizing

1.1. Footing Cross Sectional Dimensions

The footing dimensions (25 ft 4 in. x 8 ft) were selected by the reference such that the centroid of the area in contact with soil coincides with the resultant of the column loads supported by the footing to achieve uniform soil pressures.

1.2. Factored Net Pressure

The factored net pressure that will be used in the design of the concrete and reinforcement is equal to:

$$q_{nu} = \frac{P_u}{A_{footing}} = \frac{1.2\times(200+300)+1.6\times(150+225)}{25.33\times 8} = 5.92 \text{ ksf}$$

The following Figure shows the shear and moment diagrams for the combined footing based on the factored columns loads and the factored net pressure.

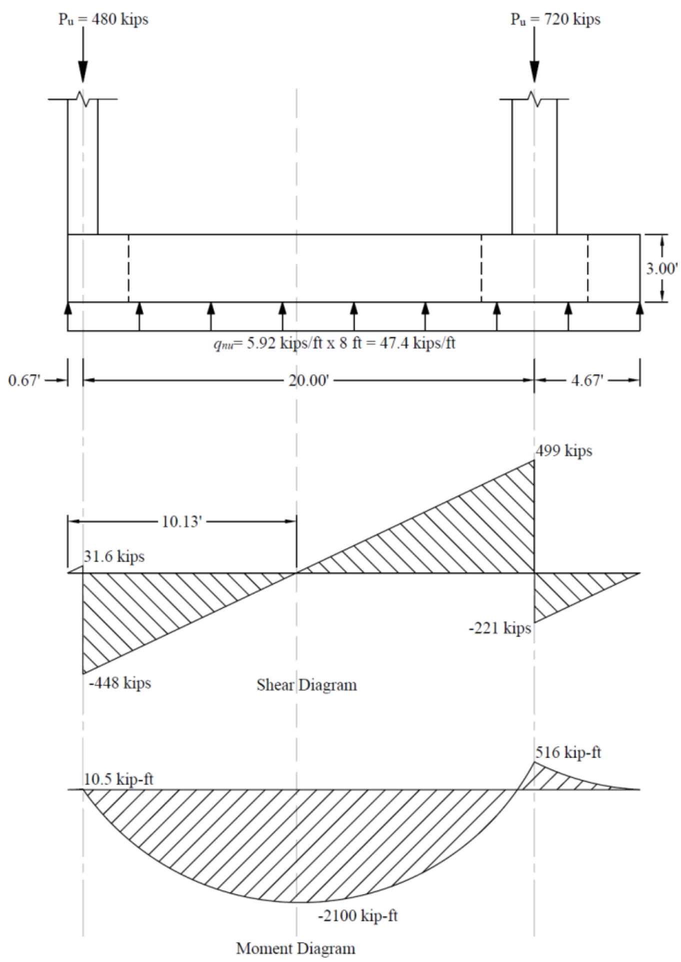


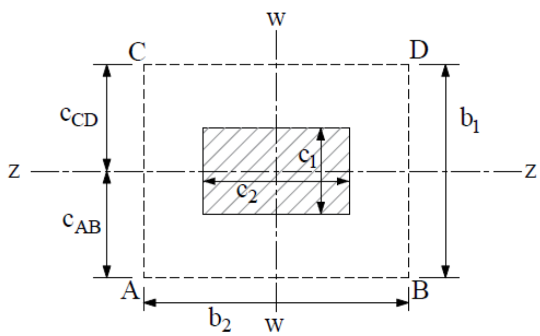
Figure 2 – Shear Force and Bending Moment Diagrams

2. Two-Way (Punching) Shear Capacity Check

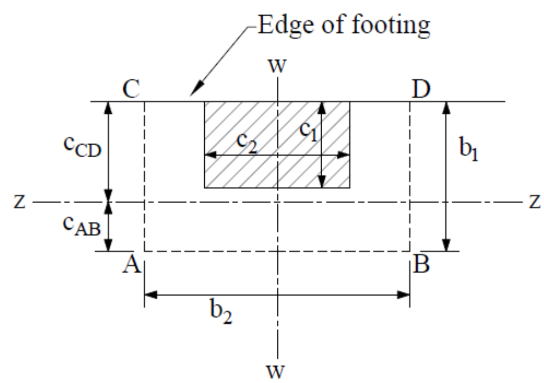
Two-way shear is critical on a rectangular section located at d/2 away from the face of the column as shown in the following Figures, Where:

b_1 = Dimension of the critical section b_o measured in the direction of the span for which moments are determined in ACI 318, Chapter 8.

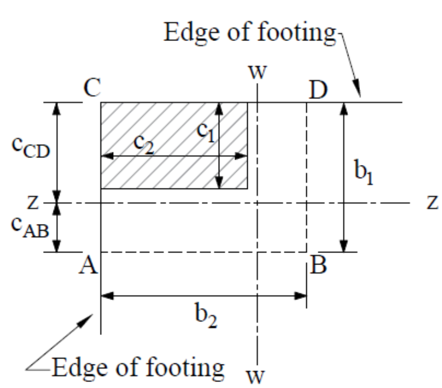
b_2 = Dimension of the critical section b_o measured in the direction perpendicular to b_1 in ACI 318, Chapter 8 (see Figure 5).



Critical shear perimeter for interior column



Critical shear perimeter for exterior column



Critical shear perimeter for corner column

Figure 3 – Critical Shear Perimeters around Columns

2.1. Interior column

The factored shear force (V_u) at the critical section is computed as the reaction at the centroid of the critical section minus the force due to soil pressure acting within the critical section (d/2 away from column face).

$$V_u = 720 - 5.92\left(\frac{56.5}{12}\right)^2 = 589\,\text{kips}$$

The factored unbalanced moment used for shear transfer, M_{unb}, is computed as the sum of the joint moments to the left and right. Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account.

$$M_{unb} = 0 \text{ kips-ft}$$

$$b_1 = c_1 + d = 24 + 32.5 = 56.5 \text{ in.}$$

$$b_2 = c_2 + d = 24 + 32.5 = 56.5 \text{ in.}$$

For the interior column, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{b_1}{2} = \frac{56.5}{2} = 28.25 \text{ in.}$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2\left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + (b_1 d)\left(\frac{b_1}{2} - c_{AB}\right)^2\right) + 2 b_2 d c_{AB}^2$$

$$J_c = 2\left(\frac{56.5 \times 32.5^3}{12} + \frac{32.5 \times 56.5^3}{12} + (56.5 \times 32.5)(0)^2\right) + 2 \times 56.5 \times 32.5 \times 28.25^2 = 4{,}231{,}103 \text{ in.}^4$$

$$\gamma_f = \frac{1}{1 + (2/3) \times \sqrt{b_1 / b_2}}$$ ***ACI 318-14 (8.4.2.3.2)***

$$\gamma_f = \frac{1}{1 + (2/3) \times \sqrt{56.5 / 56.5}} = 0.600$$

$$\gamma_v = 1 - \gamma_f = 1 - 0.600 = 0.400$$ ***ACI 318-14 (Eq. 8.4.4.2.2)***

The length of the critical perimeter for the exterior column:

$$b_o = 4 \times 56.5 = 226 \text{ in.}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d} + \frac{\gamma_v M_{unb}(b_1 - c_{AB})}{J_c}$$ ***ACI 318-14 (R.8.4.4.2.3)***

$$v_u = \frac{589 \times 1000}{226 \times 32.5} + \frac{0.400 \times (0 \times 1000) \times (56.5 - 28.25)}{4{,}231{,}103} = 80.20 \text{ psi}$$

$$v_c = \min\left[4\lambda\sqrt{f_c'}\ ,\ \left(2 + \frac{4}{\beta}\right)\lambda\sqrt{f_c'}\ ,\ \left(\frac{\alpha_s d}{b_o} + 2\right)\lambda\sqrt{f_c'}\right]$$ ***ACI 318-14 (Table 22.6.5.2)***

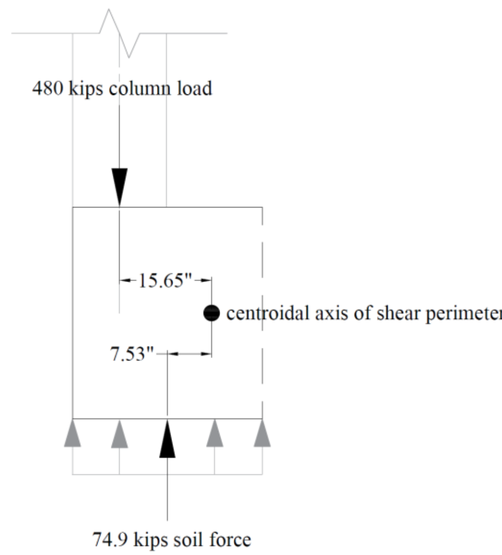
$$v_c = \min\left[4\times1\times\sqrt{3000}\ ,\ \left(2+\frac{4}{1}\right)\times1\times\sqrt{3000}\ ,\ \left(\frac{40\times32.5}{226}+2\right)\times1\times\sqrt{3000}\right]$$

$$v_c = \min\left[219.1\ ,\ 328.6\ ,\ 424.6\right] = 219.1\ \text{psi}$$

$$\phi v_c = 0.75\times219.1 = 164.3\ \text{psi}$$

Since $\phi v_c > v_u$ at the critical section, the slab has adequate two-way shear strength at this joint.

2.2. Exterior column



$$V_u = 480 - 5.92\left(\frac{32.25\times56.5}{144}\right) = 405.1\ \text{kips}$$

$$M_{unb} = 480\times15.65 - 5.92\left(\frac{32.25\times56.5}{144}\right)\times7.53 = 6950\ \text{kip-in.}$$

$$b_1 = c_1 + d/2 = 16 + 32.5/2 = 32.25\ \text{in.}$$

$$b_2 = c_2 + d = 24 + 32.5 = 56.5\ \text{in.}$$

For the exterior column, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{2(32.5\times32.25\times32.25/2)}{2\times32.25\times32.5+56.5\times32.5} = 8.60\ \text{in.}$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2\left(\frac{b_1d^3}{12}+\frac{db_1^3}{12}+(b_1d)\left(\frac{b_1}{2}-c_{AB}\right)^2\right)+b_2dc_{AB}^2$$

$$J_c = 2\left(\frac{32.25\times32.5^3}{12}+\frac{32.5\times32.25^3}{12}+(32.25\times32.5)\left(\frac{32.25}{2}-8.60\right)^2\right)+56.5\times32.5\times8.60^2 = 620{,}710\ \text{in.}^4$$

$$\gamma_f = \frac{1}{1+(2/3)\times\sqrt{b_1/b_2}}$$ ***ACI 318-14 (8.4.2.3.2)***

$$\gamma_f = \frac{1}{1+(2/3)\times\sqrt{32.25/56.5}} = 0.665$$

$$\gamma_v = 1-\gamma_f = 1-0.665 = 0.335$$ ***ACI 318-14 (Eq. 8.4.4.2.2)***

The length of the critical perimeter for the exterior column:

$$b_o = 2\times(32.25)+(56.5) = 121\text{ in.}$$

The two-way shear stress (v_u) can then be calculated as:

$$\text{v}_\text{u} = \frac{\text{V}_\text{u}}{\text{b}_\text{o}\times\text{d}} + \frac{\gamma_\text{v}\text{M}_\text{unb}\left(b_1 - c_{AB}\right)}{J_c}$$ **ACI 318-14 (R.8.4.4.2.3)**

$$\text{v}_\text{u} = \frac{405.1\times1000}{121\times32.5} + \frac{0.335\times(6950\times1000)\times(32.25-8.60)}{620,710} = 192\text{ psi}$$

$$\text{v}_\text{c} = \min\left[4\lambda\sqrt{f_c'}\ ,\ \left(2+\frac{4}{\beta}\right)\lambda\sqrt{f_c'}\ ,\ \left(\frac{\alpha_s d}{b_o}+2\right)\lambda\sqrt{f_c'}\right]$$ **ACI 318-14 (Table 22.6.5.2)**

$$\text{v}_\text{c} = \min\left[4\times1\times\sqrt{3000}\ ,\ \left(2+\frac{4}{1}\right)\times1\times\sqrt{3000}\ ,\ \left(\frac{30\times32.5}{121}+2\right)\times1\times\sqrt{3000}\right]$$

$$\text{v}_\text{c} = \min\left[219.1\ ,\ 328.6\ ,\ 550.9\right] = 219.1\text{ psi}$$

$$\phi v_c = 0.75\times219.1 = 164\text{ psi}$$

Since $\phi v_c < v_u$ at the critical section, the slab does not have adequate two-way shear strength at this joint.

Increase the footing thickness to 40 in. with effective depth, d = 36.5 in.

$$\text{V}_\text{u} = 480 - 5.92\left(\frac{34.25\times60.5}{144}\right) = 394.8\text{ kips}$$

$$\text{M}_\text{unb} = 480\times17.16 - 5.92\left(\frac{34.25\times60.5}{144}\right)\times8.04 = 7552\text{ kip-in.}$$

$$b_1 = c_1 + d/2 = 16 + 36.5/2 = 34.25\text{ in.}$$

$$b_2 = c_2 + d = 24 + 36.5 = 60.5\text{ in.}$$

For the exterior column, the location of the centroidal axis z-z is:

$$\text{c}_\text{AB} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{2(36.5\times34.25\times34.25/2)}{2\times34.25\times36.5+60.5\times36.5} = 9.09\text{ in.}$$

The polar moment J_c of the shear perimeter is:

$$\text{J}_c = 2\left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + \left(b_1 d\right)\left(\frac{b_1}{2} - c_{AB}\right)^2\right) + b_2 d c_{AB}^2$$

$$\text{J}_c = 2\left(\frac{34.25\times36.5^3}{12} + \frac{36.5\times34.25^3}{12} + (34.25\times36.5)\left(\frac{34.25}{2} - 9.09\right)^2\right) + 60.5\times36.5\times9.09^2 = 865,875\text{ in.}^4$$

$$\gamma_f = \frac{1}{1+(2/3)\times\sqrt{b_1/b_2}}$$ ***ACI 318-14 (8.4.2.3.2)***

$$\gamma_f = \frac{1}{1+(2/3)\times\sqrt{34.25/60.5}} = 0.666$$

$$\gamma_v = 1 - \gamma_f = 1 - 0.666 = 0.334$$ ***ACI 318-14 (Eq. 8.4.4.2.2)***

The length of the critical perimeter for the exterior column:

$$b_o = 2\times(34.25)+(60.5) = 129 \text{ in.}$$

The two-way shear stress (v_u) can then be calculated as:

$$\mathrm{v_u} = \frac{\mathrm{V_u}}{\mathrm{b_o\times d}} + \frac{\gamma_v \mathrm{M_{unb}}\left(b_1 - c_{AB}\right)}{J_c}$$ ***ACI 318-14 (R.8.4.4.2.3)***

$$\mathrm{v_u} = \frac{394.8\times1000}{129\times34.5} + \frac{0.334\times(7552\times1000)\times\left(34.25-9.09\right)}{865{,}875} = 157 \text{ psi}$$

$$\mathrm{v_c} = \min\left[4\lambda\sqrt{f_c'}\ ,\ \left(2+\frac{4}{\beta}\right)\lambda\sqrt{f_c'}\ ,\ \left(\frac{\alpha_s d}{b_o}+2\right)\lambda\sqrt{f_c'}\right]$$ ***ACI 318-14 (Table 22.6.5.2)***

$$\mathrm{v_c} = \min\left[4\times1\times\sqrt{3000}\ ,\ \left(2+\frac{4}{1}\right)\times1\times\sqrt{3000}\ ,\ \left(\frac{30\times32.5}{121}+2\right)\times1\times\sqrt{3000}\right]$$

$$\mathrm{v_c} = \min\left[219.1\ ,\ 328.6\ ,\ 550.9\right] = 219.1 \text{ psi}$$

$$\phi v_c = 0.75\times219.1 = 164 \text{ psi}$$

Since $\phi v_c > v_u$ at the critical section, the slab has adequate two-way shear strength at this joint.

Use a combined footing 25 ft 4 in. by 8 ft in plan, 3 ft 4 in. thick, with effective depth 36.5 in.

3. One-Way Shear Capacity Check

The critical section for one-way shear is located at distance *d* from the face of the column. The one-way shear capacity of the foundation can be calculated using the following equation:

$$\phi V_c = \phi\times2\times\lambda\times\sqrt{f_c'}\times b_w\times d$$ ***ACI 318-14 (22.5.5.1)***

Where ϕ = 0.75 ***ACI 318-14 (Table 21.2.1)***

This example focus on the calculation of two-way shear capacity for combined foundation. For more details on the one-way shear check for foundation check "Reinforced Concrete Shear Wall Foundation (Strip Footing) Analysis and Design" example.

4. Flexural Reinforcement Design

4.1. Negative Moment (Midspan)

The critical section for moment is shown in the moment diagram in Figure 2. The design moment is:

$M_u = 2100$ kip-ft

Use d = 36.5 in.

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the footing section (jd). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.9, and jd will be taken equal to $0.95d$. The assumptions will be verified once the area of steel in finalized.

Assume $jd = 0.95 \times d = 34.68$ in.

$$A_s = \frac{M_u}{\varphi f_y jd} = \frac{2100 \times 12000}{0.9 \times 60000 \times 34.68} = 13.5 \text{ in.}^2$$

Recalculate 'a' for the actual $A_s = 13.5 \text{ in.}^2 \rightarrow a = \frac{A_s f_y}{0.85 f'_c b} = \frac{13.5 \times 60000}{0.85 \times 3000 \times 8 \times 12} = 3.30$ in.

$$c = \frac{a}{\beta_1} = \frac{3.30}{0.85} = 3.88 \text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right) d_t - 0.003 = \left(\frac{0.003}{3.88}\right) \times 36.5 - 0.003 = 0.0252 > 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{2100 \times 12000}{0.9 \times 60000 \times (36.5 - 3.30/2)} = 13.4 \text{ in.}^2$$

Depending of the method of analysis the minimum area of reinforcement shall be calculated using beam provisions or one-way slab provisions. In this case both beam and slab provisions will be illustrated.

For beam provisions:

$$A_{s,\min} = \text{Greater of} \begin{Bmatrix} \dfrac{3\sqrt{f'_c}}{f_y} \\ \dfrac{200}{f_y} \end{Bmatrix} \times b \times d$$ ***ACI 318-14 (9.6.1.2)***

$$A_{s,\min} = \text{Greater of} \begin{Bmatrix} \dfrac{3\sqrt{3000}}{60,000} \\ \dfrac{200}{60,000} \end{Bmatrix} \times (8 \times 12) \times 36.5 = \frac{200}{60,000} \times (8 \times 12) \times 36.5 = 11.7 \text{ in.}^2 < 13.4 \text{ in.}^2$$

Use 17-#8 top bars with A_s = 13.43 in.^{2} at midspan.

For slab provisions:

$$A_{s,\min} = \text{Greater of} \begin{Bmatrix} \dfrac{0.0018\times 60,000}{f} \\ 0.0014 \end{Bmatrix} \times b\times h \qquad \textit{ACI 318-14 (7.6.1.1)}$$

$$A_{s,\min} = 0.0018\times(8\times 12)\times 40 = 6.91\text{ in.}^2 < 13.4\text{ in.}^2$$

Use 17-#8 top bars with A_s = 13.43 in.^{2} at midspan.

In spMats, the slab provisions for minimum reinforcement can be used since the finite element analysis calculates the required area of steel in both the x (longitudinal) and y (transverse) direction independently.

4.2. Positive Moment (At Interior Column)

For beam provisions:

Repeating the same process at Section 4.1, A_s = 3.3 in.^{2} << $A_{s,min}$ = 11.7 in.^{2}. Thus, use 15-#8 bottom bars with A_s = 11.9 in.^{2} at the interior column.

For slab provisions:

Repeating the same process at Section 4.1, A_s = 3.3 in.^{2} << $A_{s,min}$ = 6.91 in.^{2}. Thus, use 9-#8 bottom bars with A_s = 7.11 in.^{2} at the interior column.

Note code provisions permit the use of reinforcement of one third more than is required by analysis in some cases.

5. Combined Footing Analysis and Design – spMats Software

spMats uses the Finite Element Method for the structural modeling and analysis of reinforced concrete slab systems or mat foundations subject to static loading conditions.

The slab, mat, or footing is idealized as a mesh of rectangular elements interconnected at the corner nodes. The same mesh applies to the underlying soil with the soil stiffness concentrated at the nodes. Slabs of irregular geometry can be idealized to conform to geometry with rectangular boundaries. Even though slab and soil properties can vary between elements, they are assumed uniform within each element.

For illustration and comparison purposes, the following figures provide a sample of the input modules and results obtained from spMats models created for the reinforced concrete combined footing at a property line in this example. Two models were created for this example (the first model for the footing with 36 in. thickness that failed in punching shear check around the exterior column, and the second model for the same footing with a revised thickness of 40 in.).

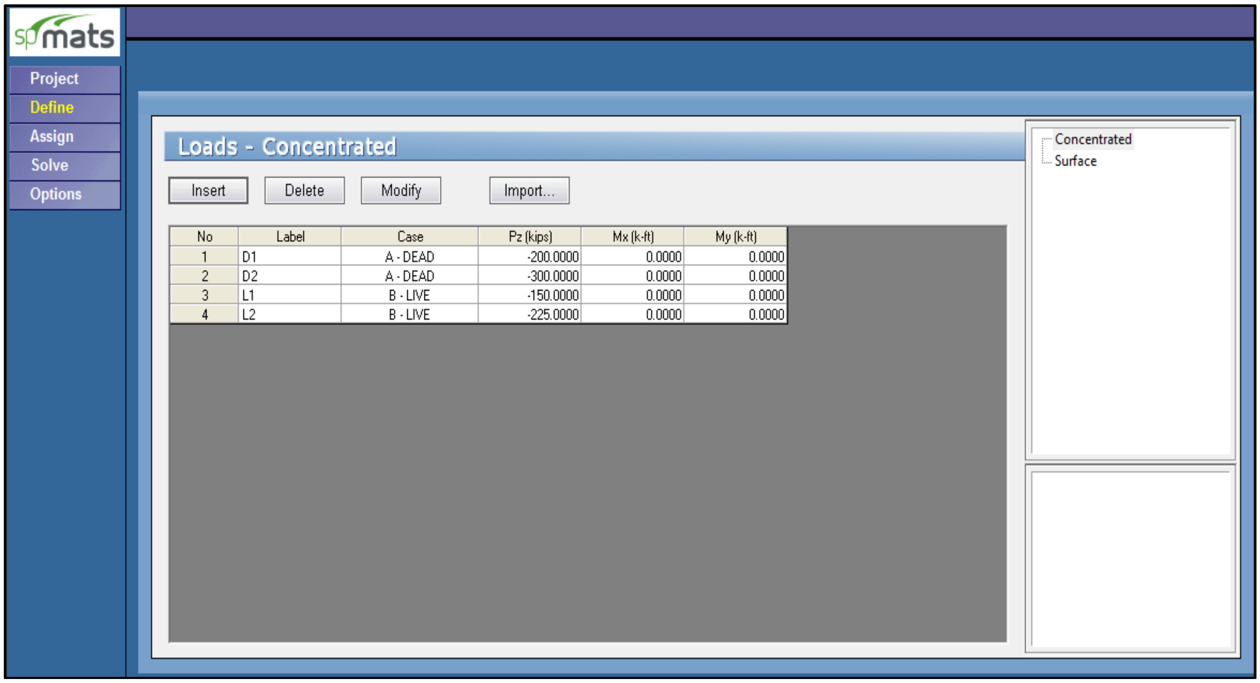


Figure 4 –Defining Service Loads (spMats)

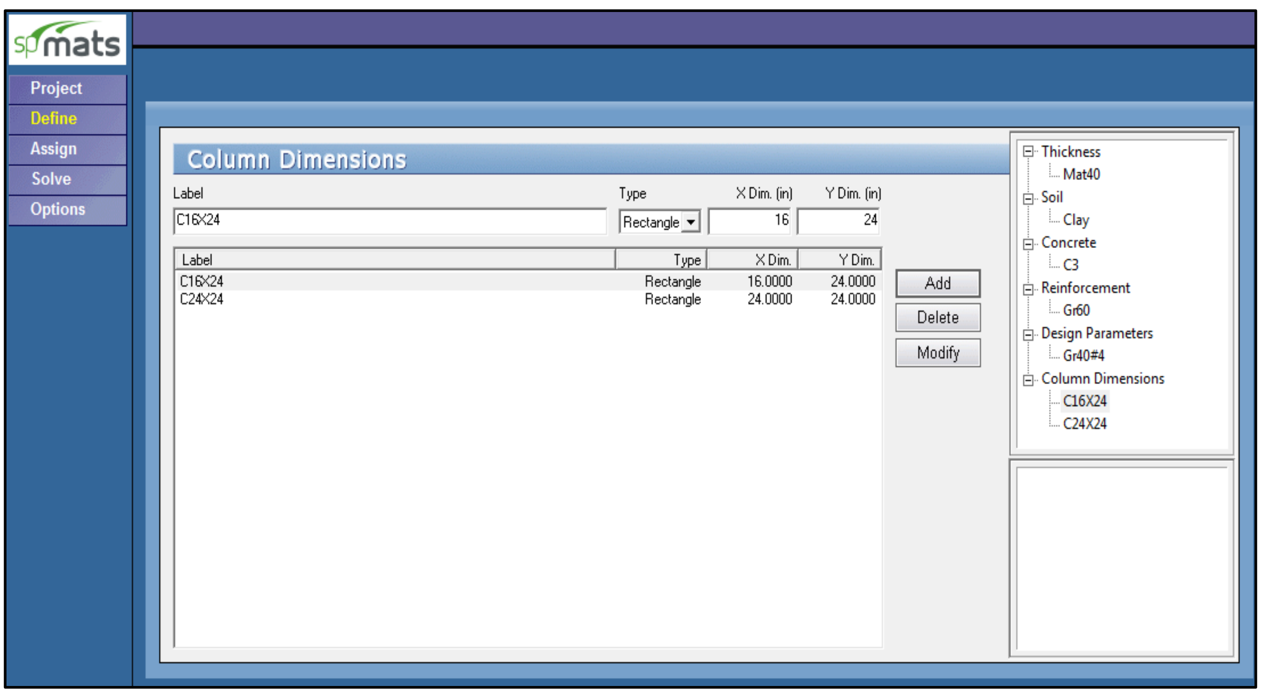


Figure 5 –Defining Columns Dimensions (spMats)

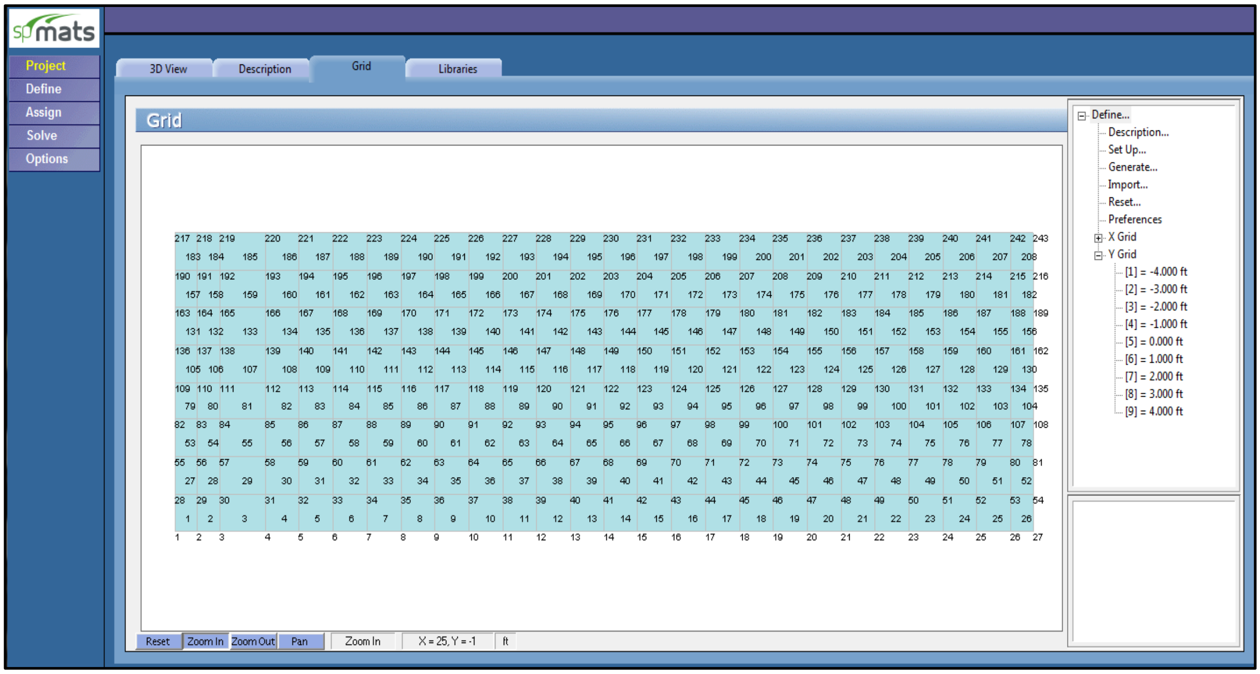


Figure 6 –Mesh Generation (spMats) Showing Node & Element Numbering

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STRUCTUREPOINT - spMats v8.12 (TM)
Licensed to: StructurePoint, License ID: 66184-1055149-4-2C6B6-2C6B6
C:\TSDA\Combined Footing\Combined Footing_36.ma8

B6 - Punching Shear Around Columns (Ultimate Load Combinations):
================================================================
Units --> Applied Shear Force Vu (kips), Applied Moments Mux, Muy (k-ft)
          Factored Shear Stress vu (psi), Factored Shear Resistance Phi*vc (psi)
          Concrete Strength f'c (psi), distances X_Offset, Y_Offset (ft)
          Average depth (in), Dimensions Bx, By (ft)
          Area (in^2), Jxx, Jyy, Jxy (in^4)

Geometry of Resisting Area
--------------------------
      _________Column_________ Average  ___Dimensions____ _____Centroid____
 Node      Label       Location  Depth      Bx       By    X_Offset Y_Offset
----- -------------- --------- -------- -------- -------- -------- --------
  109 C16X24          Edge        32.50     2.69     4.71     1.97    -0.00
  130 C24X24          Inner       32.50     4.71     4.71    -0.00    -0.00

Properties of Resisting Area
----------------------------
 Node  Column Label      Area         Jxx          Jyy          Jxy
----- -------------- ---------- ------------ ------------ ------------
  109 C16X24            3932.50   2323047.50    620710.00         0.00
  130 C24X24            7345.00   4231102.50   4231102.50         0.00

Ultimate Load Combination: U1
-----------------------------

  Factored Applied Forces:
  ------------------------
   Node  Column Label      Vu             Mux        Gamma_X        Muy        Gamma_Y
  ----- -------------- ---------- --------------- ------- --------------- -------
    109 C16X24            -480.00            -0.0   0.469          -316.9   0.335
    130 C24X24            -720.00            -0.0   0.400             0.0   0.400

  Factored Stress and Capacity:
  -----------------------------
                                                             _Critical Point__
   Node  Column Label      vu           f'c       Phi*vc    X_Offset Y_Offset Status
  ----- -------------- ----------- ---------- ---------- -------- -------- ------
    109 C16X24             -266.99    3000.00     164.32     0.00     2.35 Unsafe
    130 C24X24              -98.03    3000.00     164.32     2.35     2.35   Safe


B7 - Punching Shear Around Piles (Ultimate Load Combinations):
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    * No piles assigned
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Figure 7 – Punching Shear Output 36 in. Strip Footing (spMats)

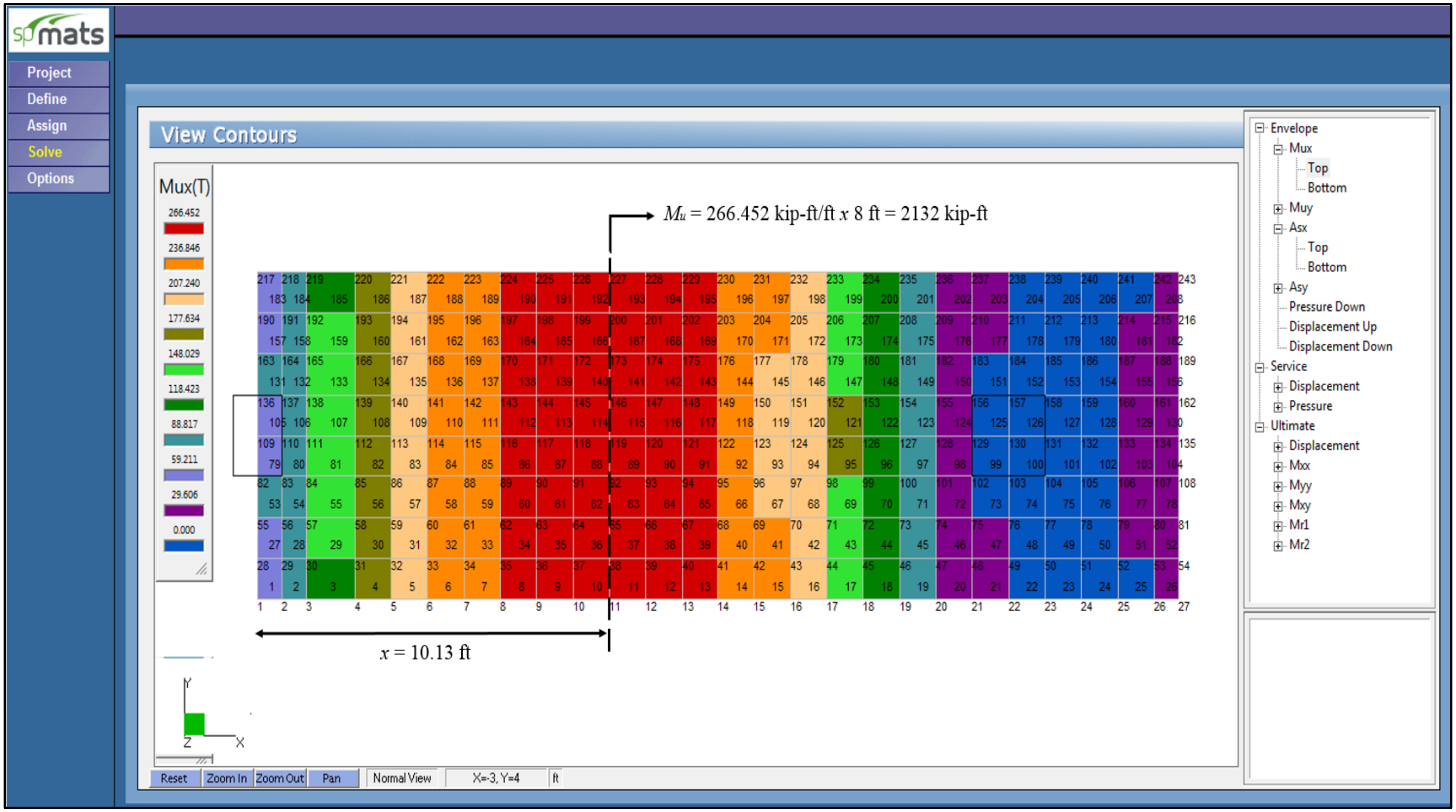

Figure 8 – Ultimate Moment Contour 40 in. Footing (spMats)

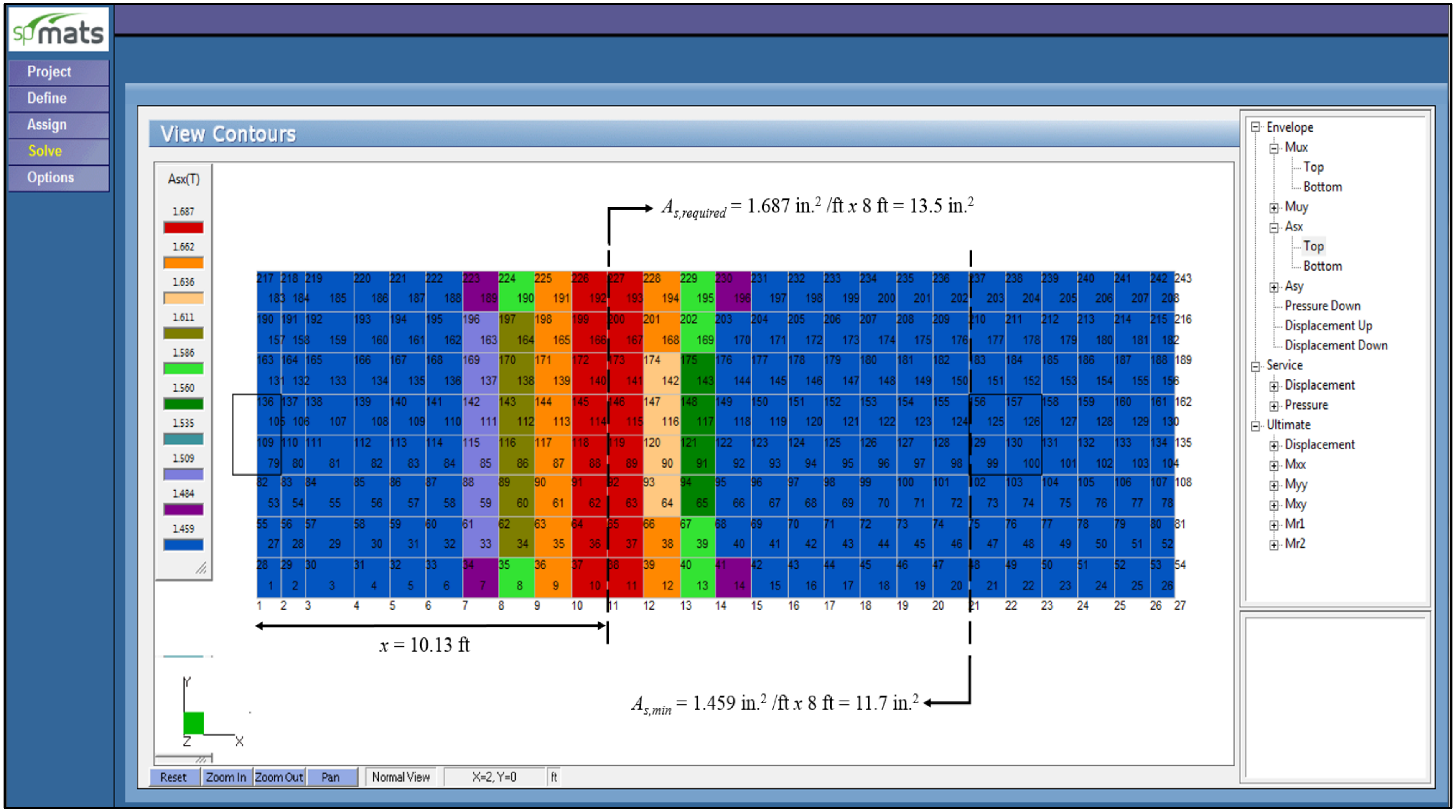

Figure 9 – Required Reinforcement Contour 40 in. Footing (spMats)

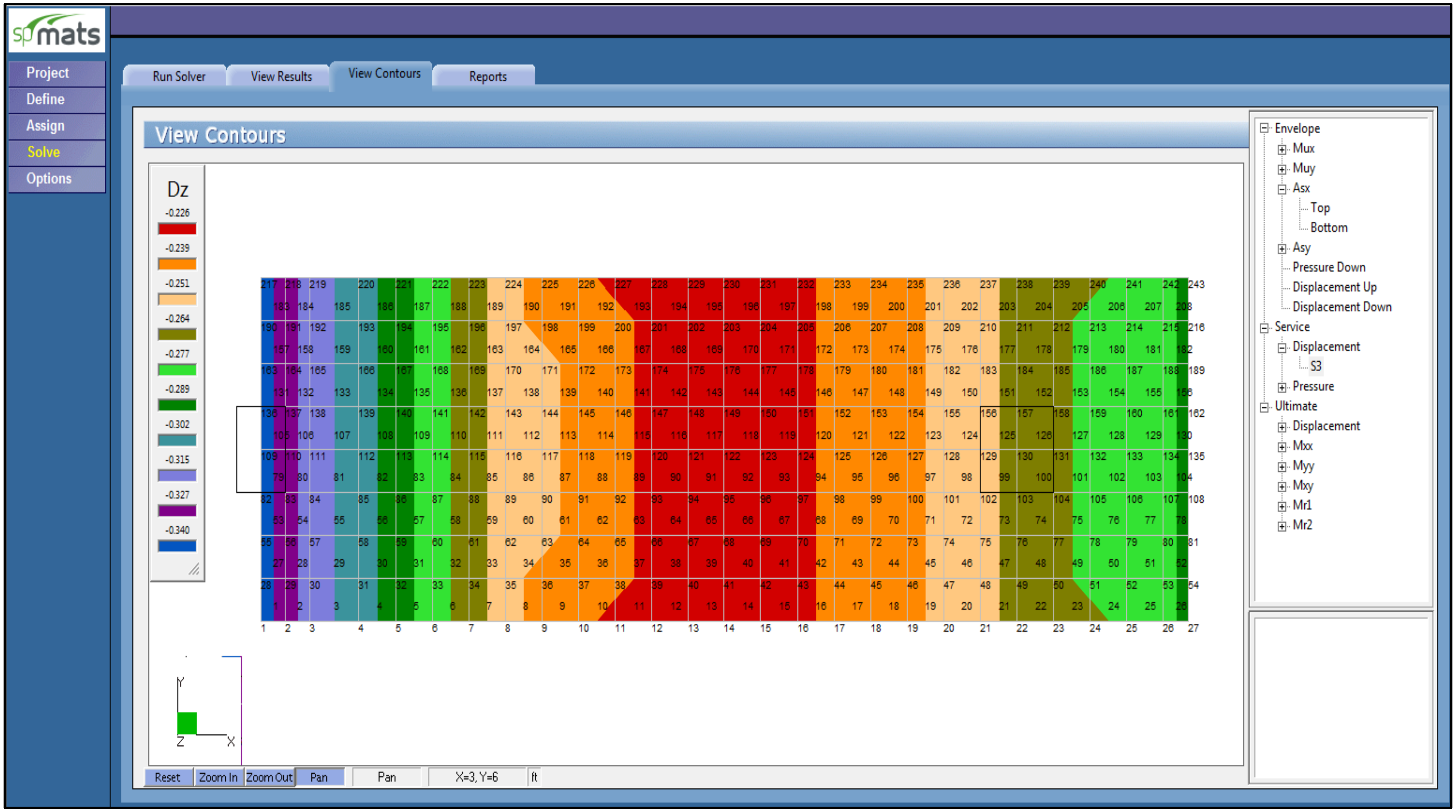

Figure 10 –Vertical Displacement Contour 40 in. Footing (spMats)

6. Design Results Comparison and Conclusions

Table 1 - Comparison of Two-Way (Punching) Shear Check Results for Footing with 36 in. Thickness

Support	b_1, in.			b_2, in.			c_{AB}, in.		
	Reference	Hand	spMats	Reference	Hand	spMats	Reference	Hand	spMats
Exterior	32.25	32.25	32.25	56.5	56.5	56.5	8.6	8.6	8.6
Interior	56.5	56.5	56.5	56.5	56.5	56.5	28.25	28.25	28.25
Support	J_c, in.^{4}			γ_v			V_u, kips		
	Reference	Hand	spMats	Reference	Hand	spMats	Reference	Hand	spMats
Exterior	621,000	620,710	620,710	0.335	0.335	0.335	480	480	480
Interior	---	4,231,103	4,231,103	---	0.4	0.4	720	720	720
Support	$M_{u,punching}$, kips-ft			v_u, psi			ϕv_c, psi		
	Reference	Hand	spMats	Reference	Hand	spMats	Reference	Hand	spMats
Exterior	579	579	943	192	192	267	164	164.3	164.3
Interior	0	0	0	80.2	80.2	98	164	164.3	164.3

Table 2 - Flexural Reinforcement Comparison - Longitudinal Direction

M_u, kips-ft			$A_{s,required}$, in.^{2}			$A_{s,min}$, in.^{2}		
Reference	Hand	spMats	Reference	Hand	spMats	Reference	Hand*	spMats*
2100	2100	2132	13.5	13.4	13.5	11.7	11.7	11.7

* Using beam provisions to find $A_{s,min}$ to be consistent with Reference approach. However, engineering judgment need to be taken to decide if the combined footing need to be treated as a one-way slab or beam.

The results of all the hand calculations and the reference used illustrated above are in agreement with the automated exact results obtained from the spMats program except for v_u values.

In spMats, the factored unbalanced moment used for shear transfer, M_{unb}, is calculated as the sum of the moments at finite element nodes within the critical section (resisting zone). Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account. The reference take into account only the moments of the vertical reaction and soil pressure with respect to the centroid of the critical section.