

Effective Flexural Stiffness for Critical Buckling Load of Concrete Columns

A primary concern in calculating the critical axial buckling load P_c is the choice of the stiffness that reasonably approximates the variation in stiffness due to cracking, creep, and concrete nonlinearity. $(EI)_{eff}$ is used in the process of determining the moment magnification at column ends and along the column length in sway and nonsway frames.

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2} \quad \text{ACI 318-19 (6.6.4.4.2)}$$

$$\delta = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1.0 \quad (\text{For nonsway frames}) \quad \text{ACI 318-19 (6.6.4.5.2)}$$

$$\delta_s = \frac{1}{1 - \frac{\Sigma P_u}{0.75 \times \Sigma P_c}} \quad (\text{For sway frames}) \quad \text{ACI 318-19 (6.6.4.6.2b)}$$

ACI 318-19 provides three options to calculate $(EI)_{eff}$ as follows:

$$(EI)_{eff} = \left\{ \begin{array}{l} \frac{0.4E_c I_g}{1 + \beta_{dns}} \quad (6.6.4.4.4a) \\ \frac{0.2E_c I_g + E_s I_{se}}{1 + \beta_{dns}} \quad (6.6.4.4.4b) \\ \frac{E_c I}{1 + \beta_{dns}} \quad (6.6.4.4.4c) \end{array} \right\} \quad \text{ACI 318-19 (6.6.4.4.4)}$$

AASHTO code provides two options to calculate EI for compression members in bridge structures as follows:

$$EI = \text{Larger of } \left\{ \begin{array}{l} \frac{\frac{E_c I_g}{5} + E_s I_s}{1 + \beta_d} \quad (5.6.4.3-1) \\ \frac{E_c I_g}{1 + \beta_d} \quad (5.6.4.3-2) \end{array} \right\} \quad \text{AASHTO 9th Edition (5.6.4.3)}$$

The moment of inertia of the column or wall section, I , in Eq. (6.6.4.4.4c) is calculated as follows:

$$0.35I_g \leq I = \left(0.80 + 25 \frac{A_{st}}{A_g} \right) \times \left(1 - \frac{M_u}{P_u h} - 0.5 \frac{P_u}{P_o} \right) \times I_g \leq 0.875I_g \quad \text{ACI 318-19 (Table 6.6.3.1.1(b))}$$

spColumn Program utilizes Eq. (6.6.4.4.4b) for the calculation of the effective flexural stiffness, $(EI)_{eff}$, of column section. The other two equations, namely, Eq. (6.6.4.4.4a), and Eq. (6.6.4.4.4c) are also permitted by the ACI 318.

Comparison of $(EI)_{eff}$ values for Individual Columns

ACI 318 states that Eq. (6.6.4.4.4a) is a simplified form of Eq. (6.6.4.4.4b) and therefore, is less 'accurate'. If the reinforcing steel is not yet chosen, I_{se} cannot be computed and Eq. (6.6.4.4.4a) is the only option to compute an initial value for $(EI)_{eff}$. On the other hand, ACI 318 states that Eq. (6.6.4.4.4c) provides improved accuracy in $(EI)_{eff}$ calculation. However, a more complex formula of moment of inertia, I , is required. In that formula, P_u , and M_u values from each load combination must be considered. Alternatively, enveloped values of P_u , and M_u can be used conservatively to compute the lowest value of I .

A different value for the magnitude of the magnified moment is possible for each option of $(EI)_{eff}$. So which equation will lead to the optimum column design based on the ACI code provisions? To answer this question Eq. (6.6.4.4.4a) is set equal to Eq. (6.6.4.4.4b) as follows:

$$\frac{0.4E_c I_g}{1 + \beta_{dns}} = \frac{0.2E_c I_g + E_s I_{se}}{1 + \beta_{dns}}$$

This will lead to the relative stiffness non-dimensional factor α to be used to illustrate the comparison:

$$\alpha = \frac{E_s I_{se}}{E_c I_g} = 0.2$$

When α is greater than 0.2, $(EI)_{eff}$ obtained from Eq. (6.6.4.4.4b) will be greater than $(EI)_{eff}$ obtained from Eq. (6.6.4.4.4a) resulting in a lower value for the magnification factor and a lower magnified moment.

Conclusions

The designer may utilize any of the options for $(EI)_{eff}$ provided in ACI 318, section 6.6.4.4.4 given the available input data at a given design stage and the desired level of accuracy. [spColumn](#) Program utilizes Eq. (6.6.4.4.4b) which is more accurate than Eq. (6.6.4.4.4a) and less accurate (but less complex) than Eq. (6.6.4.4.4c). The designer may choose between Eq. (6.6.4.4.4a) and Eq. (6.6.4.4.4c) to optimize the required moment magnification and finalize column size and reinforcement.

For bridge designer using the AASHTO the values of EI are given as:

$$EI = \text{Larger of } \left\{ \begin{array}{l} \frac{E_c I_g + E_s I_s}{5} + E_s I_s \quad (5.6.4.3-1) \\ \frac{E_c I_g}{1 + \beta_d} \\ \frac{E_c I_g}{2.5} \\ \frac{E_c I_g}{1 + \beta_d} \quad (5.6.4.3-2) \end{array} \right\} \quad \underline{\underline{\text{AASHTO 9}^{\text{th}} \text{ Edition (5.6.4.3)}}$$

References

- [1] Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19), American Concrete Institute, 2019
- [2] Wight J.K., Reinforced Concrete, Mechanics and Design, Eighth Edition, Pearson Education Inc., 2021
- [3] Darwin D., Dolan C.W., Design of Concrete Structures, Sixteenth Edition, McGraw-Hill Education, 2021
- [4] AASHTO LRFD Bridge Design Specifications, 9th Edition, American Association of State Highway and Transportation Officials, 2020
- [5] "[Effective Flexural Stiffness for Critical Buckling Load of Concrete Columns](#)" Technical Article, STRUCTUREPOINT, 2017