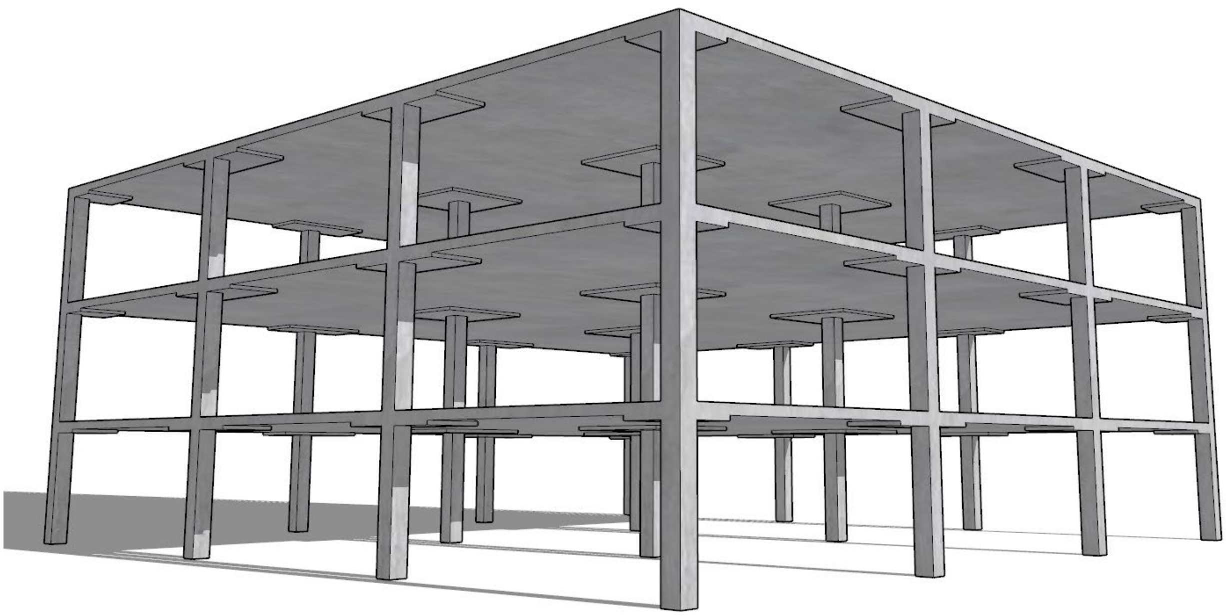
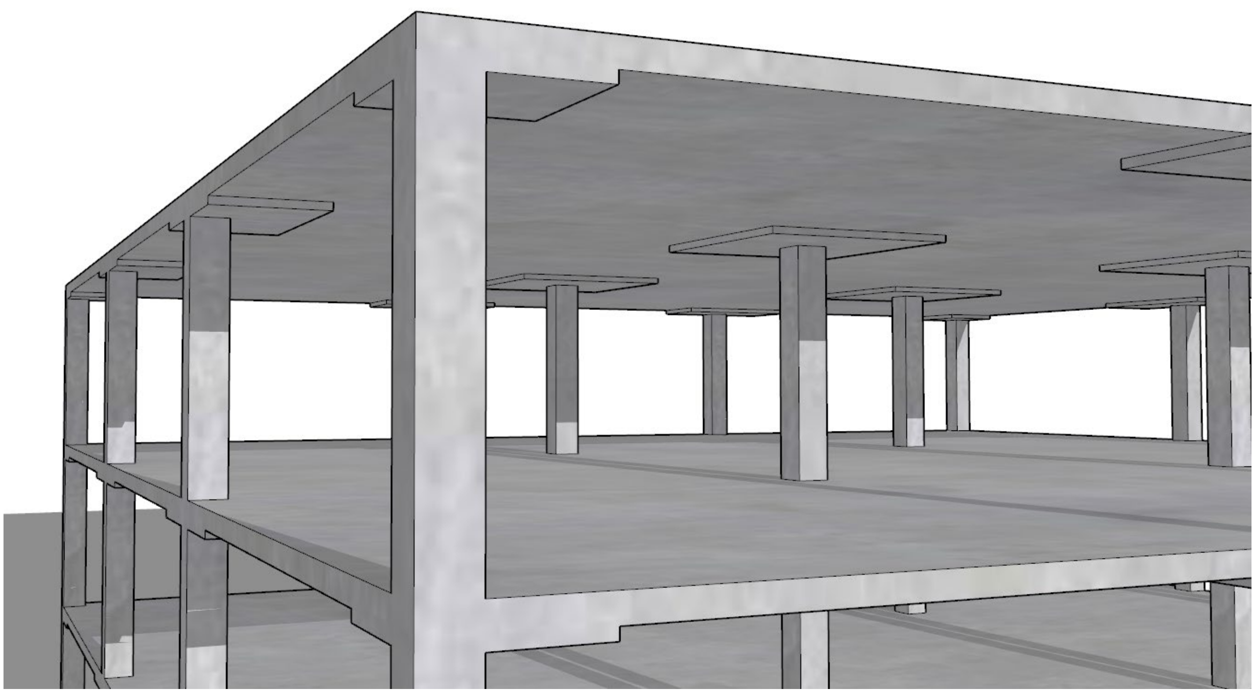
Two-Way Flat Slab (Concrete Floor with Drop Panels) System Analysis and Design (ACI 318-14)





Two-Way Flat Slab (Concrete Floor with Drop Panels) System Analysis and Design (ACI 318-14)

Design the concrete floor slab system shown below for an intermediate floor considering partition weight = 20 psf, and unfactored live load = 60 psf. The lateral loads are independently resisted by shear walls. The use of flat plate system will be checked. If the use of flat plate is not adequate, the use of flat slab system with drop panels will be investigated. Flat slab concrete floor system is similar to the flat plate system. The only exception is that the flat slab uses drop panels (thickened portions around the columns) to increase the nominal shear strength of the concrete at the critical section around the columns. The Equivalent Frame Method (EFM) shown in ACI 318 is used in this example. The hand solution from EFM is also used for a detailed comparison with the model results of spSlab engineering software program.

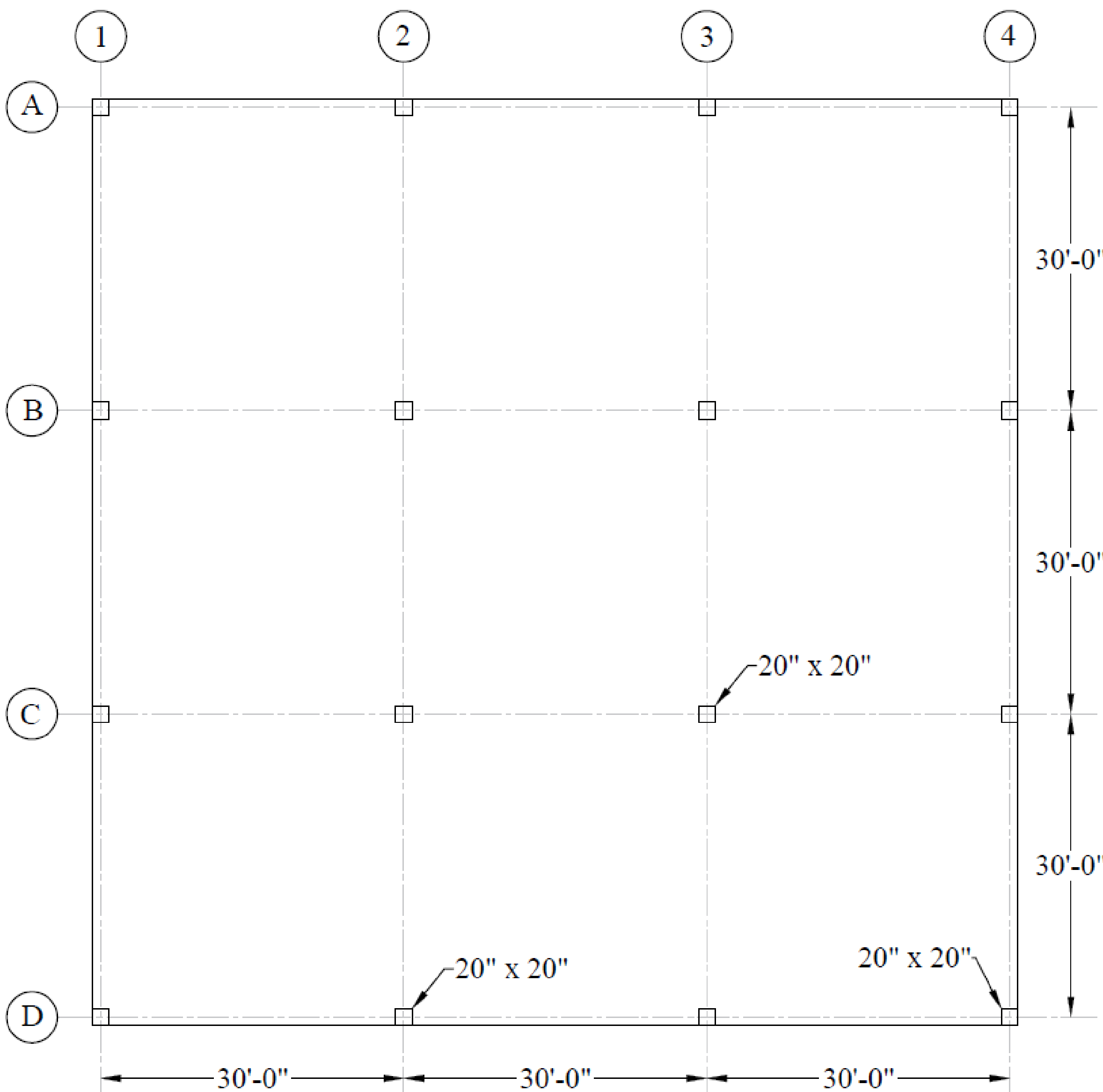


Figure 1 – Two-Way Flat Concrete Floor System

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference

- Concrete Floor Systems (Guide to Estimating and Economizing), Second Edition, 2002 David A. Fanella
- Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association.
- Simplified Design of Reinforced Concrete Buildings, Fourth Edition, 2011 Mahmoud E. Kamara and Lawrence C. Novak
- Control of Deflection in Concrete Structures (ACI 435R-95)
- spSlab Engineering Software Program Manual v5.50, STRUCTUREPOINT, 2018

Design Data

Story Height = 13 ft (provided by architectural drawings)

Superimposed Dead Load, *SDL* =20 psf for framed partitions, wood studs, 2 × 2, plastered 2 sides

ASCE/SEI 7-10 (Table C3-1)

Live Load, *LL* = 60 psf ***ASCE/SEI 7-10 (Table 4-1)***

50 psf is considered by inspection of Table 4-1 for Office Buildings – Offices (2/3 of the floor area)

80 psf is considered by inspection of Table 4-1 for Office Buildings – Corridors (1/3 of the floor area)

$LL = 2/3 \times 50 + 1/3 \times 80 = 60$ psf

$f_c' = 5{,}000$ psi (for slab)

$f_c' = 6{,}000$ psi (for columns)

$f_y = 60{,}000$ psi

1. Preliminary Member Sizing

1.1. For Flat Plate (Without Drop Panels)

1.1.1. Slab Minimum Thickness – Deflection

ACI 318-14 (8.3.1.1)

In lieu of detailed calculation for deflections, ACI 318 Code gives minimum slab thickness for two-way construction without interior beams in ***Table 8.3.1.1***.

For this flat plate slab systems the minimum slab thicknesses per ***ACI 318-14*** are:

Exterior Panels: $h_s = \dfrac{l_n}{30} = \dfrac{340}{30} = 11.33 \text{ in.}$ ***ACI 318-14 (Table 8.3.1.1)***

But not less than 5 in. ***ACI 318-14 (8.3.1.1(a))***

Interior Panels: $h_s = \dfrac{l_n}{33} = \dfrac{340}{33} = 10.30 \text{ in.}$ ***ACI 318-14 (Table 8.3.1.1)***

But not less than 5 in. ***ACI 318-14 (8.3.1.1(a))***

Where l_n = length of clear span in the long direction = 30 × 12 – 20 = 340 in.

Try 11 in. slab for all panels (self-weight = 150 pcf × 11 in. /12 = 137.5 psf)

1.1.2. Slab Shear Strength – One Way Shear

At a preliminary check level, the use of average effective depth would be sufficient. However, after determining the final depth of the slab, the exact effective depth will be used in flexural, shear and deflection calculations. Evaluate the average effective depth (Figure 2):

$$d_l = h_s - c_{clear} - d_b - \frac{d_b}{2} = 11 - 0.75 - 0.75 - \frac{0.75}{2} = 9.13 \text{ in.}$$

$$d_t = h_s - c_{clear} - \frac{d_b}{2} = 11 - 0.75 - \frac{0.75}{2} = 9.88 \text{ in.}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{9.13 + 9.88}{2} = 9.50 \text{ in.}$$

Where:

c_{clear} = 3/4 in. for # 6 steel bar **_ACI 318-14 (Table 20.6.1.3.1)_**

d_b = 0.75 in. for # 6 steel bar

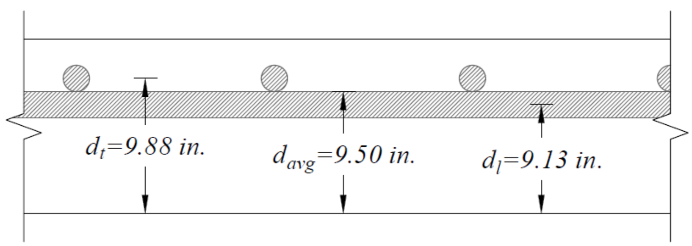


Figure 2 – Average Effective Depth for Flat Plate

Factored dead load, $q_{Du} = 1.20 \times (137.50 + 20.00) = 189.00$ psf

Factored live load, $q_{Lu} = 1.60 \times 60.00 = 96.00$ psf **_ACI 318-14 (5.3.1)_**

Total factored load, $q_u = 189.00 + 96.00 = 285.00$ psf

Check the adequacy of slab thickness for beam action (one-way shear) **_ACI 318-14 (22.5)_**

At an interior column:

Consider a 12-in. wide strip. The critical section for one-way shear is located at a distance d, from the face of support (see Figure 3):

Tributary area for one-way shear is:

$$A_{Tributary} = \left[\frac{30}{2} - \frac{20}{2\times12} - \frac{9.50}{12}\right]\times\frac{12}{12} = 13.38 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.285\times13.38 = 3.81 \text{ kips}$$

$$V_c = 2\times\lambda\times\sqrt{f'_c}\times b_w \times d$$ ***ACI 318-14 (Eq. 22.5.5.1)***

Where $\lambda = 1$ for normal weight concrete, more information can be found in "Concrete Type Classification Based on Unit Density" technical article.

$$\phi V_c = 0.75\times2.0\times1.0\times\sqrt{5{,}000}\times12\times\frac{9.50}{1{,}000} = 12.09 \text{ kips} > V_u = 3.81 \text{ kips}$$

Slab thickness of 11 in. is adequate for one-way shear.

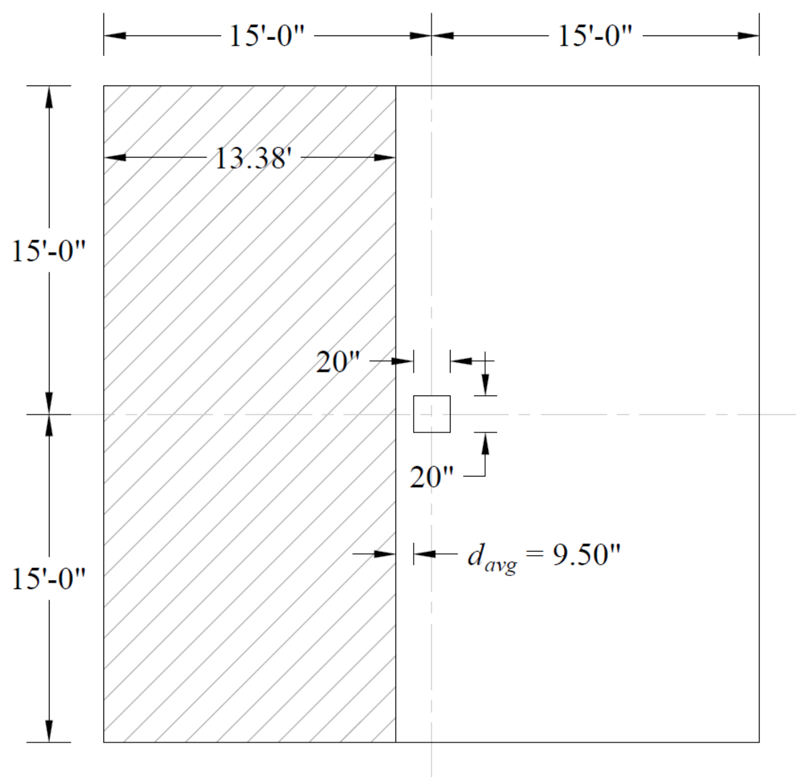


Figure 3 – Critical Section for One-Way Shear

1.1.3. Slab Shear Strength – Two-Way Shear

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior column (Figure 4):

Tributary area for two-way shear is:

$$A_{Tributary} = (30 \times 30) - \left(\frac{20 + 9.50}{12}\right)^2 = 893.96 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.285 \times 893.96 = 254.78 \text{ kips}$$

$$V_c = 4 \times \lambda \times \sqrt{f_c'} \times b_o \times d \quad \text{(For square interior column)}$$ ***ACI 318-14 (Table 22.6.5.2(a))***

$$V_c = 4 \times \sqrt{5,000} \times \left(4 \times (20 + 9.50)\right) \times \frac{9.50}{1,000} = 317.07 \text{ kips}$$

$$\phi V_c = 0.75 \times 317.07 = 237.80 \text{ kips} < V_u = 254.78 \text{ kips}$$

Slab thickness of 11 in. is not adequate for two-way shear. It is good to mention that the factored shear (V_u) used in the preliminary check does not include the effect of the unbalanced moment at supports. Including this effect will lead to an increase of V_u value as shown later in section 4.2.

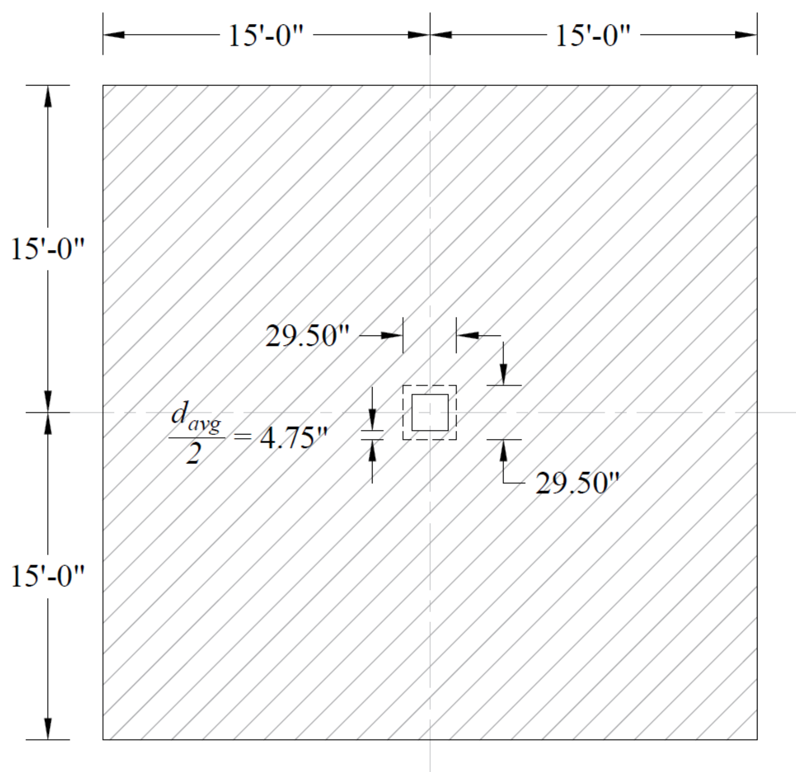


Figure 4 – Critical Section for Two-Way Shear

In this case, four options could be used: 1) to increase the slab thickness, 2) to increase columns cross sectional dimensions or cut the spacing between columns (reducing span lengths), however, this option is assumed to be not permissible in this example due to architectural limitations, 3) to use headed shear reinforcement, or 4) to use drop panels. In this example, the latter option will be used to achieve better understanding for the design of two-way slab with drop panels often called flat slab.

Check the drop panel dimensional limitations as follows:

1) The drop panel shall project below the slab at least one-fourth of the adjacent slab thickness.

ACI 318-14 (8.2.4(a))

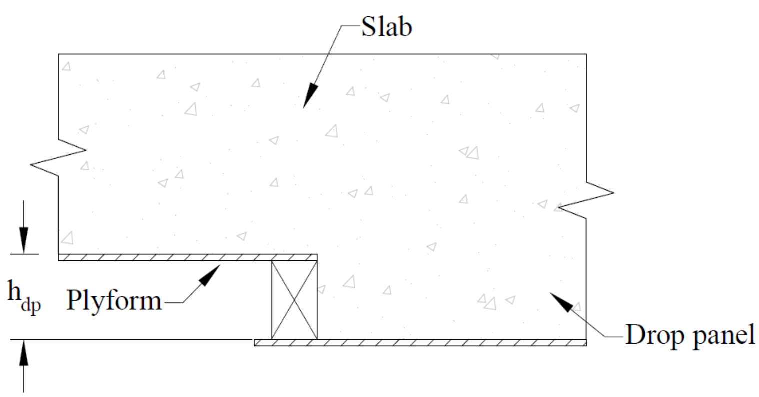
Since the slab thickness (h_s) is 10 in. (on page 9), the thickness of the drop panel should be at least:

$$h_{dp,\,min} = 0.25 \times h_s = 0.25 \times 10 = 2.50 \text{ in.}$$

Drop panel dimensions are also controlled by formwork considerations. The Figure 5 shows the standard lumber dimensions that are used when forming drop panels. Using other depths will unnecessarily increase formwork costs.

For nominal lumber size (2x), h_{dp} = 4.25 in. > $h_{dp,\,min}$ = 2.50 in.

The total thickness including the slab and the drop panel (h) = h_s + h_{dp} = 10 + 4.25 = 14.25 in.



Nominal Lumber Size (in.)	Actual Lumber Size (in.)	Plyform Thickness (in.)	h_{dp} (in.)
2x	1 1/2	3/4	2 1/4
4x	3 1/2	3/4	4 1/4
6x	5 1/2	3/4	6 1/4
8x	7 1/4	3/4	8

Figure 5 – Drop Panel Formwork Details

2) The drop panel shall extend in each direction from the centerline of support a distance not less than one-sixth the span length measured from center-to-center of supports in that direction.

ACI 318-14 (8.2.4(b))

$$\ell_{1,dp} = \frac{1}{6} \times \ell_1 + \frac{1}{6} \times \ell_1 = \frac{1}{6} \times 30 + \frac{1}{6} \times 30 = 10 \text{ ft}$$

$$\ell_{2,dp} = \frac{1}{6} \times \ell_2 + \frac{1}{6} \times \ell_2 = \frac{1}{6} \times 30 + \frac{1}{6} \times 30 = 10 \text{ ft}$$

Based on the previous discussion, Figure 6 shows the dimensions of the selected drop panels around interior, edge (exterior), and corner columns.

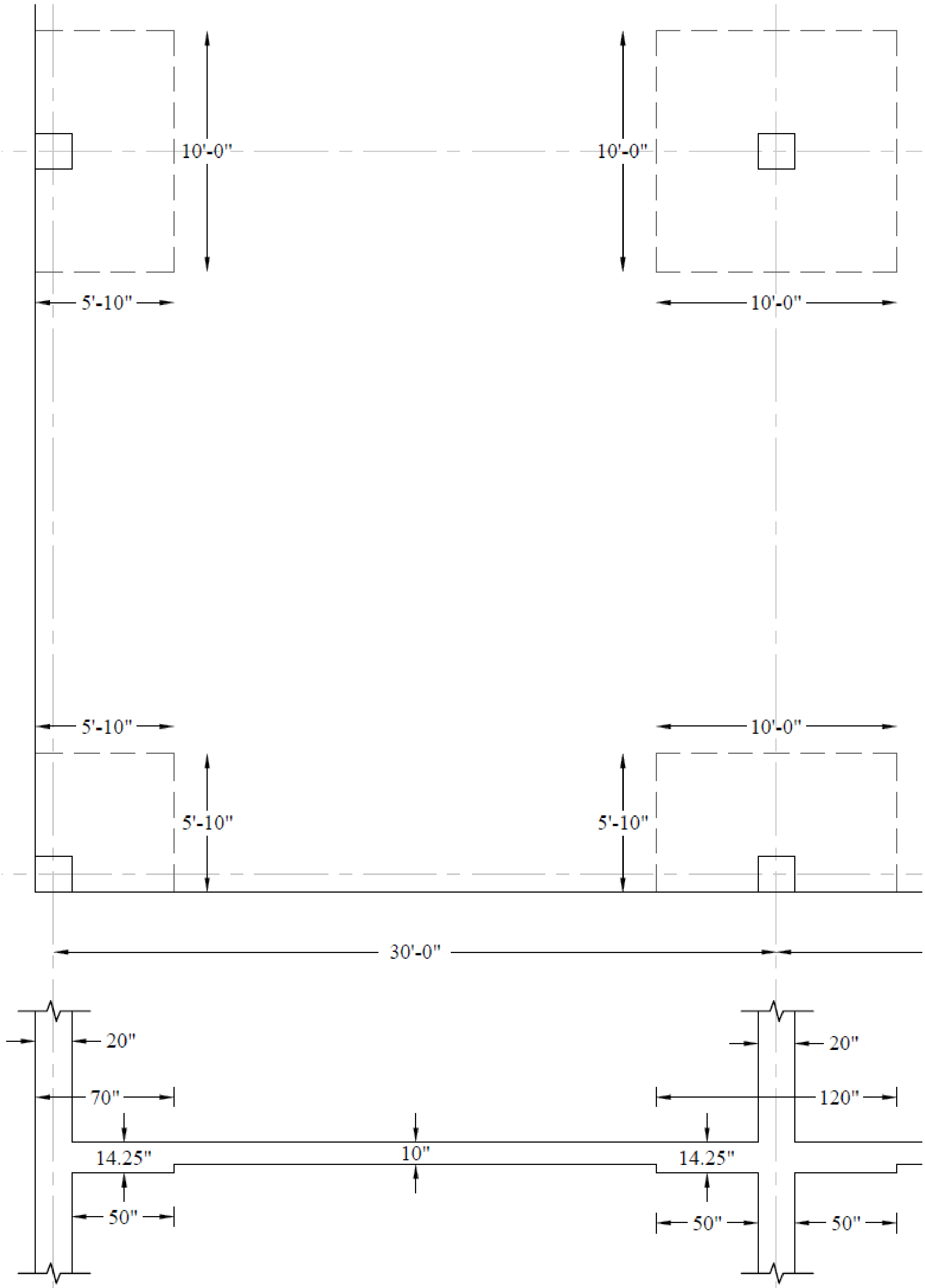


Figure 6 – Drop Panels Dimensions

1.2. For Flat Slab (with Drop Panels)

For slabs with changes in thickness and subjected to bending in two directions, it is necessary to check shear at multiple sections as defined in the ***ACI 318-14***. The critical sections shall be located with respect to:

1) Edges or corners of columns. ***ACI 318-14 (22.6.4.1(a))***

2) Changes in slab thickness, such as edges of drop panels. ***ACI 318-14 (22.6.4.1(b))***

1.2.1. Slab Minimum Thickness – Deflection

ACI 318-14 (8.3.1.1)

In lieu of detailed calculation for deflections, ACI 318 Code gives minimum slab thickness for two-way construction without interior beams in ***Table 8.3.1.1***.

For this flat plate slab systems the minimum slab thicknesses per **ACI 318-14** are:

Exterior Panels: $h_s = \frac{l_n}{33} = \frac{340}{33} = 10.30 \text{ in.}$ ***ACI 318-14 (Table 8.3.1.1)***

But not less than 4 in. ***ACI 318-14 (8.3.1.1(b))***

Interior Panels: $h_s = \frac{l_n}{36} = \frac{340}{36} = 9.44 \text{ in.}$ ***ACI 318-14 (Table 8.3.1.1)***

But not less than 4 in. ***ACI 318-14 (8.3.1.1(b))***

Where l_n = length of clear span in the long direction = 30 × 12 – 20 = 340 in.

Try 10 in. slab for all panels

Self-weight for slab section without drop panel = 150 pcf × 10 in. /12 = 125.00 psf

Self-weight for slab section with drop panel = 150 pcf × 14.25 in. /12 = 178.13 psf

1.2.2. Slab Shear Strength – One Way Shear

For critical section at distance *d* from the edge of the column (slab section with drop panel):

Evaluate the average effective depth:

$$d_l = h_s - c_{clear} - d_b - \frac{d_b}{2} = 14.25 - 0.75 - 0.75 - \frac{0.75}{2} = 12.38 \text{ in.}$$

$$d_t = h_s - c_{clear} - \frac{d_b}{2} = 14.25 - 0.75 - \frac{0.75}{2} = 13.13 \text{ in.}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{12.38 + 13.13}{2} = 12.75 \text{ in.}$$

Where:

c_{clear} = 3/4 in. for # 6 steel bar ***ACI 318-14 (Table 20.6.1.3.1)***

d_b = 0.75 in. for # 6 steel bar

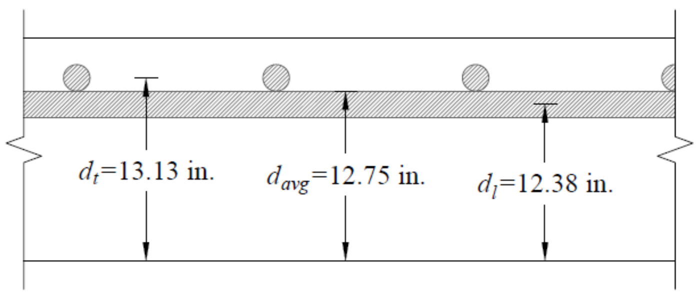


Figure 7 – Average Effective Depth for Slab Section with Drop Panel

Factored dead load, $q_{Du} = 1.20 \times (178.13 + 20.00) = 237.75$ psf

Factored live load, $q_{Lu} = 1.60 \times 60.00 = 96.00$ psf ***ACI 318-14 (5.3.1)***

Total factored load, $q_u = 237.75 + 96.00 = 333.75$ psf

Check the adequacy of slab thickness for beam action (one-way shear) from the edge of the interior column

ACI 318-14 (22.5)

Consider a 12-in. wide strip. The critical section for one-way shear is located at a distance *d*, from the edge of the column (see Figure 8)

Tributary area for one-way shear is:

$$A_{Tributary} = \left[\frac{30}{2} - \frac{20}{2\times12} - \frac{12.75}{12}\right]\times\frac{12}{12} = 13.10 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.334\times13.10 = 4.37 \text{ kips}$$

$$V_c = 2\times\lambda\times\sqrt{f_c'}\times b_w \times d$$

ACI 318-14 (Eq. 22.5.5.1)

Where $\lambda = 1$ for normal weight concrete

$$\phi V_c = 0.75\times2\times1.0\times\sqrt{5{,}000}\times12\times\frac{12.75}{1{,}000} = 16.23 \text{ kips} > V_u = 4.37 \text{ kips}$$

Slab thickness of 14.25 in. is adequate for one-way shear for the first critical section (from the edge of the column).

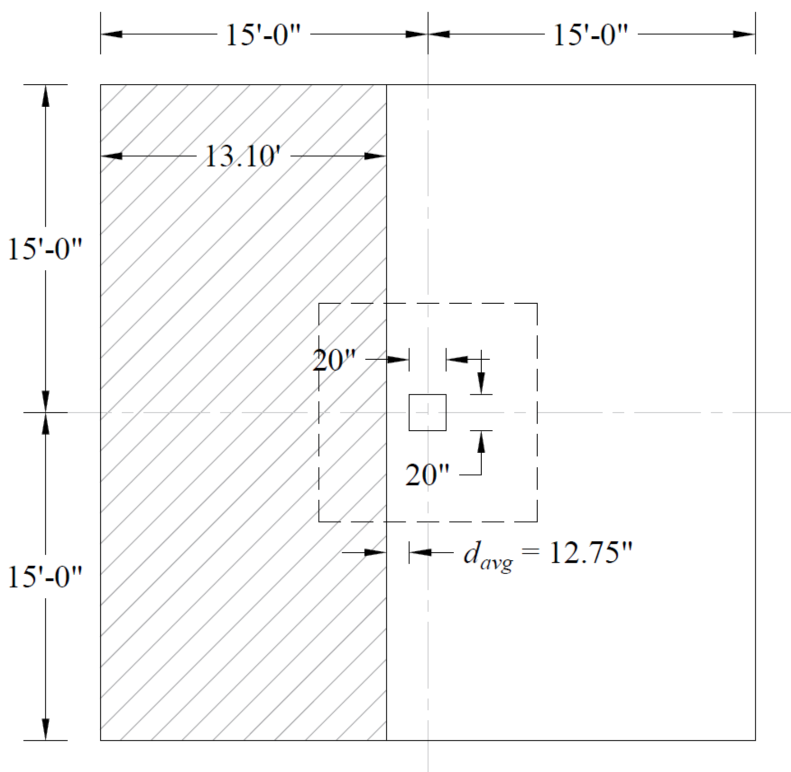


Figure 8 – Critical Section at Distance *d* from the Edge of the Column for One-Way Shear

For critical section at the edge of the drop panel (slab section without drop panel):

Evaluate the average effective depth:

$$d_l = h_s - c_{clear} - d_b - \frac{d_b}{2} = 10 - 0.75 - 0.75 - \frac{0.75}{2} = 8.13 \text{ in.}$$

$$d_t = h_s - c_{clear} - \frac{d_b}{2} = 10 - 0.75 - \frac{0.75}{2} = 8.88 \text{ in.}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{8.13 + 8.88}{2} = 8.50 \text{ in.}$$

Where:

c_{clear} = 3/4 in. for # 6 steel bar **_ACI 318-14 (Table 20.6.1.3.1)_**

d_b = 0.75 in. for # 6 steel bar

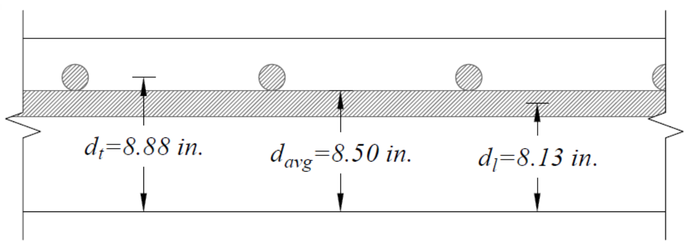


Figure 9 – Average Effective Depth for Slab Section without Drop Panel

Factored dead load, $q_{Du} = 1.20 \times (125.00 + 20.00) = 174.00$ psf

Factored live load, $q_{Lu} = 1.60 \times 60.00 = 96.00$ psf **_ACI 318-14 (5.3.1)_**

Total factored load, $q_u = 174.00 + 96.00 = 270.00$ psf

Check the adequacy of slab thickness for beam action (one-way shear) from the edge of the interior drop panel

ACI 318-14 (22.5)

Consider a 12-in. wide strip. The critical section for one-way shear is located at the face of support (see Figure 10)

Tributary area for one-way shear is:

$$A_{Tributary} = \left[\frac{30}{2} - \frac{10}{2}\right] \times \frac{12}{12} = 10.00 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.270 \times 10.00 = 2.70 \text{ kips}$$

$$V_c = 2 \times \lambda \times \sqrt{f_c'} \times b_w \times d$$

ACI 318-14 (Eq. 22.5.5.1)

Where $\lambda = 1$ for normal weight concrete

$$\phi V_c = 0.75 \times 2.0 \times 1.0 \times \sqrt{5,000} \times 12 \times \frac{8.50}{1,000} = 10.82 \text{ kips} > V_u = 2.70 \text{ kips}$$

Slab thickness of 10 in. is adequate for one-way shear for the second critical section (from the edge of the drop panel).

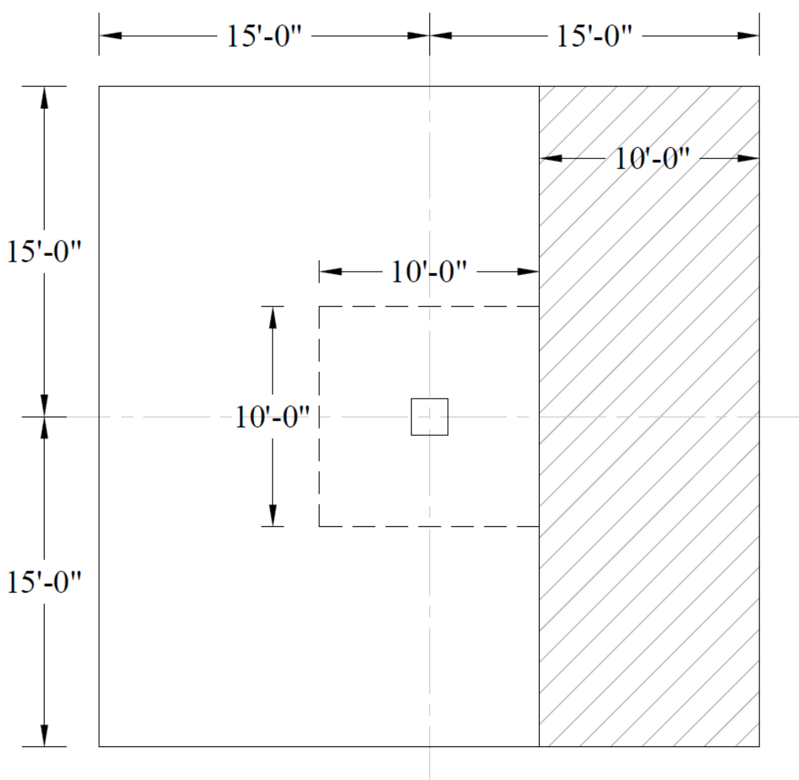


Figure 10 – Critical Section at the Face of the Drop Panel for One-Way Shear

1.2.3. Slab Shear Strength – Two-Way Shear

For critical section at distance *d*/2 from the edge of the column (slab section with drop panel):

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior column (Figure 11):

Tributary area for two-way shear is:

$$A_{Tributary} = (30 \times 30) - \left(\frac{20+12.75}{12}\right)^2 = 892.55 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.334 \times 892.55 = 297.89 \text{ kips}$$

$$V_c = 4 \times \lambda \times \sqrt{f_c'} \times b_o \times d \text{ (For square interior column)}$$ ***ACI 318-14 (Table 22.6.5.2(a))***

$$V_c = 4 \times 1.0 \times \sqrt{5,000} \times \left(4 \times \left(20+12.75\right)\right) \times \frac{12.75}{1,000} = 472.42 \text{ kips}$$

$$\phi V_c = 0.75 \times 472.42 = 354.31 \text{ kips} > V_u = 297.89 \text{ kips}$$

Slab thickness of 14.25 in. is adequate for two-way shear for the first critical section (from the edge of the column).

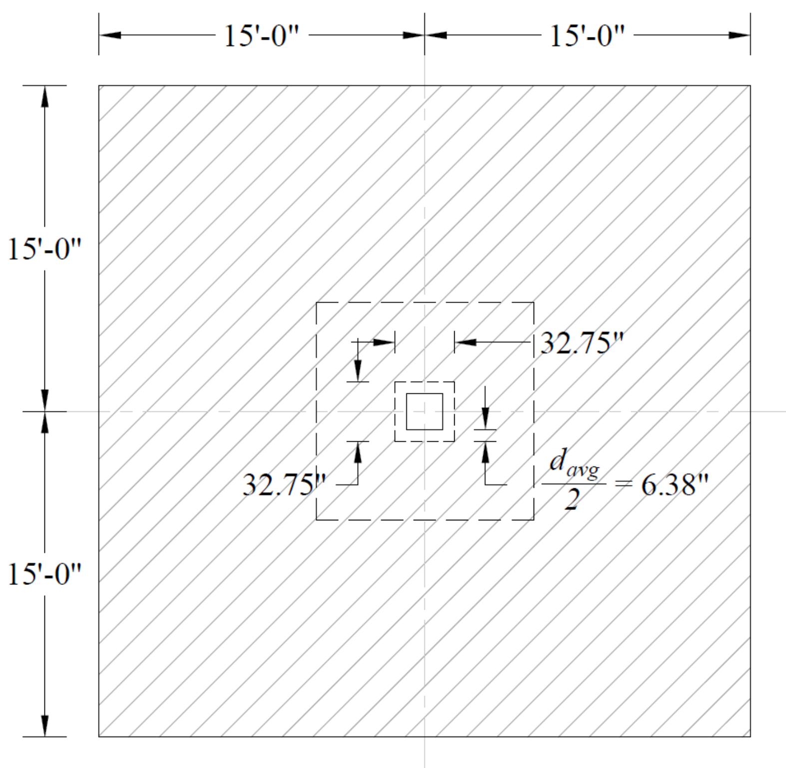


Figure 11 – Critical Section at *d*/2 from the Edge of the Column for Two-Way Shear

For critical section at the edge of the drop panel (slab section without drop panel):

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior drop panel (Figure 12):

Tributary area for two-way shear is:

$$A_{Tributary} = (30 \times 30) - \left(\frac{120 + 8.50}{12}\right)^2 = 785.33 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.270 \times 785.33 = 212.04 \text{ kips}$$

$$V_c = 4 \times \lambda \times \sqrt{f'_c} \times b_o \times d \quad \text{(For square interior column)}$$ ***ACI 318-14 (Table 22.6.5.2(a))***

$$V_c = 4 \times 1.0 \times \sqrt{5,000} \times \left(4 \times \left(120 + 8.50\right)\right) \times \frac{8.50}{1,000} = 1235.74 \text{ kips}$$

$$\phi V_c = 0.75 \times 1235.74 = 926.80 \text{ kips} > V_u = 212.04 \text{ kips}$$

Slab thickness of 10 in. is adequate for two-way shear for the second critical section (from the edge of the drop panel).

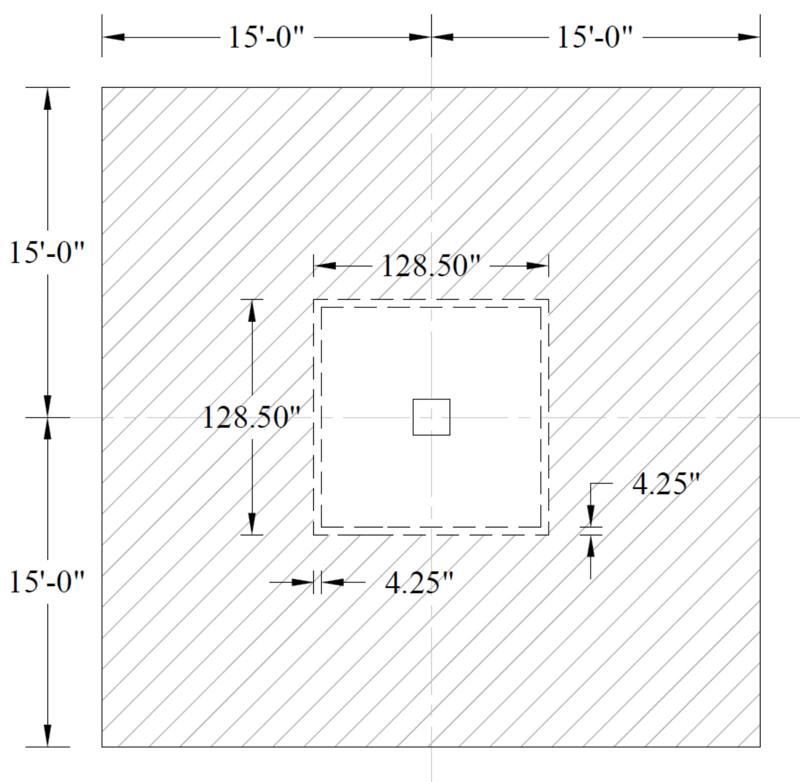


Figure 12 – Critical Section at $d/2$ from the Edge of the Drop Panel for Two-Way Shear

1.2.4. Column Dimensions - Axial Load

Check the adequacy of column dimensions for axial load:

For live load, superimposed dead load, and self-weight of the slab around an interior column:

$q_u = 270 \text{ psf}$ (see on page 12)

$A_{Tributary} = 30 \times 30 = 900 \text{ ft}^2$

For self-weight of additional slab thickness due to the presence of the drop panel around an interior column:

$q_u = 333.75 - 270 = 63.75 \text{ psf}$ (see on page 10 and 12)

$A_{Tributary} = 10 \times 10 = 100 \text{ ft}^2$

Assuming five story building

$P_u = n \times q_u \times A_{Tributary} = 5 \times (0.270 \times 900 + 0.064 \times 100) = 1246.88 \text{ kips}$

Assume 20 in. square column with 4 – No. 14 vertical bars with design axial strength, $\phi P_{n,max}$ of

$\phi P_{n,\max} = 0.80 \times \phi \times (0.85 \times f_c' \times (A_g - A_{st}) + f_y \times A_{st})$ ***ACI 318-14 (22.4.2)***

$\phi P_{n,\max} = 0.80 \times 0.65 \times (0.85 \times 6,000 \times (20 \times 20 - 4 \times 2.25) + 60,000 \times 4 \times 2.25) = 1,317.73 \text{ kips}$

$\phi P_{n,\max} = 1,317.73 \text{ kips} > P_u = 1,246.88 \text{ kips}$

Column dimensions of 20 in. × 20 in. are adequate for axial load.

2. Flexural Analysis and Design

ACI 318 states that a slab system shall be designed by any procedure satisfying equilibrium and geometric compatibility, provided that strength and serviceability criteria are satisfied. Distinction of two-systems from one-way systems is given by ***ACI 318-14 (R8.10.2.3 & R8.3.1.2)***.

ACI 318 permits the use of Direct Design Method (DDM) and Equivalent Frame Method (EFM) for the gravity load analysis of orthogonal frames and is applicable to flat plates, flat slabs, and slabs with beams. The following sections outline the solution per EFM and spSlab software. For the solution per DDM, check the flat plate example.

2.1. Equivalent Frame Method (EFM)

EFM is the most comprehensive and detailed procedure provided by the ACI 318 for the analysis and design of two-way slab systems where the structure is modeled by a series of equivalent frames (interior and exterior) on column lines taken longitudinally and transversely through the building.

The equivalent frame consists of three parts (for a detailed discussion of this method, refer to the flat plate design example):

1) Horizontal slab-beam strip.

2) Columns or other vertical supporting members.

3) Elements of the structure (Torsional members) that provide moment transfer between the horizontal and vertical members.

2.1.1. Limitations for Use of Equivalent Frame Method

In EFM, live load shall be arranged in accordance with 6.4.3 which requires slab systems to be analyzed and designed for the most demanding set of forces established by investigating the effects of live load placed in various critical patterns. ***ACI 318-14 (8.11.1.2 & 6.4.3)***

Complete analysis must include representative interior and exterior equivalent frames in both the longitudinal and transverse directions of the floor. ***ACI 318-14 (8.11.2.1)***

Panels shall be rectangular, with a ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2. ***ACI 318-14 (8.10.2.3)***

2.1.2. Frame Members of Equivalent Frame

Determine moment distribution factors and fixed-end moments for the equivalent frame members. The moment distribution procedure will be used to analyze the equivalent frame. Stiffness factors *k*, carry over factors COF, and fixed-end moment factors FEM for the slab-beams and column members are determined using the design aids tables at ***Appendix 20A of PCA Notes on ACI 318-11***. These calculations are shown below.

a) Flexural stiffness of slab-beams at both ends, K_{sb}.

$$\frac{c_{N1}}{\ell_1} = \frac{20}{(30\times 12)} = 0.056\ ,\ \frac{c_{N2}}{\ell_2} = \frac{20}{(30\times 12)} = 0.056$$

For $c_{F1} = c_{F2}$, stiffness factors, $k_{NF} = k_{FN} = 5.587$ ***PCA Notes on ACI 318-11 (Table A2 & A3)***

Thus, $K_{sb} = k_{NF} \times \frac{E_{cs}\times I_s}{\ell_1} = 5.587\times \frac{E_{cs}\times I_s}{\ell_1}$ ***PCA Notes on ACI 318-11 (Table A2 & A3)***

$$K_{sb} = 5.587\times \frac{4,287\times 10^3 \times 30,000}{360} = 1,995,955,750 \text{ in.-lb}$$

Where, $I_s = \frac{\ell_2 \times h^3}{12} = \frac{360\times (10)^3}{12} = 30,000 \text{ in.}^4$

$E_{cs} = w_c^{1.5} \times 33\times \sqrt{f_c'} = 150^{1.5} \times 33\times \sqrt{5,000} = 4,287\times 10^3 \text{ psi}$ ***ACI 318-14 (19.2.2.1.a)***

Carry-over factor $COF = 0.578$ ***PCA Notes on ACI 318-11 (Table A2 & A3)***

Fixed-end moment $FEM = \sum_{i=1}^{n} m_{NFi} \times w_i \times \ell_1^2$ ***PCA Notes on ACI 318-11 (Table A2 & A3)***

Uniform load fixed end moment coefficient, $m_{NF1} = 0.0915$

Fixed end moment coefficient for (b-a) = 0.2 when a = 0, $m_{NF2} = 0.0163$

Fixed end moment coefficient for (b-a) = 0.2 when a = 0.8, $m_{NF3} = 0.002$

b) Flexural stiffness of column members at both ends, K_c.

Referring to ***Table A7, Appendix 20A***,

For the Bottom Column (Below):

$t_a = h/2 + h_{dp} = 10/2 + 4.25 = 9.25$ in. $t_b = h/2 = 10/2 = 5.00$ in.

$H = 13$ ft $= 156$ in. $H_c = H - t_a - t_b = 156 - 9.25 - 5 = 141.75$ in.

$$\frac{t_a}{t_b} = \frac{9.25}{5.00} = 1.85 \qquad \frac{H}{H_c} = \frac{156}{141.75} = 1.101$$

Thus, $k_{AB} = 5.318$ and $C_{AB} = 0.545$ by interpolation.

$$K_{c,bottom} = \frac{5.318 \times E_{cc} \times I_c}{\ell_c}$$ ***PCA Notes on ACI 318-11 (Table A7)***

$$K_{c,bottom} = 5.318 \times \frac{4,696 \times 10^3 \times 13,333}{156} = 2,134,472,479 \text{ in.-lb}$$

Where, $I_c = \frac{c^4}{12} = \frac{(20)^4}{12} = 13,333 \text{ in.}^4$

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f_c'} = 150^{1.5} \times 33 \times \sqrt{6,000} = 4,696 \times 10^3 \text{ psi}$$ ***ACI 318-14 (19.2.2.1.a)***

$l_c = 13$ ft $= 156$ in.

For the Top Column (Above):

$$\frac{t_b}{t_a} = \frac{5.00}{9.25} = 0.54 \qquad \frac{H}{H_c} = \frac{156}{141.75} = 1.101$$

Thus, $k_{AB} = 4.879$ and $C_{AB} = 0.595$ by interpolation

$$K_c = \frac{4.878 \times E_{cc} \times I_c}{\ell_c}$$ ***PCA Notes on ACI 318-11 (Table A7)***

$$K_{c,top} = 4.879 \times \frac{4,696 \times 10^3 \times 13,333}{156} = 1,958,272,137 \text{ in.-lb}$$

c) Torsional stiffness of torsional members, K_t.

$$K_t = \frac{9 \times E_{cs} \times C}{\left[\ell_2 \times \left(1 - \frac{c_2}{\ell_2}\right)^3\right]}$$

ACI 318-14 (R.8.11.5)

$$K_t = \frac{9 \times 4,287 \times 10^3 \times 10,632}{360 \times \left(1 - \frac{20}{30 \times 12}\right)^3} = 1,352,594,724 \text{ in.-lb}$$

Where $C = \sum \left(1 - 0.63 \times \frac{x}{y}\right) \times \left(\frac{x^3 \times y}{3}\right)$

ACI 318-14 (Eq. 8.10.5.2b)

$$C = \left(1 - 0.63 \times \frac{14.25}{20}\right) \times \left(\frac{14.25^3 \times 20}{3}\right) = 10,632 \text{ in.}^4$$

c_2 = 20 in., ℓ_2 = 30 ft = 360 in.

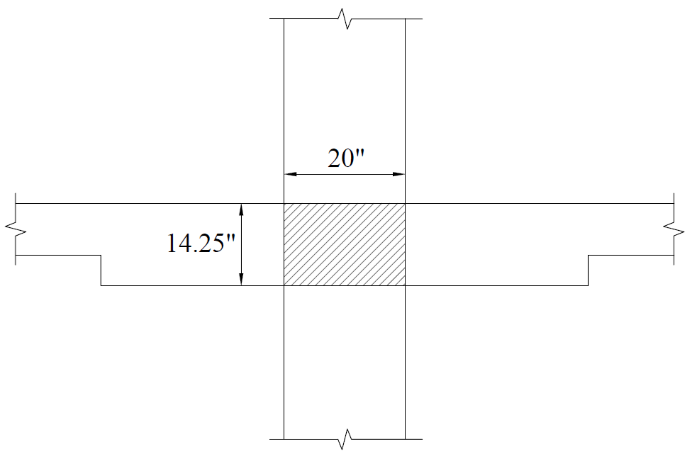


Figure 13 – Torsional Member

d) Equivalent column stiffness K_{ec}.

$$K_{ec} = \frac{\sum K_c \times \sum K_t}{\sum K_c + \sum K_t}$$

$$K_{ec} = \frac{(2134.47 + 1958.27) \times (2 \times 1352.59)}{[(2134.47 + 1958.27) + (2 \times 1352.59)]} \times 10^6 = 1,628,678,573 \text{ in.-lb}$$

Where $\sum K_t$ is for two torsional members one on each side of the column, and $\sum K_c$ is for the upper and lower columns at the slab-beam joint of an intermediate floor.

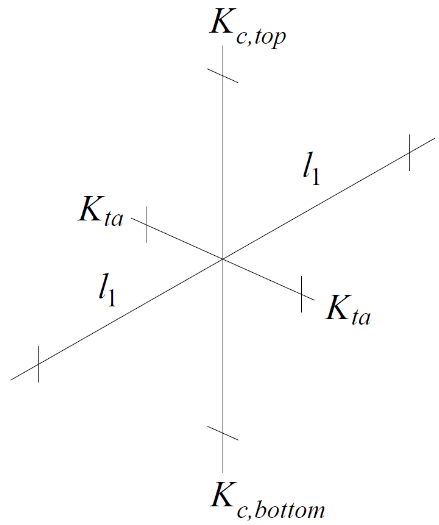


Figure 14 – Column and Edge of Slab

e) Slab-beam joint distribution factors, *DF*.

At exterior joint

$$DF = \frac{1,995.95}{(1,995.95 + 1,628.67)} = 0.551$$

At interior joint

$$DF = \frac{1,995.95}{(1,995.95 + 1,995.95 + 1,628.67)} = 0.355$$

COF for slab-beam = 0.578

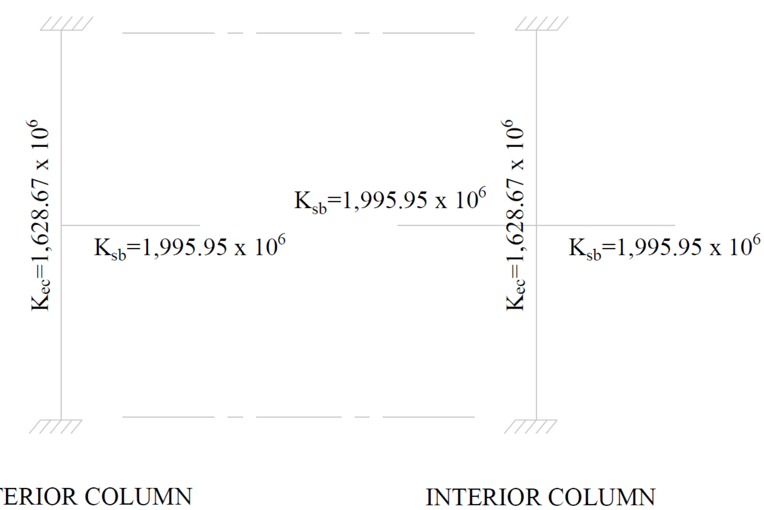


Figure 15 – Slab and Column Stiffness

2.1.3. Equivalent Frame Analysis

Determine negative and positive moments for the slab-beams using the moment distribution method. Since the unfactored live load does not exceed three-quarters of the unfactored dead load, design moments are assumed to occur at all critical sections with full factored live on all spans. ***ACI 318-14 (6.4.3.2)***

$$\frac{L}{D}=\frac{60}{(125+20)}=0.41<\frac{3}{4}$$

a) Factored load and Fixed-End Moments (FEM's).

For slab:

Factored dead load, $q_{Du}=1.20\times(125.00+20.00)=174.00$ psf

Factored live load, $q_{Lu}=1.60\times 60.00=96.00$ psf ***ACI 318-14 (5.3.1)***

Total factored load, $q_u=q_{Du}+q_{Lu}=270.00$ psf

For drop panels:

Factored dead load, $q_{Du}=1.20\times(150.00\times 4.25/12)=63.75$ psf

Factored live load, $q_{Lu}=1.60\times 0.00=0.00$ psf ***ACI 318-14 (5.3.1)***

Total factored load, $q_u=q_{Du}+q_{Lu}=63.75$ psf

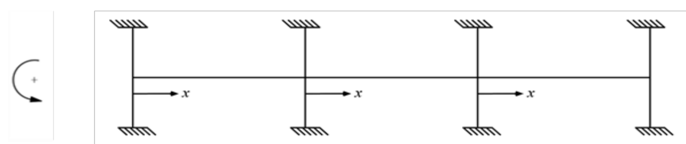
Fixed-end moment $FEM=\sum_{i=1}^{n} m_{NFi}\times w_i\times \ell_1^{\,2}$ ***PCA Notes on ACI 318-11 (Table A2 & A3)***

$$FEM=0.0915\times 0.270\times 30\times 30^2+0.0163\times 0.064\times 10\times 30^2+0.002\times 0.064\times 10\times 30^2$$

$$FEM=677.53\text{ ft-kips}$$

b) Moment distribution. Computations are shown in the Table below Counterclockwise rotational moments acting on the member ends are taken as positive.

Table 1 - Moment Distribution for Equivalent Frame



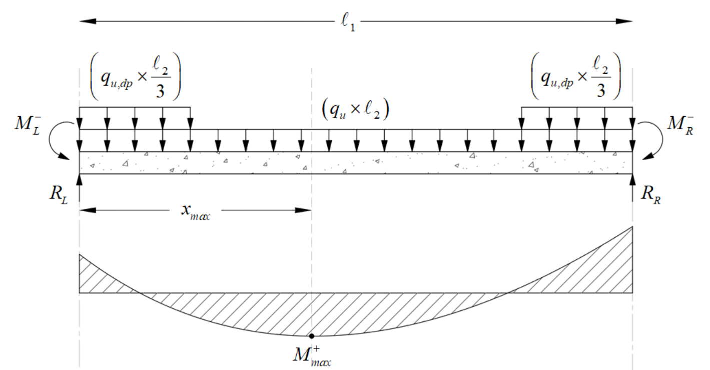
Joint	**1**	**2**		**3**		**4**
Member	**1-2**	**2-1**	**2-3**	**3-2**	**3-4**	**4-3**
DF	0.551	0.355	0.355	0.355	0.355	0.551
COF	0.578	0.578	0.578	0.578	0.578	0.578
FEM	677.53	-677.53	677.53	-677.53	677.53	-677.53
Dist	-373.09	0.00	0.00	0.00	0.00	373.09
CO	0.00	-215.68	0.00	0.00	215.68	0.00
Dist	0.00	76.59	76.59	-76.59	-76.59	0.00
CO	44.28	0.00	-44.28	44.28	0.00	-44.28
Dist	-24.38	15.72	15.72	-15.72	-15.72	24.38
CO	9.09	-14.09	-9.09	9.09	14.09	-9.09
Dist	-5.01	8.23	8.23	-8.23	-8.23	5.01
CO	4.76	-2.89	-4.76	4.76	2.89	-4.76
Dist	-2.62	2.72	2.72	-2.72	-2.72	2.62
CO	1.57	-1.52	-1.57	1.57	1.52	-1.57
Dist	-0.87	1.10	1.10	-1.10	-1.10	0.87
CO	0.63	-0.50	-0.63	0.63	0.50	-0.63
Dist	-0.35	0.40	0.40	-0.40	-0.40	0.35
CO	0.23	-0.20	-0.23	0.23	0.20	-0.23
Dist	-0.13	0.15	0.15	-0.15	-0.15	0.13
CO	0.09	-0.07	-0.09	0.09	0.07	-0.09
Dist	-0.05	0.06	0.06	-0.06	-0.06	0.05
CO	0.03	-0.03	-0.03	0.03	0.03	-0.03
Dist	-0.02	0.02	0.02	-0.02	-0.02	0.02
M^{-}_{max}	331.71	-807.52	721.85	-721.85	807.52	-331.71
V	108.83	-140.55	124.69	-124.69	140.55	-108.83
x_{max}	13.04		15.00		16.96	
M^{+}_{max}	365.13		197.37		365.13	

Maximum positive span moments are determined from the following equations:

$$M_{max}^{+} = M_1 + M_2 + M_3$$

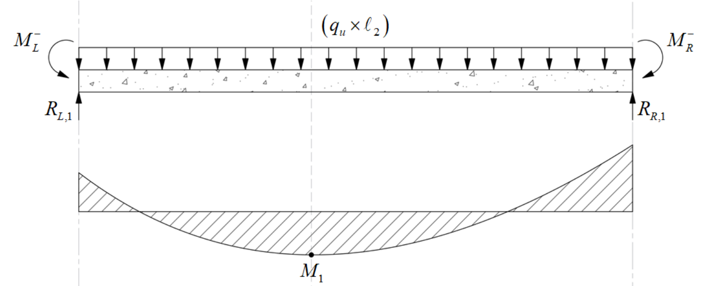
$$R_L = R_{L,1} + R_{L,2} + R_{L,3} \qquad R_R = R_{R,1} + R_{R,2} + R_{R,3}$$

$$x_{max} = \frac{\ell_1}{2} + \frac{M_L^- - M_R^-}{(q_u \times \ell_2) \times \ell_1}$$



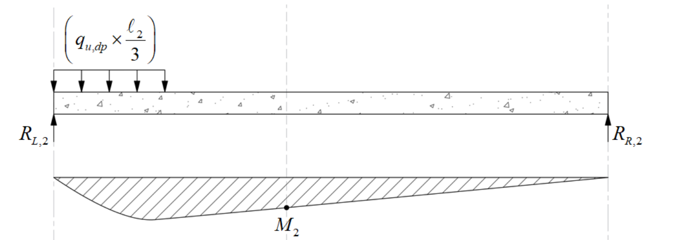
$$M_1 = \frac{(q_u \times \ell_2) \times \ell_1^2}{8} - \frac{M_L^- + M_R^-}{2} + \frac{(M_L^- - M_R^-)^2}{2 \times (q_u \times \ell_2) \times \ell_1^2}$$

$$R_{L,1} = \frac{(q_u \times \ell_2) \times \ell_1}{2} + \frac{M_L^- - M_R^-}{\ell_1} \qquad R_{R,1} = \frac{(q_u \times \ell_2) \times \ell_1}{2} - \frac{M_L^- - M_R^-}{\ell_1}$$



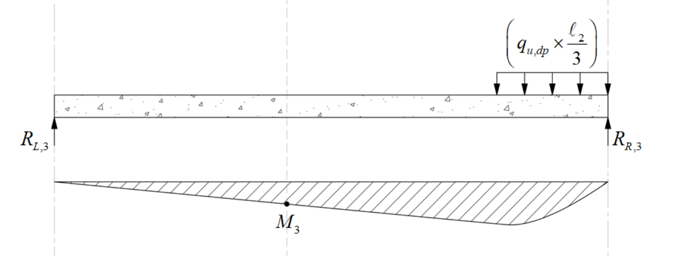
$$M_2 = R_{R,2} \times (\ell_1 - x_{max})$$

$$R_{L,2} = \frac{\left(q_{u,dp} \times \frac{\ell_2}{3}\right) \times \frac{\ell_1}{6}}{2 \times \ell_1} \times \left(2 \times \frac{5 \times \ell_1}{6} + \frac{\ell_1}{6}\right) \qquad R_{R,2} = \frac{\left(q_{u,dp} \times \frac{\ell_2}{3}\right) \times \frac{\ell_1}{6}}{2 \times \ell_1} \times \left(\frac{\ell_1}{6}\right)$$



$$M_3 = R_{L,3} \times x_{max}$$

$$R_{L,3} = \frac{\left(q_{u,dp} \times \frac{\ell_2}{3}\right) \times \frac{\ell_1}{6}}{2 \times \ell_1} \times \left(\frac{\ell_1}{6}\right) \qquad R_{R,3} = \frac{\left(q_{u,dp} \times \frac{\ell_2}{3}\right) \times \frac{\ell_1}{6}}{2 \times \ell_1} \times \left(2 \times \frac{5 \times \ell_1}{6} + \frac{\ell_1}{6}\right)$$



Maximum positive moment in spans 1-2 and 3-4:

$$M^{+}_{max} = M_1 + M_2 + M_3 = 357.16 + 4.50 + 3.46 = 365.13 \text{ ft-kip}$$

$$V_L = R_L = R_{L,1} + R_{L,2} + R_{L,3} = 105.64 + 2.92 + 0.27 = 108.83 \text{ kips}$$

$$V_R = R_R = R_{R,1} + R_{R,2} + R_{R,3} = 137.36 + 0.27 + 2.92 = 140.55 \text{ kips}$$

$$x_{max} = \frac{30}{2} + \frac{(331.71 - 807.52)}{(0.270 \times 30) \times 30} = 13.04 \text{ ft}$$

Where:

M_L^- = 337.71 ft-kip

M_R^- = 807.52 ft-kip

$$M_1 = \frac{(0.270 \times 30) \times 30^2}{8} - \frac{331.71 + 807.52}{2} + \frac{(331.71 - 807.52)^2}{2 \times (0.270 \times 30) \times 30^2} = 357.16 \text{ ft-kip}$$

$$M_2 = \frac{\left(0.064 \times \frac{30}{3}\right) \times \left(\frac{30}{6}\right)}{2 \times 30} \times \left(\frac{30}{6}\right) \times (30 - 13.04) = 4.50 \text{ ft-kip}$$

$$M_3 = \frac{\left(0.064 \times \frac{30}{3}\right) \times \left(\frac{30}{6}\right)}{2 \times 30} \times \left(\frac{30}{6}\right) \times 13.04 = 3.46 \text{ ft-kip}$$

And:

$$R_{L,1} = \frac{(0.270 \times 30) \times 30}{2} + \frac{(337.71 - 807.52)}{30} = 105.64 \text{ kips}$$

$$R_{L,2} = \frac{(0.64 \times 10) \times 5}{2 \times 30} \times \left(2 \times \frac{5 \times 30}{6} + \frac{30}{6}\right) = 2.92 \text{ kips}$$

$$R_{L,3} = \frac{(0.64 \times 10) \times 5}{2 \times 30} \times \left(\frac{30}{6}\right) = 0.27 \text{ kips}$$

$$R_{R,1} = \frac{(0.270 \times 30) \times 30}{2} - \frac{(337.71 - 807.52)}{30} = 137.36 \text{ kips}$$

$$R_{L,2} = \frac{(0.64 \times 10) \times 5}{2 \times 30} \times \left(\frac{30}{6}\right) = 0.27 \text{ kips}$$

$$R_{L,3} = \frac{(0.64 \times 10) \times 5}{2 \times 30} \times \left(2 \times \frac{5 \times 30}{6} + \frac{30}{6}\right) = 2.92 \text{ kips}$$

Maximum positive moment in span 2-3:

$$M_{max}^{+} = M_1 + M_2 + M_3 = 189.40 + 4.50 + 3.46 = 197.37 \text{ ft-kip}$$

$$V_L = R_L = R_{L,1} + R_{L,2} + R_{L,3} = 121.50 + 2.92 + 0.27 = 124.69 \text{ kips}$$

$$V_R = R_R = R_{R,1} + R_{R,2} + R_{R,3} = 121.50 + 2.92 + 0.27 = 124.69 \text{ kips}$$

$$x_{max} = \frac{30}{2} + \frac{(721.85 - 721.85)}{(0.270 \times 30) \times 30} = 15.00 \text{ ft}$$

Where:

$M_L^- = 721.85$ ft-kip

$M_R^- = 721.85$ ft-kip

$$M_1 = \frac{(0.270 \times 30) \times 30^2}{8} - \frac{721.85 + 721.85}{2} + \frac{(721.85 - 721.85)^2}{2 \times (0.270 \times 30) \times 30^2} = 189.40 \text{ ft-kip}$$

$$M_2 = \frac{\left(0.064 \times \frac{30}{3}\right) \times \left(\frac{30}{6}\right)}{2 \times 30} \times \left(\frac{30}{6}\right) \times (30 - 13.04) = 4.50 \text{ ft-kip}$$

$$M_3 = \frac{\left(0.064 \times \frac{30}{3}\right) \times \left(\frac{30}{6}\right)}{2 \times 30} \times \left(\frac{30}{6}\right) \times 13.04 = 3.46 \text{ ft-kip}$$

and:

$$R_{L,1} = \frac{(0.270 \times 30) \times 30}{2} + \frac{(721.85 - 721.85)}{30} = 121.50 \text{ kips}$$

$$R_{L,2} = \frac{(0.64 \times 10) \times 5}{2 \times 30} \times \left(2 \times \frac{5 \times 30}{6} + \frac{30}{6}\right) = 2.92 \text{ kips}$$

$$R_{L,3} = \frac{(0.64 \times 10) \times 5}{2 \times 30} \times \left(\frac{30}{6}\right) = 0.27 \text{ kips}$$

$$R_{R,1} = \frac{(0.270 \times 30) \times 30}{2} - \frac{(721.85 - 721.85)}{30} = 121.50 \text{ kips}$$

$$R_{L,2} = \frac{(0.64 \times 10) \times 5}{2 \times 30} \times \left(\frac{30}{6}\right) = 0.27 \text{ kips}$$

$$R_{L,3} = \frac{(0.64 \times 10) \times 5}{2 \times 30} \times \left(2 \times \frac{5 \times 30}{6} + \frac{30}{6}\right) = 2.92 \text{ kips}$$

2.1.4. Factored Moments Used for Design

Positive and negative factored moments for the slab system in the direction of analysis are plotted in Figure 16. The negative moments used for design are taken at the faces of supports (rectangle section or equivalent rectangle for circular or polygon sections) but not at distances greater than $0.175 \times l_1$ from the centers of supports. ***ACI 318-14 (8.11.6.1)***

$$\frac{20 \text{ in.}}{12\times 2} = 1.67 \text{ ft} < 0.175\times 30 = 5.25 \text{ ft (use face of support location)}$$

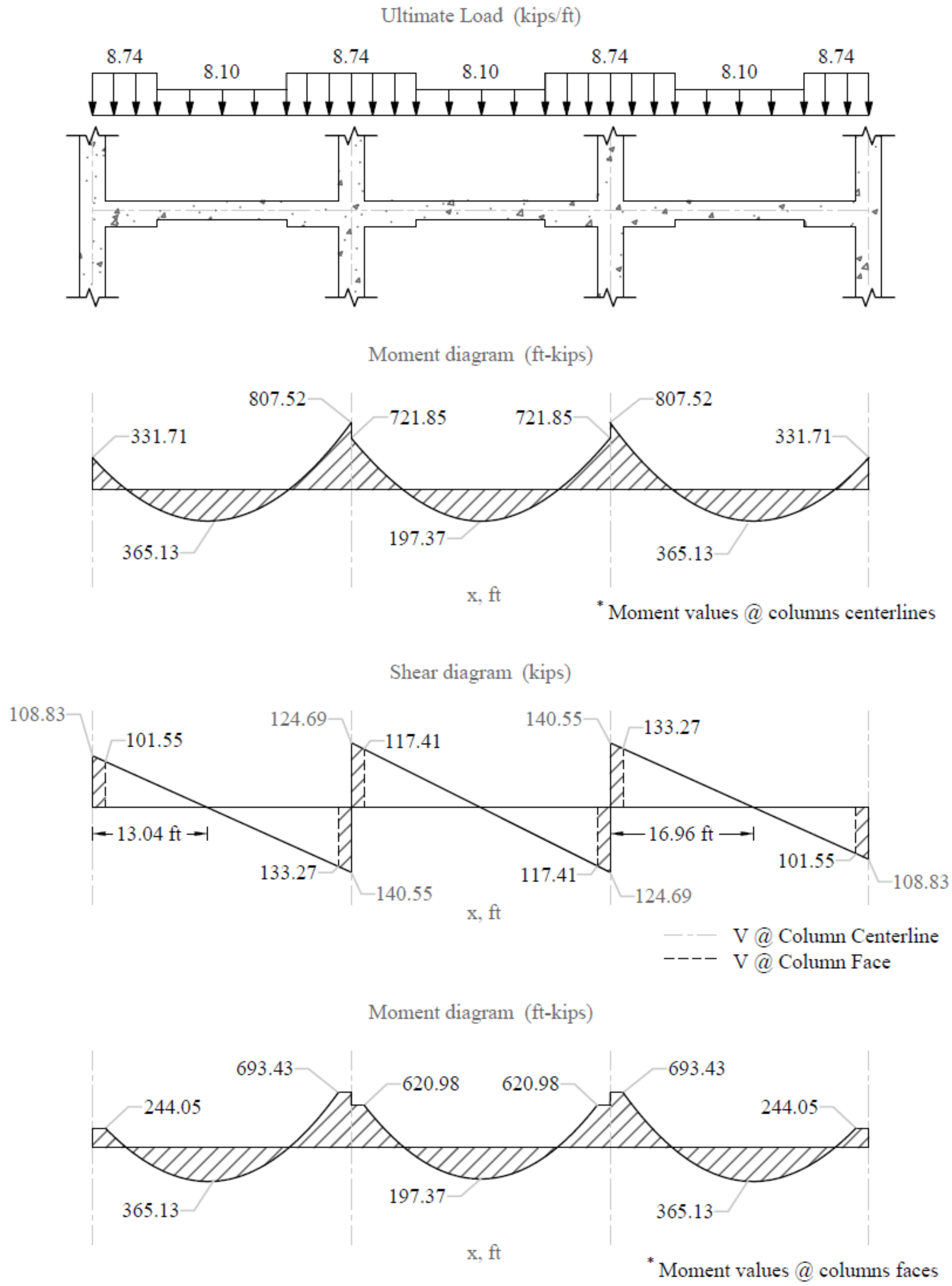


Figure 16 – Positive and Negative Design Moments for Slab-Beam (All Spans Loaded with Full Factored Live Load)

2.1.5. Factored Moments in Slab-Beam Strip

a) Check whether the moments calculated above can take advantage of the reduction permitted by ***ACI 318-14 (8.11.6.5)***:

If the slab system analyzed using EFM within the limitations of ***ACI 318-14 (8.10.2)***, it is permitted by the ACI code to reduce the calculated moments obtained from EFM in such proportion that the absolute sum of the positive and average negative design moments need not exceed the total static moment M_o given by ***Equation 8.10.3.2*** in the ***ACI 318-14***.

Check Applicability of Direct Design Method:

1. There is a minimum of three continuous spans in each direction. ***ACI 318-14 (8.10.2.1)***
2. Successive span lengths are equal. ***ACI 318-14 (8.10.2.2)***
3. Long-to-Short ratio is 30/30 = 1.00 < 2.00. ***ACI 318-14 (8.10.2.3)***
4. Column are not offset. ***ACI 318-14 (8.10.2.4)***
5. Loads are gravity and uniformly distributed with service live-to-dead ratio of 0.41 < 2.00

(Note: The self-weight of the drop panels is not uniformly distributed entirely along the span. However, the variation in load magnitude is small). ***ACI 318-14 (8.10.2.5 and 6)***

6. Check relative stiffness for slab panel. ***ACI 318-14 (8.10.2.7)***

Slab system is without beams and this requirement is not applicable.

All limitation of ***ACI 318-14 (8.10.2)*** are satisfied and the provisions of ***ACI 318-14 (8.11.6.5)*** may be applied:

$$M_o = \frac{q_u \times \ell_2 \times \ell_n^{\,2}}{8} = \frac{0.270 \times 30 \times (30 - 20/12)^2}{8} = 812.81 \text{ ft-kips}$$ ***ACI 318-14 (Eq. 8.10.3.2)***

End spans: $365.13 + \dfrac{331.71 + 807.52}{2} = 934.75$ ft-kips

Interior span: $197.37 + \dfrac{721.85 + 721.85}{2} = 919.22$ ft-kips

To illustrate proper procedure, the interior span factored moments may be reduced as follows:

Permissible reduction $\dfrac{812.81}{919.22} = 0.88$

Adjusted negative design moment = 721.85 × 0.88 = 638.29 ft-kips

Adjusted positive design moment = 197.37 × 0.88 = 174.52 ft-kips

$$M_o = 174.52 + \frac{638.29 + 638.29}{2} = 812.81 \text{ ft-kips}$$

ACI 318 allows the reduction of the moment values based on the previous procedure. Since the drop panels may cause gravity loads not to be uniform (Check limitation #5 and Figure 16), the moment values obtained from EFM will be used for comparison reasons.

b) Distribute factored moments to column and middle strips:

After the negative and positive moments have been determined for the slab-beam strip, the ACI code permits the distribution of the moments at critical sections to the column strips, beams (if any), and middle strips in accordance with the DDM. ***ACI 318-14 (8.11.6.6)***

Distribution of factored moments at critical sections is summarized in Table below.

<table>
<tr><th colspan="7">Table 2 - Distribution of factored moments</th></tr>
<tr><th colspan="2" rowspan="2">Location</th><th>Slab-beam Strip</th><th colspan="2">Column Strip</th><th colspan="2">Middle Strip</th></tr>
<tr><th>Moment (ft-kips)</th><th>Percent</th><th>Moment (ft-kips)</th><th>Percent</th><th>Moment (ft-kips)</th></tr>
<tr><th rowspan="3">End Span</th><th>Exterior Negative</th><td>244.05</td><td>100</td><td>244.05</td><td>0</td><td>0</td></tr>
<tr><th>Positive</th><td>365.13</td><td>60</td><td>219.08</td><td>40</td><td>146.05</td></tr>
<tr><th>Interior Negative</th><td>693.43</td><td>75</td><td>520.07</td><td>25</td><td>173.36</td></tr>
<tr><th rowspan="2">Interior Span</th><th>Negative</th><td>620.98</td><td>75</td><td>465.73</td><td>25</td><td>155.24</td></tr>
<tr><th>Positive</th><td>197.37</td><td>60</td><td>118.42</td><td>40</td><td>78.95</td></tr>
</table>

2.1.6. Flexural Reinforcement Requirements

a) Determine flexural reinforcement required for strip moments

The flexural reinforcement calculation for the column strip of end span – exterior negative location is provided below.

M_u = 244.05 ft-kips

Use d = 13.13 in.

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section (jd). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.90, and jd will be taken equal to $0.987 \times d$. The assumptions will be verified once the area of steel in finalized.

Assume $jd = 0.987 \times d = 12.95$ in.

Column strip width, $b = \dfrac{30 \times 12}{2} = 180 \text{ in.}$

Middle strip width, $b = \dfrac{30 \times 12}{2} = 180 \text{ in.}$

$$A_s = \frac{M_u}{\phi \times f_y \times jd} = \frac{244.05 \times 12,000}{0.90 \times 60,000 \times 12.95} = 4.19 \text{ in.}^2$$

Recalculate 'a' for the actual A_s = 4.19 in.^{2}:

$$a = \frac{A_s \times f_y}{0.85 \times f'_c \times b} = \frac{4.19 \times 60,000}{0.85 \times 5,000 \times 180} = 0.328 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.328}{0.85} = 0.386 \text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right) \times d_t - 0.003 = \left(\frac{0.003}{0.386}\right) \times 13.13 - 0.003 = 0.0989 \geq 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi \times f_y \times \left(d - \dfrac{a}{2}\right)} = \frac{244.05 \times 12,000}{0.90 \times 60,000 \times \left(13.13 - \dfrac{0.328}{2}\right)} = 4.18 \text{ in.}^2$$

The slab has two thicknesses in the column strip (14.25 in. for the slab with the drop panel and 10 in. for the slab without the drop panel).

The weighted slab thickness:

$$h_w = \frac{14.25 \times \left(\dfrac{30}{3}\right) + 10 \times \left(\dfrac{30}{2} - \dfrac{30}{3}\right)}{\left(\dfrac{30}{3}\right) + \left(\dfrac{30}{2} - \dfrac{30}{3}\right)} = 12.83 \text{ in.}$$

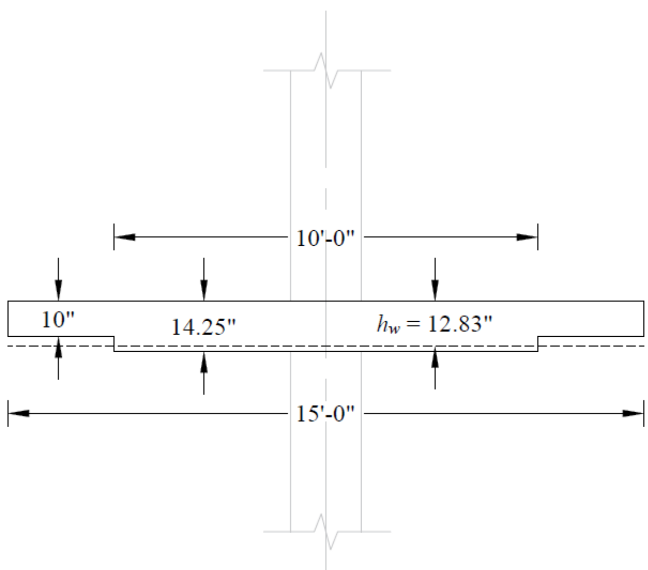


Figure 17 – The Weighted Slab Thickness

$A_{s,\min} = 0.0018 \times 180 \times 12.83 = 4.16 \text{ in.}^2 < 4.18 \text{ in.}$ ***ACI 318-14 (24.4.3.2)***

$s_{\max} = 2 \times h_w = 2 \times 12.83 = 25.67 \text{ in.} > 18 \text{ in.}$ ***ACI 318-14 (8.7.2.2)***

$s_{\max} = 18 \text{ in.}$

Provide 10 - #6 bars with $A_s = 4.40 \text{ in.}^2$ and $s = \dfrac{180}{10} = 18 \text{ in.} \leq s_{\max}$

Based on the procedure outlined above, values for all span locations are given in Table below.

Table 3 - Required Slab Reinforcement for Flexure [Equivalent Frame Method (EFM)]								
Span Location		M_u (ft-kips)	b (in.)	d (in.)	$A_{s,req}$ (in.^{2})	$A_{s,min}$ (in.^{2})	Reinforcement Provided	$A_{s,provided}$ (in.^{2})
End Span								
Column Strip	**Exterior Negative**	244.05	180	13.13	4.18	4.16	10-#6	4.40
	Positive	219.08	180	8.88	5.62	3.24	13-#6	5.72
	Interior Negative	520.07	180	13.13	9.05	4.16	21-#6	9.24
Middle Strip	**Exterior Negative**	0.00	180	8.88	0.00	3.24	10-#6 * **	4.40
	Positive	146.05	180	8.88	3.72	3.24	10-#6 **	4.40
	Interior Negative	173.36	180	8.88	4.43	3.24	11-#6	4.84
Interior Span								
Column Strip	**Positive**	118.42	180	8.88	3.01	3.24	10-#6 * **	4.40
Middle Strip	**Positive**	78.95	180	8.88	1.99	3.24	10-#6 * **	4.40

* Design governed by minimum reinforcement.
** Number of bars governed by maximum allowable spacing.

b) Calculate additional slab reinforcement at columns for moment transfer between slab and column by flexure

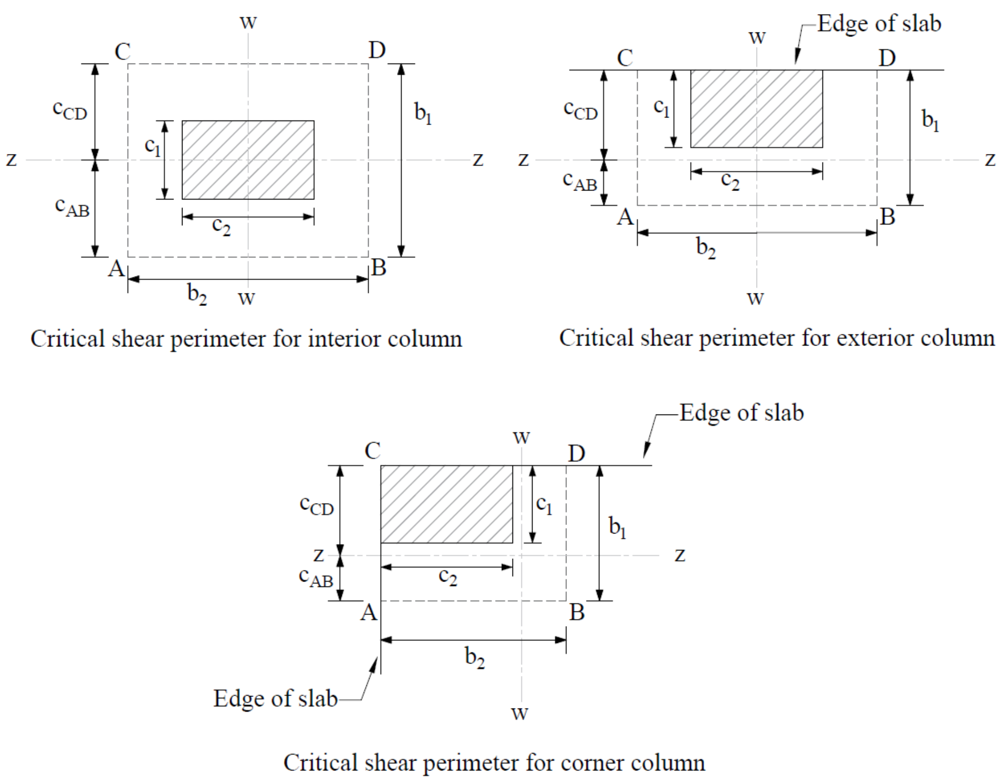
The factored slab moment resisted by the column ($\gamma_f \times M_u$) shall be assumed to be transferred by flexure. Concentration of reinforcement over the column by closer spacing or additional reinforcement shall be used to resist this moment. The fraction of slab moment not calculated to be resisted by flexure shall be assumed to be resisted by eccentricity of shear. ***ACI 318-14 (8.4.2.3)***

Portion of the unbalanced moment transferred by flexure is $\gamma_f \times M_u$ ***ACI 318-14 (8.4.2.3.1)***

Where:

$$\gamma_f = \frac{1}{1+\frac{2}{3}\times\sqrt{\frac{b_1}{b_2}}}$$ ***ACI 318-14 (8.4.2.3.2)***

- b_1 = Dimension of the critical section b_o measured in the direction of the span for which moments are determined in ACI 318, Chapter 8 (see Figure 18).
- b_2 = Dimension of the critical section b_o measured in the direction perpendicular to b_1 in ACI 318, Chapter 8 (see Figure 18).
- b_b = Effective slab width = $c_2 + 3\times h$ ***ACI 318-14 (8.4.2.3.3)***



Critical shear perimeter for interior column

Critical shear perimeter for exterior column

Critical shear perimeter for corner column

Figure 18 – Critical Shear Perimeters for Columns

For exterior support:

$$d = h - c_{clear} - \frac{d_b}{2} = 14.25 - 0.75 - \frac{0.75}{2} = 13.13 \text{ in.}$$

M_u = 331.71 ft-kips

$A_{s(prov)}$ = 4.40 in.^{2}

$$b_1 = c_1 + \frac{d}{2} = 20 + \frac{13.13}{2} = 26.56 \text{ in.}$$

$$b_2 = c_2 + d = 20 + 13.13 = 33.13 \text{ in.}$$

$$b_b = c_2 + 3 \times h = 20 + 3 \times 14.25 = 62.75 \text{ in.}$$

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{26.56}{33.13}}} = 0.63$$

$$A_s = \frac{0.85 \times f'_c \times b_b}{f_y} \times \left(d - \sqrt{d^2 - \frac{2 \times \gamma_f \times M_u}{\phi \times 0.85 \times f'_c \times b_b}} \right)$$

$$A_s = \frac{0.85 \times 5,000 \times 62.75}{60,000} \times \left(13.13 - \sqrt{13.13^2 - \frac{2 \times 0.63 \times 331.71}{0.90 \times 0.85 \times 5,000 \times 62.75}} \right) = 3.63 \text{ in.}^2$$

However, the area of steel provided to resist the flexural moment within the effective slab width b_b:

$$A_{s,provided\ within\ bb} = A_{s,provided} \times \frac{b_b}{b} = 4.40 \times \frac{62.75}{180} = 1.53 \text{ in.}^2$$

Then, the required additional reinforcement at exterior column for moment transfer between slab and column:

$$A_{s,additional} = A_s - A_{s,provided\ within\ bb} = 3.63 - 1.53 = 2.10 \text{ in.}^2$$

Provide 5 - #6 additional bars with A_s = 2.20 in.^{2}

Based on the procedure outlined above, values for all supports are given in Table below.

Table 4 - Additional Slab Reinforcement required for moment transfer between slab and column (EFM)									
Span Location		**M_u*** **(ft-kips)**	**γ_f**	**$\gamma_f M_u$ (ft-kips)**	**b_b (in.)**	**d (in.)**	**A_s req'd within b_b (in^2)**	**A_s prov. For flexure within b_b (in^2)**	**Add'l Reinf.**
End Span									
Column Strip	**Exterior Negative**	331.71	0.63	207.71	62.75	13.13	3.63	1.53	5-#6
	Interior Negative	85.68	0.60	51.41	62.75	13.13	0.88	3.37	-

* M_u is taken at the centerline of the support in Equivalent Frame Method solution.

2.1.7. Factored Moments in Columns

The unbalanced moment from the slab-beams at the supports of the equivalent frame are distributed to the support columns above and below the slab-beam in proportion to the relative stiffness of the support columns. Referring to Figure 16 the unbalanced moment at the exterior and interior joints are:

Exterior Joint = +331.71 ft-kips

Joint 2= -807.52 + 721.85 = -85.68 ft-kips

The stiffness and carry-over factors of the actual columns and the distribution of the unbalanced slab moments (M_{sc}) to the exterior and interior columns are shown in the following Figure.

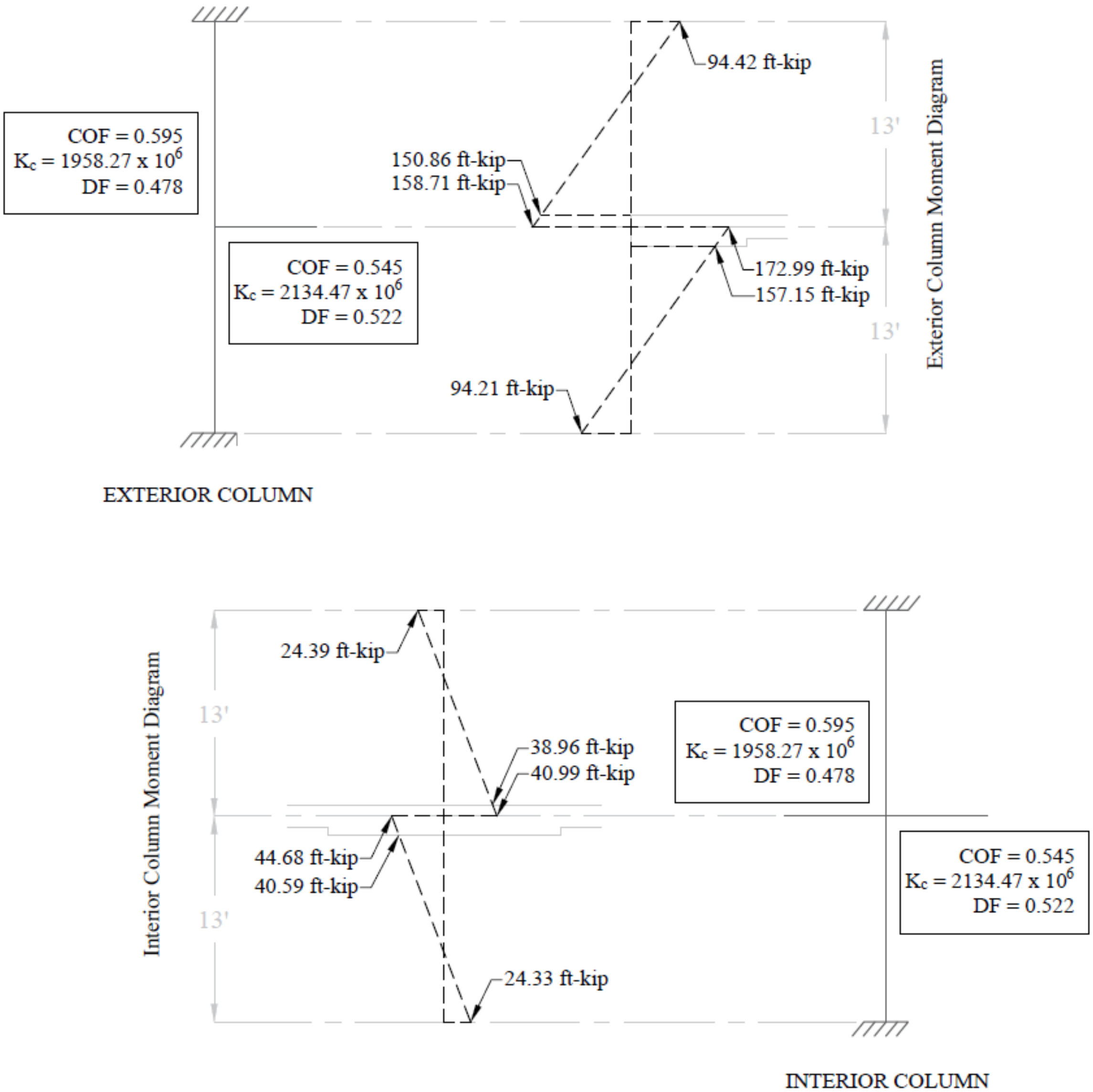


Figure 19 – Column Moments (Unbalanced Moments from Slab-Beam)

In summary:

For Top column (Above):

$M_{col,Exterior}$= 150.86 ft-kips

$M_{col,Interior}$ = 38.96 ft-kips

For Bottom column (Below):

$M_{col,Exterior}$= 157.15 ft-kips

$M_{col,Interior}$ = 40.59 ft-kips

The moments determined above are combined with the factored axial loads (for each story) and factored moments in the transverse direction for design of column sections. The moment values at the face of interior, exterior, and corner columns from the unbalanced moment values are shown in the following Table.

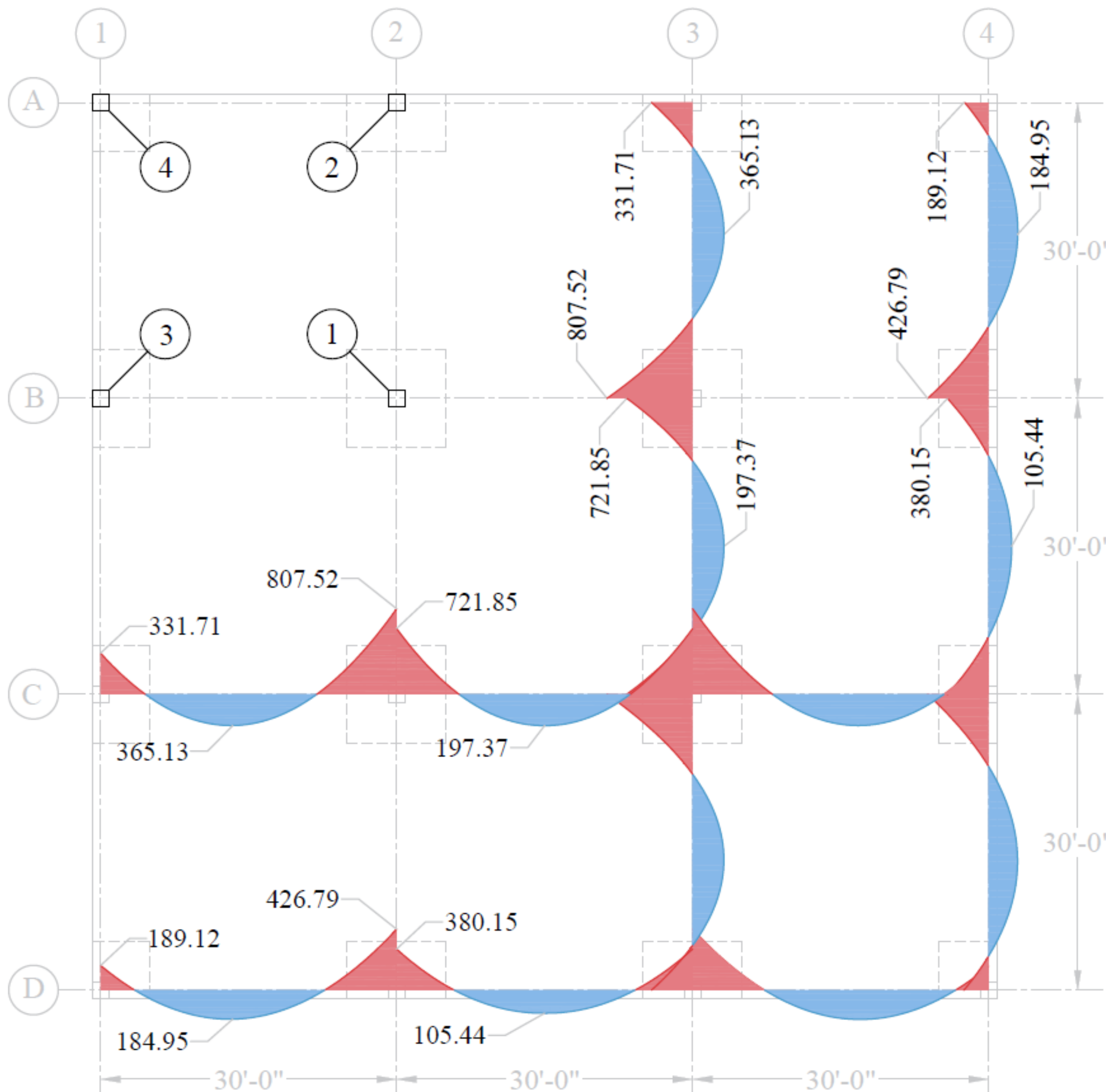


Figure 20 – Moment Diagrams (kips-ft)

Table 5 - Factored Moments in Columns				
M_u (kips-ft)	1	2	3	4
M_{ux}	40.59	157.15	22.09	97.19
M_{uy}	40.59	22.09	157.15	97.19

3. Design of Columns by spColumn

This section includes the design of interior, edge, and corner columns using spColumn software. The preliminary dimensions for these columns were calculated previously in section one. The reduction of live load per ***ASCE 7-10*** will be ignored in this example. However, the detailed procedure to calculate the reduced live loads is explained in the "One-Way Wide Module Joist Concrete Floor Design" example.

3.1. Determination of Factored Loads

Assume 5 story building

Interior Column (Column #1):

Tributary area for interior column for live load, superimposed dead load, and self-weight of the slab is

$A_{Tributary} = (30 \times 30) = 900 \text{ ft}^2$

Tributary area for interior column for self-weight of additional slab thickness due to the presence of the drop panel is

$A_{Tributary} = (10 \times 10) = 100 \text{ ft}^2$

- $P_u \;=\; 5 \times q_u \times A_{Tributary} = 5 \times (0.270 \times 900 + 0.064 \times 100) = 1246.88 \text{ kips}$
- $M_{u,x} \;=\; 40.59$ ft-kips (see the previous Table)
- $M_{u,y} \;=\; 40.59$ ft-kips (see the previous Table)

Edge (Exterior) Column (Column #2):

Tributary area for edge column for live load, superimposed dead load, and self-weight of the slab is

$$A_{Tributary} = \left(\frac{30}{2} + \frac{20/2}{12}\right) \times 30 = 475.00 \text{ ft}^2$$

Tributary area for edge column for self-weight of additional slab thickness due to the presence of the drop panel is

$$A_{Tributary} = \left(\frac{10}{2} + \frac{20/2}{12}\right) \times 10 = 58.33 \text{ ft}^2$$

- P_u = 5 × q_u × $A_{Tributary}$ = 5 × (0.270 × 475.00 + 0.064 × 58.33) = 659.85 kips
- $M_{u,x}$ = 157.15 ft-kips (see the previous Table)
- $M_{u,y}$ = 22.09 ft-kips (see the previous Table)

Edge (Exterior) Column (Column #3):

Tributary area for edge column for live load, superimposed dead load, and self-weight of the slab is

$$A_{Tributary} = \left(\frac{30}{2} + \frac{20/2}{12}\right) \times 30 = 475.00 \text{ ft}^2$$

Tributary area for edge column for self-weight of additional slab thickness due to the presence of the drop panel is

$$A_{Tributary} = \left(\frac{10}{2} + \frac{20/2}{12}\right) \times 10 = 58.33 \text{ ft}^2$$

- P_u = 5 × q_u × $A_{Tributary}$ = 5 × (0.270 × 475.00 + 0.064 × 58.33) = 659.85 kips
- $M_{u,x}$ = 22.09 ft-kips (see the previous Table)
- $M_{u,y}$ = 157.15 ft-kips (see the previous Table)

Corner Column (Column #4):

Tributary area for corner column for live load, superimposed dead load, and self-weight of the slab is

$$A_{Tributary} = \left(\frac{30}{2} + \frac{20/2}{12}\right) \times \left(\frac{30}{2} + \frac{20/2}{12}\right) = 250.69 \text{ ft}^2$$

Tributary area for corner column for self-weight of additional slab thickness due to the presence of the drop panel is

$$A_{Tributary} = \left(\frac{10}{2} + \frac{20/2}{12}\right) \times \left(\frac{10}{2} + \frac{20/2}{12}\right) = 34.03 \text{ ft}^2$$

- P_u = 5 × q_u × $A_{Tributary}$ = 5 × (0.270 × 250.69 + 0.064 × 34.03) = 349.28 kips
- $M_{u,x}$ = 97.19 ft-kips (see the previous Table)
- $M_{u,y}$ = 97.19 ft-kips (see the previous Table)

The factored loads are then input into spColumn to construct the axial load – moment interaction diagram.

3.2. Moment Interaction Diagram

Interior Column (Column #1):

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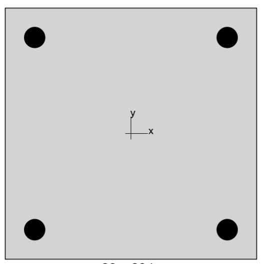

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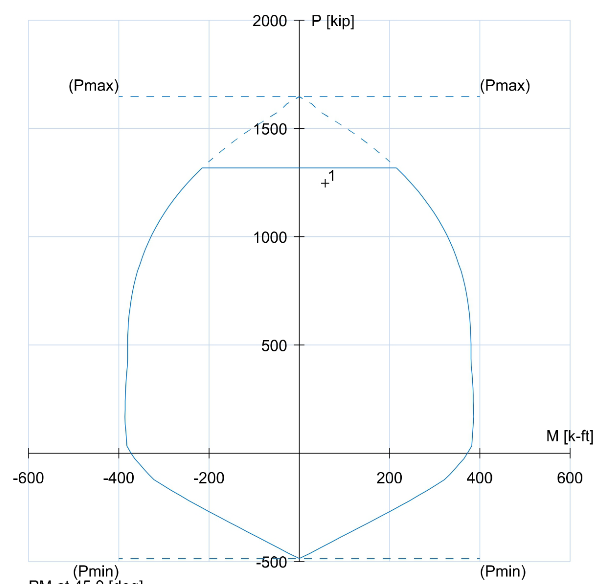

PM at 45.0 [deg]

General Information	
Project	Two-Way Sl...op Panels)
Column	Interior #1
Engineer	SP
Code	ACI 318-14
Bar Set	ASTM A615
Units	English
Run Option	Design
Run Axis	Biaxial
Slenderness	Not Considered
Column Type	Structural
Capacity Method	Moment capacity

Materials	
f'_c	6 ksi
E_c	4415.21 ksi
f_y	60 ksi
E_s	29000 ksi

Section	
Type	Rectangular
Width	20 in
Depth	20 in
A_g	400 in²
I_x	13333.3 in⁴
I_y	13333.3 in⁴

Reinforcement	
Pattern	All sides equal
Bar layout	Rectangular
Cover to	Longitudal bars
Clear cover	1.5 in
Bars	4 #14
Confinement type	Tied
Total steel area, A_s	9.00 in²
Rho	2.25 %
Min. clear spacing	13.61 in

No.	P_u	M_{ux}	M_{uy}	ϕP_n	ϕM_{nx}	ϕM_{ny}	Capacity
	kip	k-ft	k-ft	kip	k-ft	k-ft	Ratio
1	1246.9	40.6	40.6	1246.88	175.55	175.55	0.23

Max. Capacity Ratio: 0.23

Edge Column (Column #2):

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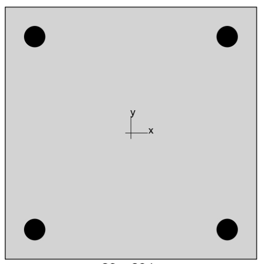

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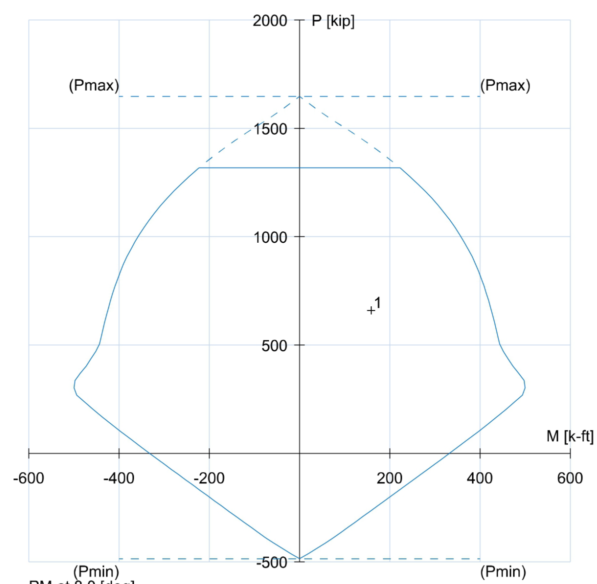

General Information

Project	Two-Way Sl...op Panles)
Column	Edge #2
Engineer	SP
Code	ACI 318-14
Bar Set	ASTM A615
Units	English
Run Option	Design
Run Axis	Biaxial
Slenderness	Not Considered
Column Type	Structural
Capacity Method	Moment capacity

Materials

f'_c	6 ksi
E_c	4415.21 ksi
f_y	60 ksi
E_s	29000 ksi

Section

Type	Rectangular
Width	20 in
Depth	20 in
A_g	400 in²
I_x	13333.3 in⁴
I_y	13333.3 in⁴

Reinforcement

Pattern	All sides equal
Bar layout	Rectangular
Cover to	Longitudinal bars
Clear cover	1.5 in
Bars	4 #14
Confinement type	Tied
Total steel area, A_s	9.00 in²
Rho	2.25 %
Min. clear spacing	13.61 in

No.	P_u	M_{ux}	M_{uy}	ϕP_n	ϕM_{nx}	ϕM_{ny}	Capacity
	kip	k-ft	k-ft	kip	k-ft	k-ft	Ratio
1	659.8	157.1	22.1	659.85	420.99	59.18	0.37

Max. Capacity Ratio: 0.37

Edge Column (Column #3):

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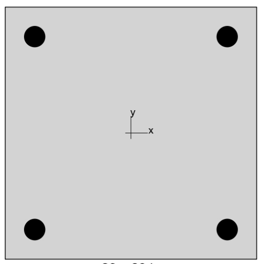

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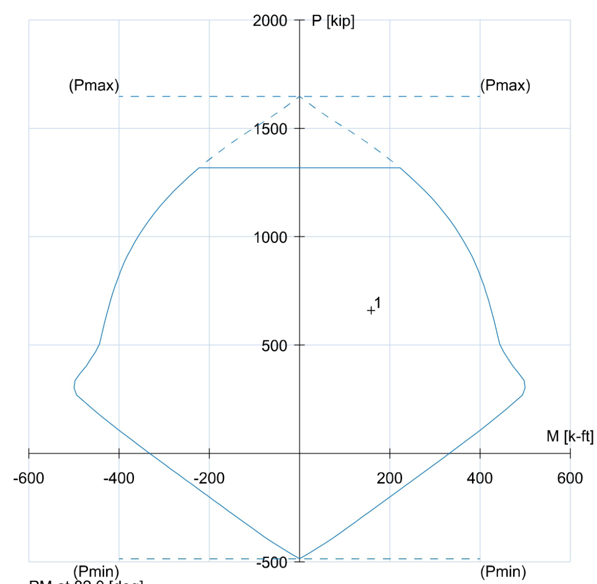

General Information

Project	Two-Way Sl...op Panles)
Column	Edge #3
Engineer	SP
Code	ACI 318-14
Bar Set	ASTM A615
Units	English
Run Option	Design
Run Axis	Biaxial
Slenderness	Not Considered
Column Type	Structural
Capacity Method	Moment capacity

Materials

f'_c	6 ksi
E_c	4415.21 ksi
f_y	60 ksi
E_s	29000 ksi

Section

Type	Rectangular
Width	20 in
Depth	20 in
A_g	400 in²
I_x	13333.3 in⁴
I_y	13333.3 in⁴

Reinforcement

Pattern	All sides equal
Bar layout	Rectangular
Cover to	Longitudinal bars
Clear cover	1.5 in
Bars	4 #14
Confinement type	Tied
Total steel area, A_s	9.00 in²
Rho	2.25 %
Min. clear spacing	13.61 in

No.	P_u	M_{ux}	M_{uy}	ϕP_n	ϕM_{nx}	ϕM_{ny}	Capacity
	kip	k-ft	k-ft	kip	k-ft	k-ft	Ratio
1	659.8	22.1	157.1	659.85	59.18	420.99	0.37

Max. Capacity Ratio: 0.37

Corner Column (Column #4):

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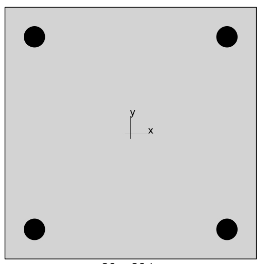

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General Information

Project	Two-Way Sl...op Panles)
Column	Corner #4
Engineer	SP
Code	ACI 318-14
Bar Set	ASTM A615
Units	English
Run Option	Design
Run Axis	Biaxial
Slenderness	Not Considered
Column Type	Structural
Capacity Method	Moment capacity

Materials

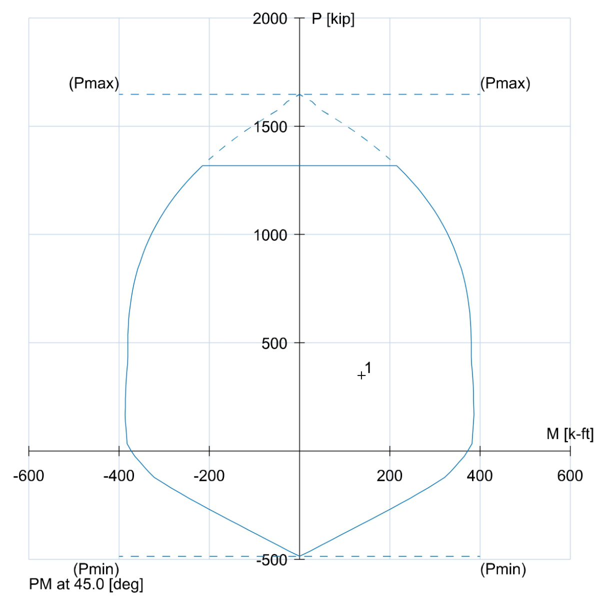
f'_c	6 ksi
E_c	4415.21 ksi
f_y	60 ksi
E_s	29000 ksi

Section

Type	Rectangular
Width	20 in
Depth	20 in
A_g	400 in²
I_x	13333.3 in⁴
I_y	13333.3 in⁴

Reinforcement

Pattern	All sides equal
Bar layout	Rectangular
Cover to	Longitudinal bars
Clear cover	1.5 in
Bars	4 #14
Confinement type	Tied
Total steel area, A_s	9.00 in²
Rho	2.25 %
Min. clear spacing	13.61 in



No.	P_u	M_{ux}	M_{uy}	ϕP_n	ϕM_{nx}	ϕM_{ny}	Capacity
	kip	k-ft	k-ft	kip	k-ft	k-ft	Ratio
1	349.3	97.2	97.2	349.28	271.20	271.20	0.36

Max. Capacity Ratio: 0.36

4. Shear Strength

Shear strength of the slab in the vicinity of columns/supports includes an evaluation of one-way shear (beam action) and two-way shear (punching) in accordance with ACI 318 Chapter 22.

4.1. One-Way (Beam Action) Shear Strength

ACI 318-14 (22.5)

One-way shear is critical at a distance *d* from the face of the column as shown in Figure 3. Figure 21 and Figure 22 show the factored shear forces (V_u) at the critical sections around each column and each drop panel, respectively. In members without shear reinforcement, the design shear capacity of the section equals to the design shear capacity of the concrete:

$$\phi V_n = \phi V_c + \phi V_s = \phi V_c,\ (\phi V_s = 0)$$ ***ACI 318-14 (Eq. 22.5.1.1)***

Where:

$$\phi V_c = \phi \times 2 \times \lambda \times \sqrt{f_c'} \times b_w \times d$$ ***ACI 318-14 (Eq. 22.5.5.1)***

Note: The calculations below follow one of two possible approaches for checking one-way shear. Refer to the conclusions section for a comparison with the other approach.

4.1.1. At Distance *d* from the Supporting Column

$$h_{weighted} = \frac{14.25\times\frac{30}{10}+10\times\left(30-\frac{30}{10}\right)}{30} = 11.42 \text{ in.}$$

$$d_w = 11.42 - 0.75 - \frac{0.75}{2} = 10.29 \text{ in.}$$

Where λ = 1 for normal weight concrete

$$\phi V_c = 0.75\times 2.0\times 1.0\times\frac{\sqrt{5,000}}{1,000}\times(30\times 12)\times 10.29 = 392.97 \text{ kips}$$

Because $\phi V_c > V_u$ at all the critical sections, the slab has adequate one-way shear strength.

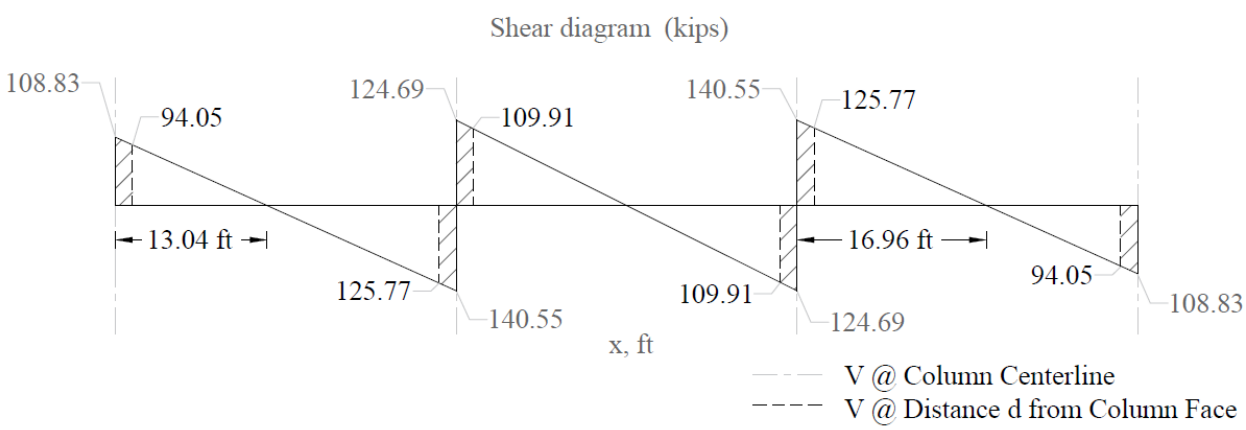


Figure 21 – One-way Shear at Critical Sections (at Distance d from the Face of the Supporting Column)

4.1.2. At the Face of the Drop Panel

h = 10 in.

$$d = 10.00 - 0.75 - \frac{0.75}{2} = 8.88 \text{ in.}$$

Where λ = 1 for normal weight concrete

$$\phi V_c = 0.75 \times 2.0 \times 1.0 \times \frac{\sqrt{5,000}}{1,000} \times (30 \times 12) \times 8.88 = 338.88 \text{ kips}$$

Because $\phi V_c > V_u$ at all the critical sections, the slab has adequate one-way shear strength.

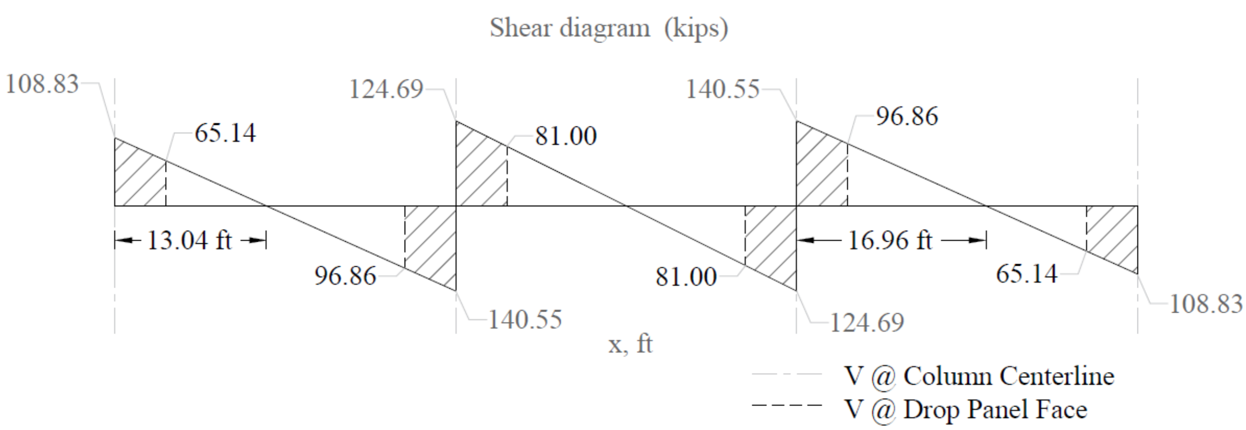


Figure 22 – One-Way Shear at Critical Sections (at the Face of the Drop Panel)

4.2. Two-Way (Punching) Shear Strength

ACI 318-14 (22.6)

4.2.1. Around the Columns Faces

Two-way shear is critical on a rectangular section located at *d*/2 away from the face of the column as shown in Figure 18.

a) Exterior column:

The factored shear force (V_u) in the critical section is computed as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section (*d/2* away from column face).

$$V_u = V - q_u \times (b_1 \times b_2) = 108.83 - 0.334 \times \left(\frac{26.56 \times 33.13}{144} \right) = 106.79 \text{ kips}$$

The factored unbalanced moment used for shear transfer, M_{unb}, is computed as the sum of the joint moments to the left and right. Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account.

$$M_{unb} = M - V_u \times \left(b_1 - c_{AB} - \frac{c_1}{2} \right) = 331.71 - 106.79 \times \left(\frac{26.56 - 8.18 - \frac{20}{2}}{12} \right) = 257.12 \text{ ft-kips}$$

For the exterior column in Figure 18 the location of the centroidal axis z-z is:

$$c_{AB} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{b_1^2}{2 \times b_1 + b_2} = \frac{26.56^2}{2 \times 26.56 + 33.13} = 8.18 \text{ in.}$$

Where

$$b_1 = c_1 + \frac{d}{2} = 20 + \frac{13.13}{2} = 26.56 \text{ in.} \qquad b_2 = c_2 + d = 20 + 13.13 = 33.13 \text{ in.}$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2 \times \left(\frac{b_1 \times d^3}{12} + \frac{d \times b_1^3}{12} + (b_1 \times d) \left(\frac{b_1}{2} - c_{AB} \right)^2 \right) + b_2 \times d \times c_{AB}^2$$

$$J_c = 2 \times \left(\frac{26.56 \times 13.13^3}{12} + \frac{13.13 \times 26.56^3}{12} + (26.56 \times 13.13) \times \left(\frac{26.56}{2} - 8.18 \right)^2 \right) + 33.13 \times 13.13 \times 8.18^2$$

$J_c = 98,243 \text{ in.}^4$

$\gamma_v = 1 - \gamma_f = 1 - 0.63 = 0.37$ **_ACI 318-14 (Eq. 8.4.4.2.2)_**

Where:

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{b_1}{b_2}}}$$ **_ACI 318-14 (8.4.2.3.2)_**

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{26.56}{33.13}}} = 0.63$$

The length of the critical perimeter for the exterior column:

$$b_o = 2 \times b_1 + b_2 = 2 \times 26.56 + 33.13 = 86.25 \text{ in.}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d} + \frac{\gamma_v \times M_{unb} \times c_{AB}}{J_c}$$ **_ACI 318-14 (R.8.4.4.2.3)_**

$$v_u = \frac{106.79 \times 1,000}{86.26 \times 13.13} + \frac{0.37 \times (257.12 \times 12 \times 1,000) \times 8.18}{98,243} = 94.34 + 96.04 = 190.38 \text{ psi}$$

$$v_c = \min \begin{Bmatrix} 4 \times \lambda \times \sqrt{f'_c} \\ \left(2 + \frac{4}{\beta}\right) \times \lambda \times \sqrt{f'_c} \\ \left(\frac{\alpha_s \times d}{b_o} + 2\right) \times \lambda \times \sqrt{f'_c} \end{Bmatrix}$$ **_ACI 318-14 (Table 22.6.5.2)_**

$$v_c = \min \begin{Bmatrix} 4 \times 1 \times \sqrt{5,000} \\ \left(2 + \frac{4}{1}\right) \times 1 \times \sqrt{5,000} \\ \left(\frac{30 \times 13.13}{86.26} + 2\right) \times 1 \times \sqrt{5,000} \end{Bmatrix} = \min \begin{Bmatrix} 282.84 \\ 424.26 \\ 464.23 \end{Bmatrix} = 282.84 \text{ psi}$$

$$\phi v_c = 0.75 \times 282.84 = 212.13 \text{ psi}$$

Because $\phi v_c > v_u$ at the critical section, the slab has adequate two-way shear strength at this joint.

b) Interior column:

$$V_u = V - q_u \times (b_1 \times b_2) = (140.55 + 124.69) - 0.334 \times \left(\frac{33.13 \times 33.13}{144}\right) = 262.70 \text{ kips}$$

$$M_{unb} = M - V_u \times \left(b_1 - c_{AB} - \frac{c_1}{2}\right) = (807.52 - 721.85) - 256.35 \times (0) = 85.67 \text{ ft-kips}$$

For the interior column in Figure 18, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{b_1}{2} = \frac{33.13}{2} = 16.56 \text{ in.}$$

Where

$$b_1 = c_1 + d = 20 + 13.13 = 33.13 \text{ in.} \qquad b_2 = c_2 + d = 20 + 13.13 = 33.13 \text{ in.}$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2 \times \left(\frac{b_1 \times d^3}{12} + \frac{d \times b_1^3}{12} + (b_1 \times d) \times \left(\frac{b_1}{2} - c_{AB}\right)^2\right) + 2 \times b_2 \times d \times c_{AB}^2$$

$$J_c = 2 \times \left(\frac{33.13 \times 13.13^3}{12} + \frac{13.13 \times 33.13^3}{12} + (33.13 \times 13.13) \times (0)^2\right) + 2 \times 33.13 \times 13.13 \times 16.56^2$$

$$J_c = 330{,}518 \text{ in.}^4$$

$$\gamma_v = 1 - \gamma_f = 1 - 0.60 = 0.40$$ ***ACI 318-14 (Eq. 8.4.4.2.2)***

Where:

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{b_1}{b_2}}}$$ ***ACI 318-14 (8.4.2.3.2)***

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{33.13}{33.13}}} = 0.60$$

The length of the critical perimeter for the interior column:

$$b_o = 2 \times (b_1 + b_2) = 2 \times (33.13 + 33.13) = 132.50 \text{ in.}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d} + \frac{\gamma_v \times M_{unb} \times c_{AB}}{J_c}$$

ACI 318-14 (R.8.4.4.2.3)

$$v_u = \frac{262.70 \times 1,000}{132.50 \times 13.13} + \frac{0.40 \times (85.67 \times 12 \times 1,000) \times 16.56}{330,518} = 151.06 + 20.61 = 171.66 \text{ psi}$$

$$v_c = \min \begin{Bmatrix} 4 \times \lambda \times \sqrt{f'_c} \\ \left(2 + \frac{4}{\beta}\right) \times \lambda \times \sqrt{f'_c} \\ \left(\frac{\alpha_s \times d}{b_o} + 2\right) \times \lambda \times \sqrt{f'_c} \end{Bmatrix}$$

ACI 318-14 (Table 22.6.5.2)

$$v_c = \min \begin{Bmatrix} 4 \times 1 \times \sqrt{5,000} \\ \left(2 + \frac{4}{1}\right) \times 1 \times \sqrt{5,000} \\ \left(\frac{40 \times 13.13}{132.50} + 2\right) \times 1 \times \sqrt{5,000} \end{Bmatrix} = \min \begin{Bmatrix} 282.84 \\ 424.26 \\ 421.60 \end{Bmatrix} = 282.84 \text{ psi}$$

$$\phi v_c = 0.75 \times 282.84 = 212.13 \text{ psi}$$

Since $\phi v_c > v_u$ at the critical section, the slab has adequate two-way shear strength at this joint.

c) Corner column:

In this example, interior equivalent frame strip was selected where it only has exterior and interior supports (no corner supports are included in this strip). However, the two-way shear strength of corner supports usually governs. Thus, the two-way shear strength for the corner column in this example will be checked for educational purposes. Same procedure is used to find the reaction and factored unbalanced moment used for shear transfer at the centroid of the critical section for the corner support for the exterior equivalent frame strip.

$$V_u = V - q_u \times (b_1 \times b_2) = 61.93 - 0.334 \times \left(\frac{26.56 \times 26.56}{144}\right) = 60.29 \text{ kips}$$

$$M_{unb} = M - V_u \times \left(b_1 - c_{AB} - \frac{c_1}{2}\right) = 187.51 - 60.29 \times \left(\frac{26.56 - 6.64 - \frac{20}{2}}{12}\right) = 137.66 \text{ ft-kips}$$

For the corner column in Figure 18, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{b_1^2}{2 \times b_1 + b_2} = \frac{26.56^2}{2 \times 26.56 + 26.56} = 6.64 \text{ in.}$$

Where

$$b_1 = c_1 + \frac{d}{2} = 20 + \frac{13.13}{2} = 26.56 \text{ in.} \qquad b_2 = c_2 + \frac{d}{2} = 20 + \frac{13.13}{2} = 26.56 \text{ in.}$$

The polar moment J_c of the shear perimeter is:

$$J_c = \left(\frac{b_1 \times d^3}{12} + \frac{d \times b_1^3}{12} + (b_1 \times d) \times \left(\frac{b_1}{2} - c_{AB}\right)^2\right) + b_2 \times d \times c_{AB}^{\ 2}$$

$$J_c = \left(\frac{26.56 \times 13.13^3}{12} + \frac{13.13 \times 26.56^3}{12} + (26.56 \times 13.13) \times \left(\frac{26.56}{2} - 6.64\right)^2\right) + 26.56 \times 13.13 \times 6.64^2$$

$$J_c = 56{,}251 \text{ in.}^4$$

$$\gamma_v = 1 - \gamma_f = 1 - 0.60 = 0.40$$ ***ACI 318-14 (Eq. 8.4.4.2.2)***

Where:

$$\gamma_f = \frac{1}{1+\frac{2}{3}\times\sqrt{\frac{b_1}{b_2}}}$$ ***ACI 318-14 (8.4.2.3.2)***

$$\gamma_f = \frac{1}{1+\frac{2}{3}\times\sqrt{\frac{33.13}{33.13}}} = 0.60$$

The length of the critical perimeter for the corner column:

$$b_o = b_1 + b_2 = 26.56 + 26.56 = 53.13 \text{ in.}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d} + \frac{\gamma_v \times M_{unb} \times c_{AB}}{J_c}$$ ***ACI 318-14 (R.8.4.4.2.3)***

$$v_u = \frac{60.29\times 1,000}{53.13\times 13.13} + \frac{0.40\times(137.66\times 12\times 1,000)\times 6.64}{56,251} = 86.47 + 78.00 = 164.48 \text{ psi}$$

$$v_c = \min\left\{\begin{array}{l} 4\times\lambda\times\sqrt{f'_c} \\ \left(2+\frac{4}{\beta}\right)\times\lambda\times\sqrt{f'_c} \\ \left(\frac{\alpha_s\times d}{b_o}+2\right)\times\lambda\times\sqrt{f'_c} \end{array}\right\}$$ ***ACI 318-14 (Table 22.6.5.2)***

$$v_c = \min\left\{\begin{array}{l} 4\times 1\times\sqrt{5,000} \\ \left(2+\frac{4}{1}\right)\times 1\times\sqrt{5,000} \\ \left(\frac{20\times 13.13}{53.13}+2\right)\times 1\times\sqrt{5,000} \end{array}\right\} = \min\left\{\begin{array}{l} 282.84 \\ 424.26 \\ 490.82 \end{array}\right\} = 282.84 \text{ psi}$$

$$\phi v_c = 0.75\times 282.84 = 212.13 \text{ psi}$$

Since $\phi v_c > v_u$ at the critical section, the slab has adequate two-way shear strength at this joint.

4.2.2. Around Drop Panels

Two-way shear is critical on a rectangular section located at *d*/2 away from the face of the drop panel.

Note: The two-way shear stress calculations around drop panels do not have the term for unbalanced moment since drop panels are a thickened portion of the slab and are not considered as a support.

a) Exterior drop panel:

$$V_u = V - q_u \times (b_1 \times b_2) = 108.83 - 0.270 \times \left(\frac{74.44 \times 128.88}{144}\right) = 90.84 \text{ kips}$$

The length of the critical perimeter for the exterior drop panel:

$$b_o = 2 \times 74.44 + 128.88 = 277.75 \text{ in.}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d}$$ ***ACI 318-14 (R.8.4.4.2.3)***

$$v_u = \frac{90.84 \times 1,000}{277.75 \times 8.88} = 36.85 \text{ psi}$$

$$v_c = \min \begin{Bmatrix} 4 \times \lambda \times \sqrt{f'_c} \\ \left(2 + \frac{4}{\beta}\right) \times \lambda \times \sqrt{f'_c} \\ \left(\frac{\alpha_s \times d}{b_o} + 2\right) \times \lambda \times \sqrt{f'_c} \end{Bmatrix}$$ ***ACI 318-14 (Table 22.6.5.2)***

$$v_c = \min \begin{Bmatrix} 4 \times 1 \times \sqrt{5,000} \\ \left(2 + \frac{4}{1}\right) \times 1 \times \sqrt{5,000} \\ \left(\frac{30 \times 8.88}{277.75} + 2\right) \times 1 \times \sqrt{5,000} \end{Bmatrix} = \min \begin{Bmatrix} 282.84 \\ 424.26 \\ 209.20 \end{Bmatrix} = 209.20 \text{ psi}$$

$$\phi v_c = 0.75 \times 209.20 = 156.90 \text{ psi}$$

Since $\phi v_c > v_u$ at the critical section, the slab has adequate two-way shear strength around this drop panel.

b) Interior drop panel:

$$V_u = V - q_u \times (b_1 \times b_2) = 140.55 + 124.69 - 0.270 \times \left(\frac{128.88 \times 128.88}{144}\right) = 234.10 \text{ kips}$$

The length of the critical perimeter for the interior drop panel:

$$b_o = 2 \times (128.88 + 128.88) = 515.50 \text{ in.}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d}$$ ***ACI 318-14 (R.8.4.4.2.3)***

$$v_u = \frac{234.10 \times 1{,}000}{515.50 \times 8.88} = 51.17 \text{ psi}$$

$$v_c = \min \left\{ \begin{array}{l} 4 \times \lambda \times \sqrt{f'_c} \\ \left(2 + \frac{4}{\beta}\right) \times \lambda \times \sqrt{f'_c} \\ \left(\frac{\alpha_s \times d}{b_o} + 2\right) \times \lambda \times \sqrt{f'_c} \end{array} \right\}$$ ***ACI 318-14 (Table 22.6.5.2)***

$$v_c = \min \left\{ \begin{array}{l} 4 \times 1 \times \sqrt{5{,}000} \\ \left(2 + \frac{4}{1}\right) \times 1 \times \sqrt{5{,}000} \\ \left(\frac{40 \times 8.88}{515.50} + 2\right) \times 1 \times \sqrt{5{,}000} \end{array} \right\} = \min \left\{ \begin{array}{l} 282.84 \\ 424.26 \\ 190.12 \end{array} \right\} = 190.12 \text{ psi}$$

$$\phi v_c = 0.75 \times 190.12 = 142.59 \text{ psi}$$

Since $\phi v_c > v_u$ at the critical section, the slab has adequate two-way shear strength around this drop panel.

c) Corner drop panel:

$$V_u = V - q_u \times (b_1 \times b_2) = 61.93 - 0.270 \times \left(\frac{74.44 \times 74.44}{144}\right) = 51.54 \text{ kips}$$

The length of the critical perimeter for the corner drop panel:

$$b_o = 74.44 + 74.44 = 148.88 \text{ in.}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d}$$ ***ACI 318-14 (R.8.4.4.2.3)***

$$v_u = \frac{51.54 \times 1{,}000}{148.88 \times 8.88} = 39.01 \text{ psi}$$

$$v_c = \min \begin{Bmatrix} 4 \times \lambda \times \sqrt{f'_c} \\ \left(2 + \frac{4}{\beta}\right) \times \lambda \times \sqrt{f'_c} \\ \left(\frac{\alpha_s \times d}{b_o} + 2\right) \times \lambda \times \sqrt{f'_c} \end{Bmatrix}$$ ***ACI 318-14 (Table 22.6.5.2)***

$$v_c = \min \begin{Bmatrix} 4 \times 1 \times \sqrt{5{,}000} \\ \left(2 + \frac{4}{1}\right) \times 1 \times \sqrt{5{,}000} \\ \left(\frac{20 \times 8.88}{148.88} + 2\right) \times 1 \times \sqrt{5{,}000} \end{Bmatrix} = \min \begin{Bmatrix} 282.84 \\ 424.26 \\ 225.73 \end{Bmatrix} = 225.73 \text{ psi}$$

$$\phi v_c = 0.75 \times 225.73 = 169.30 \text{ psi}$$

Since $\phi v_c > v_u$ at the critical section, the slab has adequate two-way shear strength around this drop panel.

5. Serviceability Requirements (Deflection Check)

Since the slab thickness was selected below the minimum slab thickness tables in ACI 318-14, the deflection calculations of immediate and time-dependent deflections are required and shown below including a comparison with spSlab model results.

5.1. Immediate (Instantaneous) Deflections

The calculation of deflections for two-way slabs is challenging even if linear elastic behavior can be assumed. Elastic analysis for three service load levels (D, $D + L_{sustained}$, $D+L_{Full}$) is used to obtain immediate deflections of the two-way slab in this example. However, other procedures may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests. ***ACI 318-14 (24.2.3)***

The effective moment of inertia (I_e) is used to account for the cracking effect on the flexural stiffness of the slab. I_e for uncracked section ($M_{cr} > M_a$) is equal to I_g. When the section is cracked ($M_{cr} < M_a$), then the following equation should be used:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \times I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \times I_{cr} \le I_g$$ ***ACI 318-14 (Eq. 24.2.3.5a)***

Where:

M_a = Maximum moment in member due to service loads at stage deflection is calculated.

The values of the maximum moments for the three service load levels are calculated from structural analysis as shown previously in this document. These moments are shown in Figure 23.

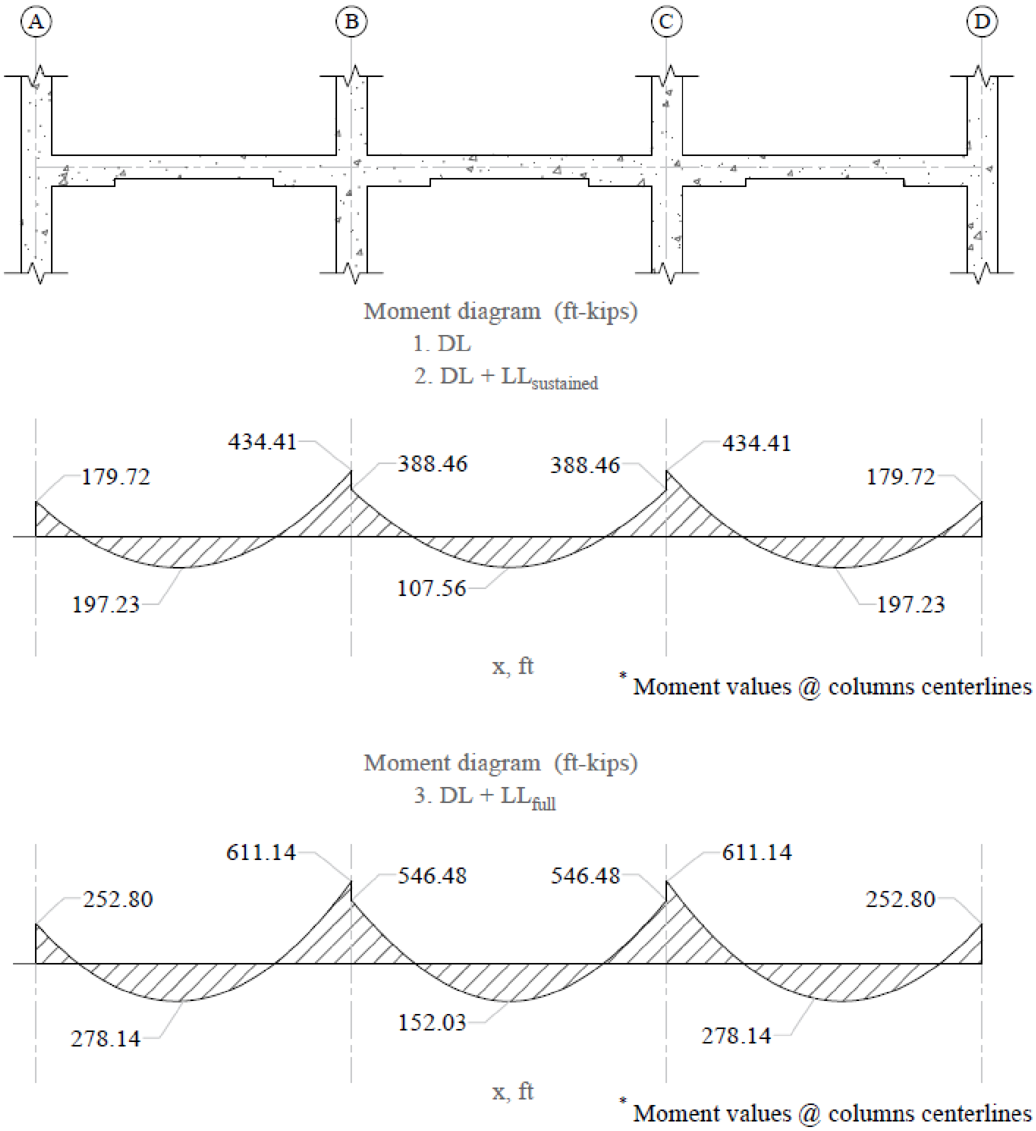


Figure 23 – Maximum Moments for the Three Service Load Levels (No live load is sustained in this example)

For positive moment (midspan) section:

M_{cr} = cracking moment.

$$M_{cr} = \frac{f_r \times I_g}{y_t} = \frac{530.33 \times 30,000}{5} \times \frac{1}{12 \times 1,000} = 265.17 \text{ ft-kip}$$ ***ACI 318-14 (Eq. 24.2.3.5b)***

f_r = Modulus of rapture of concrete.

$$f_r = 7.5 \times \lambda \times \sqrt{f'_c} = 7.5 \times 1.0 \times \sqrt{5,000} = 530.33 \text{ psi}$$ ***ACI 318-14 (Eq. 19.2.3.1)***

I_g = Moment of inertia of the gross uncracked concrete section

$$I_g = \frac{l_2 \times h^3}{12} = \frac{(30 \times 12) \times 10^3}{12} = 30,000 \text{ in.}^2$$

y_t = Distance from centroidal axis of gross section, neglecting reinforcement, to tension face, in.

$$y_t = \frac{h}{2} = \frac{10}{2} = 5 \text{ in.}$$

I_{cr} = moment of inertia of the cracked section transformed to concrete. ***PCA Notes on ACI 318-11 (9.5.2.2)***

As calculated previously, the positive reinforcement for the end span frame strip is 23 #6 bars located at 1.125 in. along the section from the bottom of the slab. Two of these bars are not continuous and will be conservatively excluded from the calculation of I_{cr} since they might not be adequately developed or tied (21 bars are used). The below shows all the parameters needed to calculate the moment of inertia of the cracked section transformed to concrete at midspan.

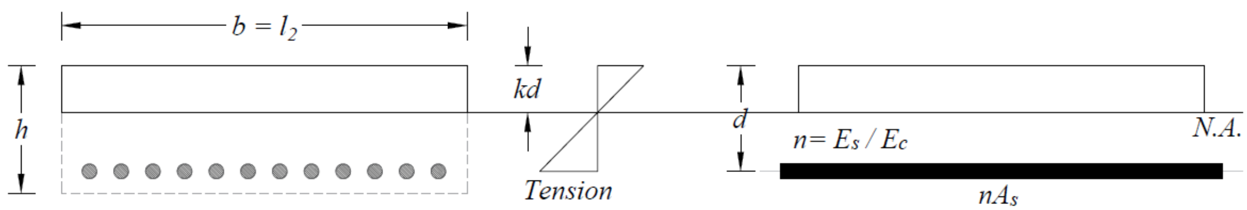


Figure 24 – Cracked Transformed Section (Positive Moment Section)

E_{cs} = Modulus of elasticity of slab concrete.

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f'_c} = 150^{1.5} \times 33 \times \sqrt{5,000} = 4,287 \times 10^3 \text{ psi}$$ ***ACI 318-14 (19.2.2.1.a)***

$$n = \frac{E_s}{E_{cs}} = \frac{29,000,000}{4,287,000} = 6.76$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$B = \frac{b}{n \times A_s} = \frac{30 \times 12}{6.76 \times (21 \times 0.44)} = 5.76 \text{ in.}^{-1}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$kd = \frac{\sqrt{2 \times d \times B + 1} - 1}{B} = \frac{\sqrt{2 \times 8.88 \times 5.76 + 1} - 1}{5.76} = 1.59 \text{ in.}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr} = \frac{b \times (kd)^3}{3} + n \times A_s \times (d - kd)^2$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr} = \frac{30 \times 12 \times (1.59)^3}{3} + 6.76 \times (21 \times 0.44) \times (8.88 - 1.59)^2 = 3,799.59 \text{ in.}^4$$

For negative moment section (near the interior support of the end span):

The negative reinforcement for the end span frame strip near the interior support is 32 #6 bars located at 1.125 in. along the section from the top of the slab.

$$M_{cr} = \frac{f_r \times I_g}{y_t} = \frac{530.33 \times 53,445}{5} \times \frac{1}{12 \times 1,000} = 401.42 \text{ ft-kip}$$ ***ACI 318-14 (Eq. 24.2.3.5b)***

$$f_r = 7.5 \times \lambda \times \sqrt{f'_c} = 7.5 \times 1.0 \times \sqrt{5,000} = 530.33 \text{ psi}$$ ***ACI 318-14 (Eq. 19.2.3.1)***

$$I_g = 53,445 \text{ in.}^2$$

$$y_t = 5.88 \text{ in.}$$

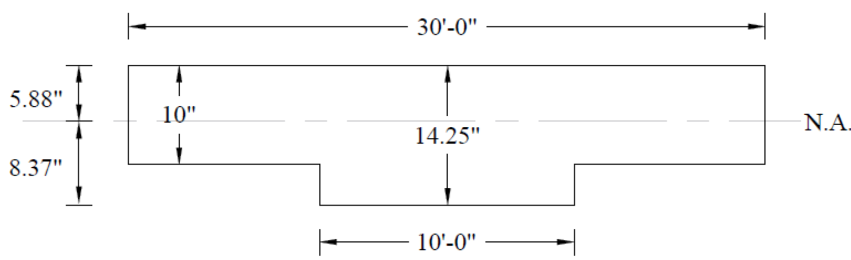


Figure 25 – I_g Calculations for Slab Section Near Support

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f'_c} = 150^{1.5} \times 33 \times \sqrt{5,000} = 4,287 \times 10^3 \text{ psi}$$ ***ACI 318-14 (19.2.2.1.a)***

$$n = \frac{E_s}{E_{cs}} = \frac{29,000,000}{4,287,000} = 6.76$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$B = \frac{b_b}{n \times A_s} = \frac{10 \times 12}{6.76 \times (32 \times 0.44)} = 1.26 \text{ in.}^{-1}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$kd = \frac{\sqrt{2 \times d \times B + 1} - 1}{B} = \frac{\sqrt{2 \times 13.13 \times 1.26 + 1} - 1}{1.26} = 3.84 \text{ in.}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr} = \frac{b_b \times (kd)^3}{3} + n \times A_s \times (d - kd)^2$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr} = \frac{10 \times 12 \times (3.84)^3}{3} + 6.76 \times (32 \times 0.44) \times (13.13 - 3.84)^2 = 10,476 \text{ in.}^4$$

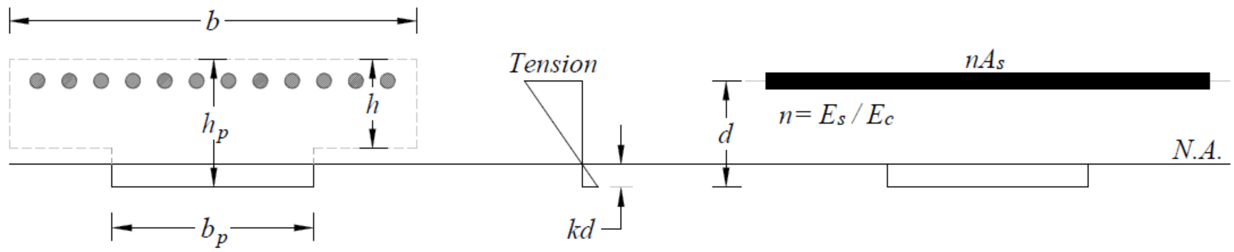


Figure 26 – Cracked Transformed Section (Negative Moment Section)

The effective moment of inertia procedure described in the Code is considered sufficiently accurate to estimate deflections. The effective moment of inertia, I_e, was developed to provide a transition between the upper and lower bounds of I_g and I_{cr} as a function of the ratio M_{cr}/M_a. For conventionally reinforced (nonprestressed) members, the effective moment of inertia, I_e, shall be calculated by Eq. (24.2.3.5a) unless obtained by a more comprehensive analysis.

I_e shall be permitted to be taken as the value obtained from Eq. (24.2.3.5a) at midspan for simple and continuous spans, and at the support for cantilevers. ***ACI 318-14 (24.2.3.7)***

For continuous one-way slabs and beams. I_e shall be permitted to be taken as the average of values obtained from Eq. (24.2.3.5a) for the critical positive and negative moment sections. ***ACI 318-14 (24.2.3.6)***

For the middle span (span with two ends continuous) with service load level ($D+LL_{full}$):

Since M_{cr} = 401.42 ft-kips < M_a = 546.48 ft-kips

$$I_e^- = \left(\frac{M_{cr}}{M_a}\right)^3 \times I_g + \left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right] \times I_{cr}$$ ***ACI 318-14 (24.2.3.5a)***

Where I_e^- is the effective moment of inertia for the critical negative moment section (near the support).

$$I_e^- = \left(\frac{401.42}{546.48}\right)^3 \times 53,445 + \left[1-\left(\frac{401.42}{546.48}\right)^3\right] \times 10,476 = 27,506 \text{ in.}^4$$

$$I_e^+ = I_g = 30,000 \text{ in.}^4 \text{ , since } M_{cr} = 265.17 \text{ ft-kips} > M_a = 152.03 \text{ ft-kips}$$

Where I_e^+ is the effective moment of inertia for the critical positive moment section (midspan).

Since midspan stiffness (including the effect of cracking) has a dominant effect on deflections, midspan section is heavily represented in calculation of I_e and this is considered satisfactory in approximate deflection calculations. Both the midspan stiffness (I_e^+) and averaged span stiffness ($I_{e,avg}$) can be used in the calculation of immediate (instantaneous) deflection.

The averaged effective moment of inertia ($I_{e,avg}$) is given by:

$$I_{e,avg} = 0.70 \times I_e^+ + 0.15 \times \left(I_{e,l}^- + I_{e,r}^-\right) \text{ for interior span}$$ ***PCA Notes on ACI 318-11 (9.5.2.4(2))***

$$I_{e,avg} = 0.85 \times I_e^+ + 0.15 \times I_e^- \text{ for end span}$$ ***PCA Notes on ACI 318-11 (9.5.2.4(1))***

However, these expressions lead to improved results only for continuous prismatic members. The drop panels in this example result in non-prismatic members and the following expressions should be used according to ACI 318-89:

$$I_{e,avg} = 0.50 \times I_e^+ + 0.25 \times \left(I_{e,l}^- + I_{e,r}^-\right) \text{ for interior span}$$ ***ACI 435R-95 (2.14)***

For the middle span (span with two ends continuous) with service load level ($D+LL_{full}$):

$$I_{e,avg} = 0.50 \times 30{,}000 + 0.25 \times \left(27{,}506 + 27{,}506\right) = 28{,}753 \text{ in.}^4$$

$$I_{e,avg} = 0.50 \times I_e^+ + 0.50 \times I_e^- \text{ for end span}$$ ***ACI 435R-95 (2.14)***

For the end span (span with one end continuous) with service load level ($D+LL_{full}$):

$$I_{e,avg} = 0.50 \times 26{,}502 + 0.50 \times 22{,}649 = 24{,}577 \text{ in.}^4$$

Where:

- $I_{e,l}^-$ = The effective moment of inertia for the critical negative moment section near the left support.
- $I_{e,r}^-$ = The effective moment of inertia for the critical negative moment section near the right support.
- I_e^+ = The effective moment of inertia for the critical positive moment section (midspan).

The following Table provides a summary of the required parameters and calculated values needed for deflections for exterior and interior spans.

<table>
<tr><th colspan="15">Table 6 - Averaged Effective Moment of Inertia Calculations</th></tr>
<tr><th colspan="15">For Frame Strip</th></tr>
<tr><th rowspan="2">Span</th><th rowspan="2">zone</th><th rowspan="2">I_g (in.^{4})</th><th rowspan="2">I_{cr} (in.^{4})</th><th colspan="3">M_a (ft-kip)</th><th rowspan="2">M_{cr} (k-ft)</th><th colspan="3">I_e (in.^{4})</th><th colspan="3">$I_{e,avg}$ (in.^{4})</th></tr>
<tr><th>D</th><th>D + LL_{Sus}</th><th>D + L_{full}</th><th>D</th><th>D + LL_{Sus}</th><th>D + L_{full}</th><th>D</th><th>D + LL_{Sus}</th><th>D + L_{full}</th></tr>
<tr><td rowspan="3">Ext</td><td>Left</td><td>53,445</td><td>10,476</td><td>179.72</td><td>179.72</td><td>252.80</td><td>401.42</td><td>53,445</td><td>53,445</td><td>53,445</td><td rowspan="3">37,190</td><td rowspan="3">37,190</td><td rowspan="3">24,577</td></tr>
<tr><td>Midspan</td><td>30,000</td><td>3,800</td><td>197.23</td><td>197.23</td><td>278.14</td><td>265.17</td><td>30,000</td><td>30,000</td><td>26,502</td></tr>
<tr><td>Right</td><td>53,445</td><td>10,476</td><td>434.41</td><td>434.41</td><td>611.14</td><td>401.42</td><td>44,379</td><td>44,379</td><td>22,653</td></tr>
<tr><td rowspan="3">Int</td><td>Left</td><td>53,445</td><td>10,476</td><td>388.46</td><td>388.46</td><td>546.48</td><td>401.42</td><td>53,445</td><td>53,445</td><td>27,506</td><td rowspan="3">41,723</td><td rowspan="3">41,723</td><td rowspan="3">28,753</td></tr>
<tr><td>Mid</td><td>30,000</td><td>3,800</td><td>107.56</td><td>107.56</td><td>152.03</td><td>265.17</td><td>30,000</td><td>30,000</td><td>30,000</td></tr>
<tr><td>Right</td><td>53,445</td><td>10,476</td><td>388.46</td><td>388.46</td><td>546.48</td><td>401.42</td><td>53,445</td><td>53,445</td><td>27,506</td></tr>
</table>

Deflections in two-way slab systems shall be calculated taking into account size and shape of the panel, conditions of support, and nature of restraints at the panel edges. For immediate deflections in two-way slab systems, the midpanel deflection is computed as the sum of deflection at midspan of the column strip or column line in one direction (Δ_{cx} or Δ_{cy}) and deflection at midspan of the middle strip in the orthogonal direction (Δ_{mx} or Δ_{my}). Figure 27 shows the deflection computation for a rectangular panel. The average Δ for panels that have different properties in the two direction is calculated as follows:

$$\Delta = \frac{\left(\Delta_{cx} + \Delta_{my}\right) + \left(\Delta_{cy} + \Delta_{mx}\right)}{2}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 8)

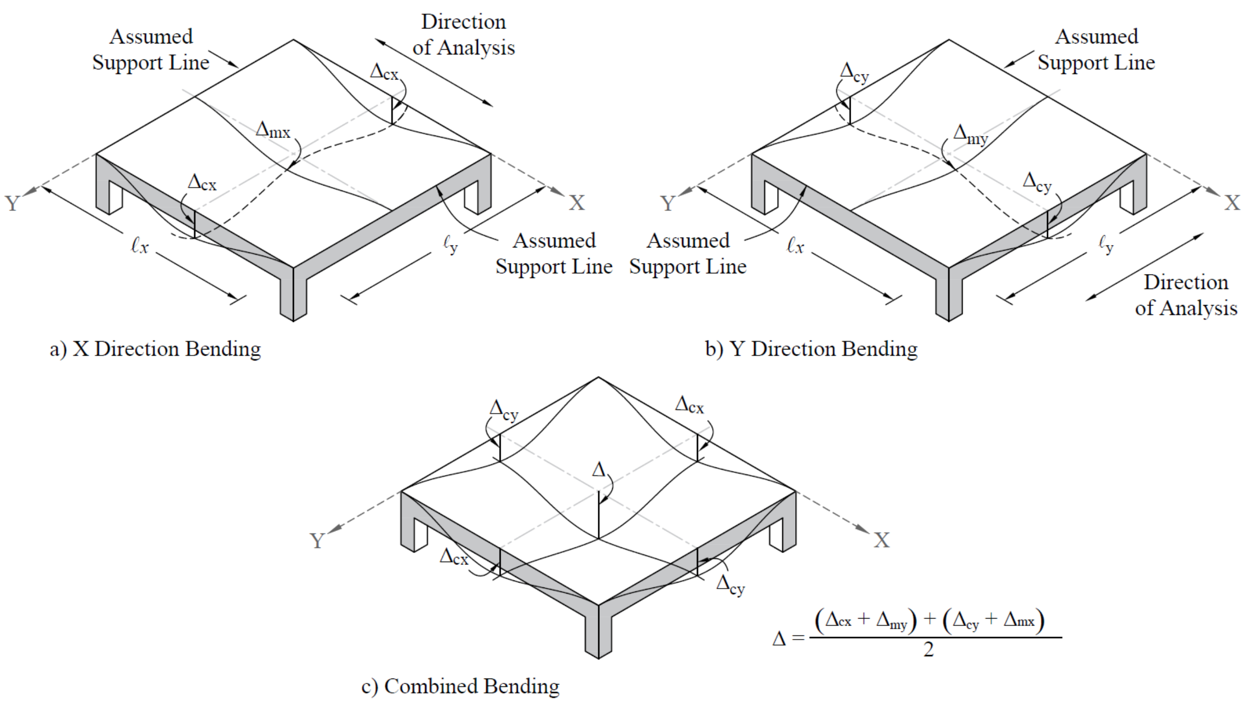


Figure 27 – Deflection Computation for a Rectangular Panel

To calculate each term of the previous equation, the following procedure should be used. Figure 28 shows the procedure of calculating the term Δ_{cx}. Same procedure can be used to find the other terms.

$$\Delta_{cx} = \Delta_{cx,fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$

$$\Delta_{c,fixed} = LDF_c \times \Delta_{frame,fixed} \times \frac{I_{frame,average}}{I_{c,g}}$$

$$LDF_c = \frac{LDF^{+} + \frac{LDF_l^{-} + LDF_R^{-}}{2}}{2}$$

$$\Delta_{frame,fixed} = \frac{w \times l^4}{384 \times E_c \times I_{frame,average}}$$

$$\Delta\theta_{c,R} = \theta_{c,R} \times \left(\frac{l}{8}\right) \times \left(\frac{I_g}{I_e}\right)_{frame}$$

$$\theta_{c,R} = \frac{(M_{net,R})_{frame}}{K_{ec}}$$

$$\Delta\theta_{c,L} = \theta_{c,L} \times \left(\frac{l}{8}\right) \times \left(\frac{I_g}{I_e}\right)_{frame}$$

$$\theta_{c,L} = \frac{(M_{net,L})_{frame}}{K_{ec}}$$

Figure 28 – Δ_{cx} Calculation Procedure

For end span - service dead load case:

$$\Delta_{frame,fixed} = \frac{w \times l^4}{384 \times E_c \times I_{frame,averaged}}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 10)***

Where:

Δ_{frame_fixed} = Deflection of column strip assuming fixed-end condition.

$$w = \left(20 + 150 \times \frac{10}{12}\right) \times 30 = 4,350 \ \frac{\text{lb}}{\text{ft}}$$

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f_c'} = 150^{1.5} \times 33 \times \sqrt{5,000} = 4,287 \times 10^3 \text{ psi}$$ ***ACI 318-14 (19.2.2.1.a)***

$I_{frame,averaged}$ = The averaged effective moment of inertia ($I_{e,avg}$) for the frame strip for service dead load case from Table 6 = 37,190 in.^{4}

$$\Delta_{frame,fixed} = \frac{4,350 \times 30^4 \times 12^4}{384 \times (4,287 \times 10^3) \times 37,190} = 0.0995 \text{ in.}$$

$$\Delta_{c,fixed} = LDF_c \times \Delta_{frame,fixed} \times \frac{I_{frame,averaged}}{I_{c,g}}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 11)***

For this example and like in the spSlab program, the effective moment of inertia at midspan will be used.

LDF_c is the load distribution factor for the column strip. The load distribution factor for the column strip can be found from the following equation:

$$LDF_c = \frac{LDF^+ + \dfrac{LDF_l^- + LDF_R^-}{2}}{2}$$

And the load distribution factor for the middle strip can be found from the following equation:

$$LDF_m = 1 - LDF_c$$

For the end span, LDF for exterior negative region (LDF_L^-), interior negative region (LDF_R^-), and positive region (LDF_L^+) are 1.00, 0.75, and 0.60, respectively (From Table 2 of this document). Thus, the load distribution factor for the column strip for the end span is given by:

$$LDF_c = \frac{0.6 + \dfrac{1.0 + 0.75}{2}}{2} = 0.738$$

$I_{c,g}$ = The gross moment of inertia (I_g) for the column strip for service dead load = 15,000 in.^{4}

$$\Delta_{c,fixed} = 0.738 \times 0.0995 \times \frac{30,000}{15,000} = 0.1467 \text{ in.}$$

$$\theta_{c,L} = \frac{(M_{net,L})_{frame}}{K_{ec}}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 12)***

Where:

$\theta_{c,L}$ = Rotation of the left support

$(M_{net,L})_{frame}$ = 179.72 ft-kips = Net frame strip negative moment of the left support

K_{ec} = Effective column stiffness = 1,628.67 × 10^6 in.-lb (calculated above).

$$\theta_{c,L} = \frac{179.72 \times 12 \times 1,000}{1,628.67 \times 10^6} = 0.0013 \text{ rad}$$

$$\Delta\theta_{c,L} = \theta_{c,L} \times \left(\frac{l}{8}\right) \times \left(\frac{I_g}{I_e}\right)_{frame}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 14)***

Where:

$\Delta\theta_{c,L}$ = Midspan deflection due to rotation of left support.

$(I_g / I_e)_{frame}$ = Gross to effective moment of inertia ratio for frame strip.

$$\Delta\theta_{c,L} = 0.0013 \times \frac{30 \times 12}{8} \times \frac{30{,}000}{37{,}190} = 0.0481 \text{ in.}$$

$$\theta_{c,R} = \frac{(M_{net,R})_{frame}}{K_{ec}} = \frac{(434.41 - 388.46) \times 12 \times 1{,}000}{1{,}628.67 \times 10^6} = 0.0003 \text{ rad}$$

Where:

$\theta_{c,R}$ = rotation of the span right support.

$(M_{net,R})_{frame}$ = Net frame strip negative moment of the right support.

$$\Delta\theta_{c,R} = \theta_{c,R} \times \left(\frac{l}{8}\right) \times \left(\frac{I_g}{I_e}\right)_{frame} = 0.0003 \times \frac{30 \times 12}{8} \times \frac{30{,}000}{37{,}190} = 0.0123 \text{ in.}$$

Where:

$\Delta\theta_{c,R}$ = Midspan deflection due to rotation of right support.

$$\Delta_{cx} = \Delta_{cx,fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 9)***

$$\Delta_{cx} = 0.1467 + 0.0123 + 0.003 = 0.2070 \text{ in.}$$

Following the same procedure, Δ_{mx} can be calculated for the middle strip. This procedure is repeated for the equivalent frame in the orthogonal direction to obtain Δ_{cy}, and Δ_{my} for the end and middle spans for the other load levels ($D+LL_{sus}$ *and* $D+LL_{full}$).

Since in this example the panel is squared, $\Delta_{cx} = \Delta_{cy}$= 0.2170 in. and $\Delta_{mx} = \Delta_{my}$= 0.1126 in.

The average Δ for the corner panel is calculated as follows:

$$\Delta = \frac{(\Delta_{cx} + \Delta_{my}) + (\Delta_{cy} + \Delta_{mx})}{2} = (\Delta_{cx} + \Delta_{my}) = (\Delta_{cy} + \Delta_{mx}) = 0.2170 + 0.1126 = 0.3196 \text{ in.}$$

Table 7 - Immediate (Instantaneous) Deflections in the x-direction

Column Strip

Span	LDF	D						
		$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{c\text{-}fixed}$ (in.)	θ_{c1} (rad)	θ_{c2} (rad)	$\Delta\theta_{c1}$ (in.)	$\Delta\theta_{c2}$ (in.)	Δ_{cx}, (in.)
Ext	0.738	0.0995	0.1467	0.0013	0.0003	0.0481	0.0123	0.2070
Int	0.675	0.0886	0.1197	-0.0003	-0.0003	-0.0110	-0.0110	0.0978

Span	LDF	D+LL$_{sus}$						
		$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{c\text{-}fixed}$ (in.)	θ_{c1} (rad)	θ_{c2} (rad)	$\Delta\theta_{c1}$ (in.)	$\Delta\theta_{c2}$ (in.)	Δ_{cx} (in.)
Ext	0.738	0.0995	0.1467	0.0013	0.0003	0.0481	0.0123	0.2070
Int	0.675	0.0886	0.1197	-0.0003	-0.0003	-0.0110	-0.0110	0.0978

Span	LDF	D+LL$_{full}$						
		$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{c\text{-}fixed}$ (in.)	θ_{c1} (rad)	θ_{c2} (rad)	$\Delta\theta_{c1}$ (in.)	$\Delta\theta_{c2}$ (in.)	Δ_{cx} (in.)
Ext	0.738	0.2128	0.2772	0.0019	0.0005	0.1023	0.0262	0.4057
Int	0.675	0.1819	0.2455	-0.0005	-0.0005	-0.0224	-0.0224	0.2008

Span	LDF	LL
		Δ_{cx} (in.)
Ext	0.738	0.1987
Int	0.675	0.1030

Middle Strip

LDF	D						
	$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{m\text{-}fixed}$ (in.)	θ_{m1} (rad)	θ_{m2} (rad)	$\Delta\theta_{m1}$ (in.)	$\Delta\theta_{m2}$ (in.)	Δ_{mx} (in.)
0.263	0.0995	0.0522	0.0013	0.0003	0.0481	0.0123	0.1126
0.325	0.0886	0.0576	-0.0003	-0.0003	-0.0110	-0.0110	0.0357

LDF	D+LL$_{sus}$						
	$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{m\text{-}fixed}$ (in.)	θ_{m1} (rad)	θ_{m2} (rad)	$\Delta\theta_{m1}$ (in.)	$\Delta\theta_{m2}$ (in.)	Δ_{mx} (in.)
0.263	0.0995	0.0522	0.0013	0.0003	0.0481	0.0123	0.1126
0.325	0.0886	0.0576	-0.0003	-0.0003	-0.0110	-0.0110	0.0357

LDF	D+LL$_{full}$						
	$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{m\text{-}fixed}$ (in.)	θ_{m1} (rad)	θ_{m2} (rad)	$\Delta\theta_{m1}$ (in.)	$\Delta\theta_{m2}$ (in.)	Δ_{mx} (in.)
0.263	0.2128	0.0987	0.0019	0.0005	0.1023	0.0262	0.2272
0.325	0.1819	0.1182	-0.0005	-0.0005	-0.0224	-0.0224	0.0735

LDF	LL
	Δ_{mx} (in.)
0.263	0.1146
0.325	0.0378

5.2. Time-Dependent (Long-Term) Deflections (Δ_{lt})

The additional time-dependent (long-term) deflection resulting from creep and shrinkage (Δ_{cs}) may be estimated as follows:

$$\Delta_{cs} = \lambda_{\Delta} \times (\Delta_{sust})_{Inst}$$ **PCA Notes on ACI 318-11 (9.5.2.5 Eq. 4)**

The total time-dependent (long-term) deflection is calculated as:

$$(\Delta_{total})_{lt} = (\Delta_{sust})_{Inst} \times (1+\lambda_{\Delta}) + \left[(\Delta_{total})_{Inst} - (\Delta_{sust})_{Inst}\right]$$ **CSA A23.3-04 (N9.8.2.5)**

Where:

$(\Delta_{sust})_{Inst}$ = Immediate (instantaneous) deflection due to sustained load, in.

$$\lambda_{\Delta} = \frac{\xi}{1+50\times\rho'}$$ **ACI 318-14 (24.2.4.1.1)**

$(\Delta_{total})_{lt}$ = Time-dependent (long-term) total deflection, in.

$(\Delta_{total})_{Inst}$ = Total immediate (instantaneous) deflection, in.

For the exterior span

ξ = 2, consider the sustained load duration to be 60 months or more. **ACI 318-14 (Table 24.2.4.1.3)**

ρ' = 0, conservatively.

$$\lambda_{\Delta} = \frac{2}{1+50\times 0} = 2$$

$$\Delta_{cs} = 2\times 0.2070 = 0.4140 \text{ in.}$$

$$(\Delta_{total})_{lt} = 0.2070\times(1+2)+(0.4057-0.2070) = 0.8197 \text{ in.}$$

The following Table shows long-term deflections for the exterior and interior spans for the analysis in the x-direction, for column and middle strips.

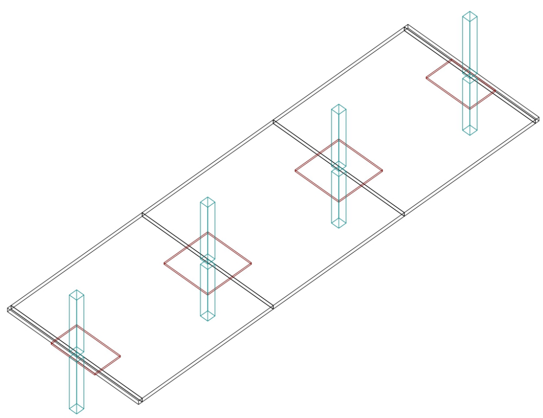
Table 8 - Long-Term Deflections					
Column Strip					
Span	**$(\Delta_{sust})_{Inst}$ (in.)**	**λ_Δ**	**Δ_{cs} (in.)**	**$(\Delta_{total})_{Inst}$ (in.)**	**$(\Delta_{total})_{lt}$ (in.)**
Exterior	0.2070	2	0.4140	0.4057	0.8197
Interior	0.0978	2	0.1955	0.2008	0.3963
Middle Strip					
Exterior	0.1126	2	0.2251	0.2272	0.4523
Interior	0.0357	2	0.0714	0.0735	0.1449

6. spSlab Software Program Model Solution

spSlab program utilizes the Equivalent Frame Method described and illustrated in details here for modeling, analysis and design of two-way concrete floor slab systems with drop panels. spSlab uses the exact geometry and boundary conditions provided as input to perform an elastic stiffness (matrix) analysis of the equivalent frame taking into account the torsional stiffness of the slabs framing into the column. It also takes into account the complications introduced by a large number of parameters such as vertical and torsional stiffness of transverse beams, the stiffening effect of drop panels, column capitals, and effective contribution of columns above and below the floor slab using the of equivalent column concept ***(ACI 318-14 (R8.11.4))***.

spSlab Program models the equivalent frame as a design strip. The design strip is, then, separated by spSlab into column and middle strips. The program calculates the internal forces (Shear Force & Bending Moment), moment and shear capacity vs. demand diagrams for column and middle strips, instantaneous and long-term deflection results, and required flexural reinforcement for column and middle strips.

spSlab v5.50
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams, One-way and Two-way Slab Systems



Contents

1. Input Echo

1.1. General Information

File Name	...\Two-Way Flat Slab with Drop Panel ACI 318-...
Project	Two-Way Flat Slab with Drop Panels ACI 318-14
Frame	Interior Frame
Engineer	SP
Code	ACI 318-14
Reinforcement Database	ASTM A615
Mode	Design
Number of supports =	4 + Left cantilever + Right cantilever
Floor System	Two-Way

1.2. Solve Options

Live load pattern ratio = 0%
Minimum free edge distance for punching shear = 4 times slab thickness.
Circular critical section around circular supports used (if possible).
Deflections are based on cracked section properties.
In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
User-defined slab strip widths NOT selected.
User-defined distribution factors NOT selected.
One-way shear in drop panel selected.
Distribution of shear to strips NOT selected.
Beam T-section design NOT selected.
Longitudinal beam contribution in negative reinforcement design over support NOT selected.
Transverse beam contribution in negative reinforcement design over support NOT selected.

1.3. Material Properties

1.3.1. Concrete: Slabs / Beams

w_c	150	lb/ft^3
f'_c	5	ksi
E_c	4286.8	ksi
f_r	0.53033	ksi

1.3.2. Concrete: Columns

w_c	150	lb/ft^3
f'_c	6	ksi
E_c	4696	ksi
f_r	0.58095	ksi

1.3.3. Reinforcing Steel

f_y	60 ksi
f_{yt}	60 ksi
E_s	29000 ksi
Epoxy coated bars	No

1.4. Reinforcement Database

Size	Db	Ab	Wb	Size	Db	Ab	Wb
	in	in²	lb/ft		in	in²	lb/ft
#3	0.38	0.11	0.38	#4	0.50	0.20	0.67
#5	0.63	0.31	1.04	#6	0.75	0.44	1.50
#7	0.88	0.60	2.04	#8	1.00	0.79	2.67
#9	1.13	1.00	3.40	#10	1.27	1.27	4.30
#11	1.41	1.56	5.31	#14	1.69	2.25	7.65
#18	2.26	4.00	13.60				

1.5. Span Data

1.5.1. Slabs

Notes:
*a - Deflection check required for panels where slab thickness (t) is less than minimum (Hmin).
Deflection check required for panels where code-specified Hmin for two-way construction doesn't apply due to:
*i - cantilever end span (LC, RC) support condition

Span	Loc	L1	t	wL	wR	L2L	L2R	H_{min}
		ft	in	ft	ft	ft	ft	in
1	Int	0.833	10.00	15.000	15.000	30.000	30.000	--- LC *i
2	Int	30.000	10.00	15.000	15.000	30.000	30.000	10.30 *a
3	Int	30.000	10.00	15.000	15.000	30.000	30.000	9.44
4	Int	30.000	10.00	15.000	15.000	30.000	30.000	10.30 *a
5	Int	0.833	10.00	15.000	15.000	30.000	30.000	--- RC *i

1.6. Support Data

1.6.1. Columns

Support	c1a	c2a	Ha	c1b	c2b	Hb	Red %
	in	in	ft	in	in	ft	
1	20.00	20.00	13.000	20.00	20.00	13.000	100
2	20.00	20.00	13.000	20.00	20.00	13.000	100
3	20.00	20.00	13.000	20.00	20.00	13.000	100
4	20.00	20.00	13.000	20.00	20.00	13.000	100

1.6.2. Drop Panels

Notes:
*b - Standard drop.

Support	h	Ll	Lr	Wl	Wr	
	in	ft	ft	ft	ft	
1	4.25	0.833	5.000	5.000	5.000	*b
2	4.25	5.000	5.000	5.000	5.000	*b
3	4.25	5.000	5.000	5.000	5.000	*b
4	4.25	5.000	0.833	5.000	5.000	*b

1.6.3. Boundary Conditions

Support	Spring K_z kip/in	Spring K_{ry} kip-in/rad	Far End Above	Far End Below
1	0	0	Fixed	Fixed
2	0	0	Fixed	Fixed
3	0	0	Fixed	Fixed
4	0	0	Fixed	Fixed

1.7. Load Data

1.7.1. Load Cases and Combinations

Case	SELF	Dead	Live
Type	DEAD	DEAD	LIVE
U1	1.200	1.200	1.600

1.7.2. Area Loads

Case/Patt	Span	Wa lb/ft²
SELF	1	125.00
	2	125.00
	3	125.00
	4	125.00
	5	125.00
Dead	1	20.00
	2	20.00
	3	20.00
	4	20.00
	5	20.00
Live	1	60.00
	2	60.00
	3	60.00
	4	60.00
	5	60.00

1.7.3. Line Loads

Case/Patt	Span	Wa lb/ft	La ft	Wb lb/ft	Lb ft
SELF	1	531.25	0.000	531.25	0.833
	2	531.25	0.000	531.25	5.000
	2	531.25	25.000	531.25	30.000
	3	531.25	0.000	531.25	5.000
	3	531.25	25.000	531.25	30.000
	4	531.25	0.000	531.25	5.000
	4	531.25	25.000	531.25	30.000
	5	531.25	0.000	531.25	0.833

1.8. Reinforcement Criteria

1.8.1. Slabs and Ribs

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Bar Size		#6	#6	#6	#6
Bar spacing	in	1.00	18.00	1.00	18.00
Reinf ratio	%	0.18	2.00	0.18	2.00
Clear Cover	in	0.75		0.75	

There is NOT more than 12 in of concrete below top bars.

1.8.2. Beams

	Units	Top Bars		Bottom Bars		Stirrups	
		Min.	Max.	Min.	Max.	Min.	Max.
Bar Size		#5	#8	#5	#8	#3	#5
Bar spacing	in	1.00	18.00	1.00	18.00	6.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00		
Clear Cover	in	1.50		1.50			
Layer dist.	in	1.00		1.00			
No. of legs						2	6
Side cover	in					1.50	
1st Stirrup	in					3.00	

There is NOT more than 12 in of concrete below top bars.

2. Design Results*

*Unless otherwise noted, all results are in the direction of analysis only. Another analysis in the perpendicular direction has to be carried out for two-way slab systems.

2.1. Strip Widths and Distribution Factors

Notes:
*Used for bottom reinforcement. **Used for top reinforcement.

		Width			Moment Factor		
Span	Strip	Left **	Right **	Bottom *	Left **	Right **	Bottom *
		ft	ft	ft	ft	ft	ft
1	Column	15.00	15.00	15.00	1.000	1.000	0.600
	Middle	15.00	15.00	15.00	0.000	0.000	0.400
2	Column	15.00	15.00	15.00	1.000	0.750	0.600
	Middle	15.00	15.00	15.00	0.000	0.250	0.400
3	Column	15.00	15.00	15.00	0.750	0.750	0.600
	Middle	15.00	15.00	15.00	0.250	0.250	0.400
4	Column	15.00	15.00	15.00	0.750	1.000	0.600
	Middle	15.00	15.00	15.00	0.250	0.000	0.400
5	Column	15.00	15.00	15.00	1.000	1.000	0.600
	Middle	15.00	15.00	15.00	0.000	0.000	0.400

2.2. Top Reinforcement

Notes:
*3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

Span	Strip	Zone	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars	
			ft	k-ft	ft	in^2	in^2	in^2	in		
1	Column	Left	15.00	0.28	0.241	3.240	31.950	0.007	18.000	10-#6	*3 *5
		Midspan	15.00	0.90	0.447	3.240	47.250	0.015	18.000	10-#6	*3 *5
		Right	15.00	2.10	0.687	4.158	31.500	0.036	18.000	10-#6	*3
	Middle	Left	15.00	0.00	0.000	3.240	31.950	0.000	18.000	10-#6	*3 *5
		Midspan	15.00	0.00	0.344	3.240	31.950	0.000	18.000	10-#6	*3 *5
		Right	15.00	0.00	0.687	3.240	31.950	0.000	18.000	10-#6	*3 *5
2	Column	Left	15.00	244.81	0.833	4.158	31.500	4.225	18.000	10-#6	
		Midspan	15.00	0.00	15.000	0.000	31.950	0.000	0.000	---	
		Right	15.00	517.57	29.167	4.158	31.500	9.137	8.571	21-#6	
	Middle	Left	15.00	1.37	2.059	3.240	31.950	0.034	18.000	10-#6	*3 *5
		Midspan	15.00	0.00	15.000	0.000	31.950	0.000	0.000	---	
		Right	15.00	172.52	29.167	3.240	31.950	4.406	16.364	11-#6	
3	Column	Left	15.00	463.59	0.833	4.158	31.500	8.147	8.571	21-#6	
		Midspan	15.00	0.00	15.000	0.000	31.950	0.000	0.000	---	
		Right	15.00	463.59	29.167	4.158	31.500	8.147	8.571	21-#6	
	Middle	Left	15.00	154.53	0.833	3.240	31.950	3.938	16.364	11-#6	*5
		Midspan	15.00	0.00	15.000	0.000	31.950	0.000	0.000	---	
		Right	15.00	154.53	29.167	3.240	31.950	3.938	16.364	11-#6	*5
4	Column	Left	15.00	517.57	0.833	4.158	31.500	9.137	8.571	21-#6	
		Midspan	15.00	0.00	15.000	0.000	31.950	0.000	0.000	---	
		Right	15.00	244.81	29.167	4.158	31.500	4.225	18.000	10-#6	
	Middle	Left	15.00	172.52	0.833	3.240	31.950	4.406	16.364	11-#6	
		Midspan	15.00	0.00	15.000	0.000	31.950	0.000	0.000	---	
		Right	15.00	1.37	27.941	3.240	31.950	0.034	18.000	10-#6	*3 *5
5	Column	Left	15.00	2.10	0.146	4.158	31.500	0.036	18.000	10-#6	*3
		Midspan	15.00	0.90	0.386	3.240	47.250	0.015	18.000	10-#6	*3 *5
		Right	15.00	0.28	0.593	3.240	31.950	0.007	18.000	10-#6	*3 *5
	Middle	Left	15.00	0.00	0.146	3.240	31.950	0.000	18.000	10-#6	*3 *5
		Midspan	15.00	0.00	0.490	3.240	31.950	0.000	18.000	10-#6	*3 *5
		Right	15.00	0.00	0.833	3.240	31.950	0.000	18.000	10-#6	*3 *5

2.3. Top Bar Details

		Left				Continuous		Right			
Span	Strip	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
			ft		ft		ft		ft		ft
1	Column	---		---		10-#6	0.83	---		---	
	Middle	---		---		10-#6	0.83	---		---	

		Left				Continuous		Right			
Span	Strip	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
			ft		ft		ft		ft		ft
2	Column	10-#6	10.18	---		---		11-#6	10.18	10-#6	6.50
	Middle	10-#6	7.07	---		---		11-#6	9.27	---	
3	Column	11-#6	10.18	10-#6	6.50	---		11-#6	10.18	10-#6	6.50
	Middle	11-#6	9.77	---		---		11-#6	9.77	---	
4	Column	11-#6	10.18	10-#6	6.50	---		10-#6	10.18	---	
	Middle	11-#6	9.27	---		---		10-#6	7.07	---	
5	Column	---		---		10-#6	0.83	---		---	
	Middle	---		---		10-#6	0.83	---		---	

2.4. Top Bar Development Lengths

		Left				Continuous		Right			
Span	Strip	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen
			in		in		in		in		in
1	Column	---		---		10-#6	12.00	---		---	
	Middle	---		---		10-#6	12.00	---		---	
2	Column	10-#6	24.44	---		---		11-#6	25.17	10-#6	25.17
	Middle	10-#6	12.00	---		---		11-#6	23.17	---	
3	Column	11-#6	22.44	10-#6	22.44	---		11-#6	22.44	10-#6	22.44
	Middle	11-#6	20.71	---		---		11-#6	20.71	---	
4	Column	11-#6	25.17	10-#6	25.17	---		10-#6	24.44	---	
	Middle	11-#6	23.17	---		---		10-#6	12.00	---	
5	Column	---		---		10-#6	12.00	---		---	
	Middle	---		---		10-#6	12.00	---		---	

2.5. Bottom Reinforcement

Notes:
*3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

Span	Strip	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars	
		ft	k-ft	ft	in^2	in^2	in^2	in		
1	Column	15.00	0.00	0.344	0.000	31.950	0.000	0.000	---	
	Middle	15.00	0.00	0.344	0.000	31.950	0.000	0.000	---	
2	Column	15.00	219.68	13.000	3.240	31.950	5.641	13.846	13-#6	
	Middle	15.00	146.45	13.000	3.240	31.950	3.728	18.000	10-#6	*5
3	Column	15.00	120.14	15.000	3.240	31.950	3.049	18.000	10-#6	*3 *5
	Middle	15.00	80.09	15.000	3.240	31.950	2.024	18.000	10-#6	*3 *5
4	Column	15.00	219.68	17.000	3.240	31.950	5.641	13.846	13-#6	
	Middle	15.00	146.45	17.000	3.240	31.950	3.728	18.000	10-#6	*5

Span	Strip	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars
		ft	k-ft	ft	in²	in²	in²	in	
5	Column	15.00	0.00	0.490	0.000	31.950	0.000	0.000	---
	Middle	15.00	0.00	0.490	0.000	31.950	0.000	0.000	---

2.6. Bottom Bar Details

Span	Strip	Long Bars			Short Bars		
		Bars	Start	Length	Bars	Start	Length
			ft	ft		ft	ft
1	Column	---			---		
	Middle	---			---		
2	Column	13-#6	0.00	30.00	---		
	Middle	8-#6	0.00	30.00	2-#6	0.00	25.50
3	Column	10-#6	0.00	30.00	---		
	Middle	8-#6	0.00	30.00	2-#6	4.50	21.00
4	Column	13-#6	0.00	30.00	---		
	Middle	8-#6	0.00	30.00	2-#6	4.50	25.50
5	Column	---			---		
	Middle	---			---		

2.7. Bottom Bar Development Lengths

Span	Strip	Long Bars		Short Bars	
		Bars	DevLen	Bars	DevLen
			in		in
1	Column	---		---	
	Middle	---		---	
2	Column	13-#6	25.10	---	
	Middle	8-#6	21.57	2-#6	21.57
3	Column	10-#6	17.64	---	
	Middle	8-#6	12.00	2-#6	12.00
4	Column	13-#6	25.10	---	
	Middle	8-#6	21.57	2-#6	21.57
5	Column	---		---	
	Middle	---		---	

2.8. Flexural Capacity

Span	Strip	Top						Bottom				
		x	$A_{s,top}$	ΦM_n-	M_u-	Comb Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb Pat	Status
		ft	in²	k-ft	k-ft			in²	k-ft	k-ft		
1	Column	0.000	4.40	-172.31	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.241	4.40	-256.46	-0.28	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.417	4.40	-256.46	-0.76	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.447	4.40	-254.75	-0.90	U1 All	OK	0.00	0.00	0.00	U1 All	OK

Span	Strip	Top						Bottom				
		x	$A_{s,top}$	ΦM_n-	M_u-	Comb Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb Pat	Status
		ft	in²	k-ft	k-ft			in²	k-ft	k-ft		
		0.687	4.40	-254.75	-2.10	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.833	4.40	-254.75	-3.03	U1 All	---	0.00	0.00	0.00	U1 All	---
	Middle	0.000	4.40	-172.31	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.241	4.40	-172.31	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.417	4.40	-172.31	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.447	4.40	-172.31	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.687	4.40	-172.31	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.833	4.40	-172.31	0.00	U1 All	---	0.00	0.00	0.00	U1 All	---
2	Column	0.000	4.40	-254.75	-335.03	U1 All	---	5.72	222.67	0.00	U1 All	---
		0.625	4.40	-254.75	-266.67	U1 All	---	5.72	222.67	0.00	U1 All	---
		0.833	4.40	-254.75	-244.81	U1 All	OK	5.72	222.67	0.00	U1 All	OK
		5.000	4.40	-254.75	0.00	U1 All	OK	5.72	222.67	61.82	U1 All	OK
		5.000	4.40	-172.31	0.00	U1 All	OK	5.72	222.67	61.84	U1 All	OK
		8.146	4.40	-172.31	0.00	U1 All	OK	5.72	222.67	160.99	U1 All	OK
		10.183	0.00	0.00	0.00	U1 All	OK	5.72	222.67	199.55	U1 All	OK
		10.750	0.00	0.00	0.00	U1 All	OK	5.72	222.67	206.72	U1 All	OK
		13.000	0.00	0.00	0.00	U1 All	OK	5.72	222.67	219.68	U1 All	OK
		15.000	0.00	0.00	0.00	U1 All	OK	5.72	222.67	210.54	U1 All	OK
		19.250	0.00	0.00	0.00	U1 All	OK	5.72	222.67	126.57	U1 All	OK
		19.817	0.00	0.00	0.00	U1 All	OK	5.72	222.67	108.71	U1 All	OK
		21.914	4.84	-189.16	0.00	U1 All	OK	5.72	222.67	29.13	U1 All	OK
		23.500	4.84	-189.16	-60.24	U1 All	OK	5.72	222.67	0.00	U1 All	OK
		25.000	7.99	-307.67	-166.19	U1 All	OK	5.72	222.67	0.00	U1 All	OK
		25.000	7.99	-454.84	-166.23	U1 All	OK	5.72	222.67	0.00	U1 All	OK
		25.598	9.24	-523.14	-211.55	U1 All	OK	5.72	222.67	0.00	U1 All	OK
		29.167	9.24	-523.14	-517.57	U1 All	OK	5.72	222.67	0.00	U1 All	OK
		29.375	9.24	-523.14	-537.19	U1 All	---	5.72	222.67	0.00	U1 All	---
		30.000	9.24	-523.14	-597.13	U1 All	---	5.72	222.67	0.00	U1 All	---
	Middle	0.000	4.40	-172.31	2.45	U1 All	---	4.40	172.31	0.00	U1 All	---
		0.833	4.40	-172.31	0.00	U1 All	OK	4.40	172.31	0.00	U1 All	OK
		2.059	4.40	-172.31	-1.37	U1 All	OK	4.40	172.31	0.00	U1 All	OK
		6.067	4.40	-172.31	0.00	U1 All	OK	4.40	172.31	67.21	U1 All	OK
		7.067	0.00	0.00	0.00	U1 All	OK	4.40	172.31	88.25	U1 All	OK
		10.750	0.00	0.00	0.00	U1 All	OK	4.40	172.31	137.81	U1 All	OK
		13.000	0.00	0.00	0.00	U1 All	OK	4.40	172.31	146.45	U1 All	OK
		15.000	0.00	0.00	0.00	U1 All	OK	4.40	172.31	140.36	U1 All	OK
		19.250	0.00	0.00	0.00	U1 All	OK	4.40	172.31	84.38	U1 All	OK
		20.729	0.00	0.00	0.00	U1 All	OK	4.40	172.31	51.16	U1 All	OK
		22.660	4.84	-189.16	-1.38	U1 All	OK	4.40	172.31	0.00	U1 All	OK
		23.702	4.84	-189.16	-18.69	U1 All	OK	4.40	172.31	0.00	U1 All	OK
		25.500	4.84	-189.16	-56.75	U1 All	OK	3.52	138.39	0.00	U1 All	OK
		29.167	4.84	-189.16	-172.52	U1 All	OK	3.52	138.39	0.00	U1 All	OK
		29.583	4.84	-189.16	-189.32	U1 All	---	3.52	138.39	0.00	U1 All	---
		30.000	4.84	-189.16	-206.93	U1 All	---	3.52	138.39	0.00	U1 All	---
3	Column	0.000	9.24	-523.14	-539.24	U1 All	---	4.40	172.31	0.00	U1 All	---
		0.833	9.24	-523.14	-463.59	U1 All	OK	4.40	172.31	0.00	U1 All	OK
		4.630	9.24	-523.14	-176.57	U1 All	OK	4.40	172.31	0.00	U1 All	OK
		5.000	8.37	-475.78	-153.60	U1 All	OK	4.40	172.31	0.00	U1 All	OK
		5.000	8.37	-321.84	-153.56	U1 All	OK	4.40	172.31	0.00	U1 All	OK

		Top							Bottom					
Span	**Strip**	**x**	**$A_{s,top}$**	**ΦM_n-**	**M_u-**	**Comb**	**Pat**	**Status**	**$A_{s,bot}$**	**ΦM_n+**	**M_u+**	**Comb**	**Pat**	**Status**
		ft	in^2	k-ft	k-ft				in^2	k-ft	k-ft			
		6.500	4.84	-189.16	-69.29	U1	All	OK	4.40	172.31	0.00	U1	All	OK
		8.313	4.84	-189.16	0.00	U1	All	OK	4.40	172.31	11.45	U1	All	OK
		10.183	0.00	0.00	0.00	U1	All	OK	4.40	172.31	63.73	U1	All	OK
		10.750	0.00	0.00	0.00	U1	All	OK	4.40	172.31	76.24	U1	All	OK
		15.000	0.00	0.00	0.00	U1	All	OK	4.40	172.31	120.14	U1	All	OK
		19.250	0.00	0.00	0.00	U1	All	OK	4.40	172.31	76.24	U1	All	OK
		19.817	0.00	0.00	0.00	U1	All	OK	4.40	172.31	63.73	U1	All	OK
		21.687	4.84	-189.16	0.00	U1	All	OK	4.40	172.31	11.45	U1	All	OK
		23.500	4.84	-189.16	-69.29	U1	All	OK	4.40	172.31	0.00	U1	All	OK
		25.000	8.37	-321.84	-153.56	U1	All	OK	4.40	172.31	0.00	U1	All	OK
		25.000	8.37	-475.78	-153.60	U1	All	OK	4.40	172.31	0.00	U1	All	OK
		25.370	9.24	-523.14	-176.57	U1	All	OK	4.40	172.31	0.00	U1	All	OK
		29.167	9.24	-523.14	-463.59	U1	All	OK	4.40	172.31	0.00	U1	All	OK
		30.000	9.24	-523.14	-539.24	U1	All	---	4.40	172.31	0.00	U1	All	---
	Middle	0.000	4.84	-189.16	-179.75	U1	All	---	3.52	138.39	0.00	U1	All	---
		0.833	4.84	-189.16	-154.53	U1	All	OK	3.52	138.39	0.00	U1	All	OK
		4.500	4.84	-189.16	-61.59	U1	All	OK	3.52	138.39	0.00	U1	All	OK
		5.500	4.84	-189.16	-41.32	U1	All	OK	4.40	172.31	0.00	U1	All	OK
		8.045	4.84	-189.16	0.00	U1	All	OK	4.40	172.31	1.71	U1	All	OK
		9.771	0.00	0.00	0.00	U1	All	OK	4.40	172.31	35.79	U1	All	OK
		10.750	0.00	0.00	0.00	U1	All	OK	4.40	172.31	50.83	U1	All	OK
		15.000	0.00	0.00	0.00	U1	All	OK	4.40	172.31	80.09	U1	All	OK
		19.250	0.00	0.00	0.00	U1	All	OK	4.40	172.31	50.83	U1	All	OK
		20.229	0.00	0.00	0.00	U1	All	OK	4.40	172.31	35.79	U1	All	OK
		21.955	4.84	-189.16	0.00	U1	All	OK	4.40	172.31	1.71	U1	All	OK
		24.500	4.84	-189.16	-41.32	U1	All	OK	4.40	172.31	0.00	U1	All	OK
		25.500	4.84	-189.16	-61.59	U1	All	OK	3.52	138.39	0.00	U1	All	OK
		29.167	4.84	-189.16	-154.53	U1	All	OK	3.52	138.39	0.00	U1	All	OK
		30.000	4.84	-189.16	-179.75	U1	All	---	3.52	138.39	0.00	U1	All	---
4	Column	0.000	9.24	-523.14	-597.13	U1	All	---	5.72	222.67	0.00	U1	All	---
		0.625	9.24	-523.14	-537.19	U1	All	---	5.72	222.67	0.00	U1	All	---
		0.833	9.24	-523.14	-517.57	U1	All	OK	5.72	222.67	0.00	U1	All	OK
		4.402	9.24	-523.14	-211.55	U1	All	OK	5.72	222.67	0.00	U1	All	OK
		5.000	7.99	-454.84	-166.23	U1	All	OK	5.72	222.67	0.00	U1	All	OK
		5.000	7.99	-307.67	-166.19	U1	All	OK	5.72	222.67	0.00	U1	All	OK
		6.500	4.84	-189.16	-60.24	U1	All	OK	5.72	222.67	0.00	U1	All	OK
		8.086	4.84	-189.16	0.00	U1	All	OK	5.72	222.67	29.13	U1	All	OK
		10.183	0.00	0.00	0.00	U1	All	OK	5.72	222.67	108.71	U1	All	OK
		10.750	0.00	0.00	0.00	U1	All	OK	5.72	222.67	126.57	U1	All	OK
		15.000	0.00	0.00	0.00	U1	All	OK	5.72	222.67	210.54	U1	All	OK
		17.000	0.00	0.00	0.00	U1	All	OK	5.72	222.67	219.68	U1	All	OK
		19.250	0.00	0.00	0.00	U1	All	OK	5.72	222.67	206.72	U1	All	OK
		19.817	0.00	0.00	0.00	U1	All	OK	5.72	222.67	199.55	U1	All	OK
		21.854	4.40	-172.31	0.00	U1	All	OK	5.72	222.67	160.99	U1	All	OK
		25.000	4.40	-172.31	0.00	U1	All	OK	5.72	222.67	61.84	U1	All	OK
		25.000	4.40	-254.75	0.00	U1	All	OK	5.72	222.67	61.82	U1	All	OK
		29.167	4.40	-254.75	-244.81	U1	All	OK	5.72	222.67	0.00	U1	All	OK
		29.375	4.40	-254.75	-266.68	U1	All	---	5.72	222.67	0.00	U1	All	---
		30.000	4.40	-254.75	-335.03	U1	All	---	5.72	222.67	0.00	U1	All	---
	Middle	0.000	4.84	-189.16	-206.93	U1	All	---	3.52	138.39	0.00	U1	All	---

Span	Strip	Top						Bottom				
		x	$A_{s,top}$	ΦM_n-	M_u-	Comb Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb Pat	Status
		ft	in²	k-ft	k-ft			in²	k-ft	k-ft		
		0.417	4.84	-189.16	-189.32	U1 All	---	3.52	138.39	0.00	U1 All	---
		0.833	4.84	-189.16	-172.52	U1 All	OK	3.52	138.39	0.00	U1 All	OK
		4.500	4.84	-189.16	-56.75	U1 All	OK	3.52	138.39	0.00	U1 All	OK
		6.298	4.84	-189.16	-18.69	U1 All	OK	4.40	172.31	0.00	U1 All	OK
		7.340	4.84	-189.16	-1.38	U1 All	OK	4.40	172.31	0.00	U1 All	OK
		9.271	0.00	0.00	0.00	U1 All	OK	4.40	172.31	51.16	U1 All	OK
		10.750	0.00	0.00	0.00	U1 All	OK	4.40	172.31	84.38	U1 All	OK
		15.000	0.00	0.00	0.00	U1 All	OK	4.40	172.31	140.36	U1 All	OK
		17.000	0.00	0.00	0.00	U1 All	OK	4.40	172.31	146.45	U1 All	OK
		19.250	0.00	0.00	0.00	U1 All	OK	4.40	172.31	137.81	U1 All	OK
		22.933	0.00	0.00	0.00	U1 All	OK	4.40	172.31	88.25	U1 All	OK
		23.933	4.40	-172.31	0.00	U1 All	OK	4.40	172.31	67.21	U1 All	OK
		27.941	4.40	-172.31	-1.37	U1 All	OK	4.40	172.31	0.00	U1 All	OK
		29.167	4.40	-172.31	0.00	U1 All	OK	4.40	172.31	0.00	U1 All	OK
		30.000	4.40	-172.31	2.45	U1 All	---	4.40	172.31	0.00	U1 All	---
5	Column	0.000	4.40	-254.75	-3.03	U1 All	---	0.00	0.00	0.00	U1 All	---
		0.146	4.40	-254.75	-2.10	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.386	4.40	-256.46	-0.90	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.417	4.40	-256.46	-0.76	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.593	4.40	-172.31	-0.28	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.833	4.40	-172.31	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	Middle	0.000	4.40	-172.31	0.00	U1 All	---	0.00	0.00	0.00	U1 All	---
		0.146	4.40	-172.31	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.386	4.40	-172.31	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.417	4.40	-172.31	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.593	4.40	-172.31	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.833	4.40	-172.31	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK

2.9. Slab Shear Capacity

Span	b	d	V_{ratio}	ΦV_c	V_u	X_u
	in	in		kip	kip	ft
1	360.00	8.88	1.000	338.88	7.28	0.00
	360.00	10.29	1.000	392.97	7.28	0.00
2	360.00	10.29	1.000	392.97	95.23	1.57
	360.00	8.88	1.000	338.88	96.72	25.00
	360.00	10.29	1.000	392.97	126.66	28.43
3	360.00	10.29	1.000	392.97	110.94	1.57
	360.00	8.88	1.000	338.88	81.00	25.00
	360.00	10.29	1.000	392.97	110.94	28.43
4	360.00	10.29	1.000	392.97	126.66	1.57
	360.00	8.88	1.000	338.88	96.72	5.00
	360.00	10.29	1.000	392.97	95.23	28.43
5	360.00	10.29	1.000	392.97	0.00	0.83
	360.00	8.88	1.000	338.88	0.00	0.83

2.10. Flexural Transfer of Negative Unbalanced Moment at Supports

Support	Width	Width-c	d	M_{unb}	Comb	Patt	γ_f	$A_{s,req}$	$A_{s,prov}$	Add Bars
	in	in	in	k-ft				in²	in²	
1	62.75	62.75	13.13	329.55	U1	All	0.626	3.605	1.534	5-#6
2	62.75	62.75	13.13	85.07	U1	All	0.600	0.871	3.221	---
3	62.75	62.75	13.13	85.07	U1	All	0.600	0.871	3.221	---
4	62.75	62.75	13.13	329.55	U1	All	0.626	3.605	1.534	5-#6

2.11. Punching Shear Around Columns

2.11.1. Critical Section Properties

Support	Type	b_1	b_2	b_0	d_{avg}	CG	$c_{(left)}$	$c_{(right)}$	A_c	J_c
		in	in	in	in	in	in	in	in²	in⁴
1	Rect	26.56	33.13	86.25	13.13	8.38	18.38	8.18	1132	98239
2	Rect	33.13	33.13	132.50	13.13	0.00	16.56	16.56	1739.1	3.3052e+005
3	Rect	33.13	33.13	132.50	13.13	0.00	16.56	16.56	1739.1	3.3052e+005
4	Rect	26.56	33.13	86.25	13.13	-8.38	8.18	18.38	1132	98239

2.11.2. Punching Shear Results

Support	V_u	v_u	M_{unb}	Comb	Patt	γ_v	v_u	ΦV_c
	kip	psi	k-ft				psi	psi
1	114.58	101.2	249.52	U1	All	0.374	194.4	212.1
2	262.99	151.2	-85.07	U1	All	0.400	171.7	212.1
3	262.99	151.2	85.07	U1	All	0.400	171.7	212.1
4	114.58	101.2	-249.52	U1	All	0.374	194.4	212.1

2.12. Punching Shear Around Drops

2.12.1. Critical Section Properties

Support	Type	b_1	b_2	b_0	d_{avg}	CG	$c_{(left)}$	$c_{(right)}$	A_c	J_c
		in	in	in	in	in	in	in	in²	in⁴
1	Rect	74.44	128.88	277.75	8.88	44.49	54.49	19.95	2465	1.468e+006
2	Rect	128.88	128.88	515.50	8.88	0.00	64.44	64.44	4575.1	1.2679e+007
3	Rect	128.88	128.88	515.50	8.88	0.00	64.44	64.44	4575.1	1.2679e+007
4	Rect	74.44	128.88	277.75	8.88	-44.49	19.95	54.49	2465	1.468e+006

2.12.2. Punching Shear Results

Support	V_u	Comb	Pat	v_u	ΦV_c
	kip			psi	psi
1	98.24	U1	All	39.9	156.9
2	233.91	U1	All	51.1	142.6
3	233.91	U1	All	51.1	142.6
4	98.24	U1	All	39.9	156.9

2.13. Material TakeOff

2.13.1. Reinforcement in the Direction of Analysis

Top Bars	2261.0 lb	<=>	24.67 lb/ft	<=>	0.822 lb/ft²
Bottom Bars	2919.9 lb	<=>	31.85 lb/ft	<=>	1.062 lb/ft²
Stirrups	0.0 lb	<=>	0.00 lb/ft	<=>	0.000 lb/ft²
Total Steel	5180.9 lb	<=>	56.52 lb/ft	<=>	1.884 lb/ft²

Concrete	2403.8	ft^3	<=>	26.22 ft^3/ft	<=>	0.874 ft^3/ft^2	

3. Deflection Results: Summary

3.1. Section Properties

3.1.1. Frame Section Properties

Notes:
M+ve values are for positive moments (tension at bottom face).
M-ve values are for negative moments (tension at top face).

Span	Zone	M_{+ve} I_g in⁴	M_{+ve} I_{cr} in⁴	M_{+ve} M_{cr} k-ft	M_{-ve} I_g in⁴	M_{-ve} I_{cr} in⁴	M_{-ve} M_{cr} k-ft
1	Left	30000	0	265.17	30000	3641	-265.17
	Midspan	30000	0	265.17	30000	3641	-265.17
	Right	53445	0	282.33	53445	7174	-401.42
2	Left	53445	3164	282.33	53445	7174	-401.42
	Midspan	30000	3800	265.17	30000	0	-265.17
	Right	53445	3164	282.33	53445	10477	-401.42
3	Left	53445	2799	282.33	53445	10477	-401.42
	Midspan	30000	3319	265.17	30000	0	-265.17
	Right	53445	2799	282.33	53445	10477	-401.42
4	Left	53445	3164	282.33	53445	10477	-401.42
	Midspan	30000	3800	265.17	30000	0	-265.17
	Right	53445	3164	282.33	53445	7174	-401.42
5	Left	53445	0	282.33	53445	7174	-401.42
	Midspan	30000	0	265.17	30000	3641	-265.17
	Right	30000	0	265.17	30000	3641	-265.17

3.1.2. Frame Effective Section Properties

Span	Zone	Weight	Dead M_{max} k-ft	Dead I_e in⁴	Sustained M_{max} k-ft	Sustained I_e in⁴	Dead+Live M_{max} k-ft	Dead+Live I_e in⁴
1	Right	1.000	-1.69	53445	-1.69	53445	-2.32	53445
	Span Avg	----	----	53445	----	53445	----	53445
2	Middle	0.500	197.23	30000	197.23	30000	278.14	26502
	Right	0.500	-434.41	44379	-434.41	44379	-611.14	22653
	Span Avg	----	----	37189	----	37189	----	24578
3	Left	0.250	-388.46	53445	-388.46	53445	-546.48	27506
	Middle	0.500	107.56	30000	107.56	30000	152.03	30000
	Right	0.250	-388.46	53445	-388.46	53445	-546.48	27506
	Span Avg	----	----	41723	----	41723	----	28753
4	Left	0.500	-434.41	44379	-434.41	44379	-611.14	22653
	Middle	0.500	197.23	30000	197.23	30000	278.14	26502
	Span Avg	----	----	37189	----	37189	----	24578
5	Left	1.000	-1.69	53445	-1.69	53445	-2.32	53445
	Span Avg	----	----	53445	----	53445	----	53445

3.1.3. Strip Section Properties at Midspan

Notes:
Load distirubtion factor, LDL, averages moment distribution factors listed in Design Results.
Ratio refers to proportion of strip to frame deflections under fix-end condtions.

Span	Column Strip			Middle Strip		
	I_g in^4	LDF	Ratio	I_g in^4	LDF	Ratio
1	15000	0.800	1.600	15000	0.200	0.400
2	15000	0.738	1.475	15000	0.262	0.525
3	15000	0.675	1.350	15000	0.325	0.650
4	15000	0.738	1.475	15000	0.262	0.525
5	15000	0.800	1.600	15000	0.200	0.400

3.2. Instantaneous Deflections

3.2.1. Extreme Instantaneous Frame Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live Sustained	Live Unsustained	Live Total	Total Sustained	Total Dead+Live
1	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.012	---	-0.008	-0.008	-0.012	-0.021
		Loc	ft	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	in	0.163	---	0.143	0.143	0.163	0.306
		Loc	ft	13.750	---	14.000	14.000	13.750	13.750
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
3	Down	Def	in	0.060	---	0.068	0.068	0.060	0.128
		Loc	ft	15.000	---	15.000	15.000	15.000	15.000
	Up	Def	in	-0.002	---	-0.001	-0.001	-0.002	-0.003
		Loc	ft	1.324	---	1.078	1.078	1.324	1.078
4	Down	Def	in	0.163	---	0.143	0.143	0.163	0.306
		Loc	ft	16.250	---	16.000	16.000	16.250	16.250
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
5	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.012	---	-0.008	-0.008	-0.012	-0.021
		Loc	ft	0.833	---	0.833	0.833	0.833	0.833

3.2.2. Extreme Instantaneous Column Strip Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live Sustained	Live Unsustained	Live Total	Total Sustained	Total Dead+Live
1	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.012	---	-0.008	-0.008	-0.012	-0.021
		Loc	ft	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	in	0.207	---	0.188	0.188	0.207	0.395
		Loc	ft	14.000	---	14.250	14.250	14.000	14.000
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
3	Down	Def	in	0.089	---	0.096	0.096	0.089	0.185
		Loc	ft	15.000	---	15.000	15.000	15.000	15.000

					Live			Total	
Span	Direction	Value	Units	Dead	Sustained	Unsustained	Total	Sustained	Dead+Live
	Up	Def	in	-0.002	---	-0.001	-0.001	-0.002	-0.002
		Loc	ft	1.078	---	0.833	0.833	1.078	1.078
4	Down	Def	in	0.207	---	0.188	0.188	0.207	0.395
		Loc	ft	16.000	---	15.750	15.750	16.000	16.000
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
5	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.012	---	-0.008	-0.008	-0.012	-0.021
		Loc	ft	0.833	---	0.833	0.833	0.833	0.833

3.2.3. Extreme Instantaneous Middle Strip Deflections and Corresponding Locations

					Live			Total	
Span	Direction	Value	Units	Dead	Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.012	---	-0.008	-0.008	-0.012	-0.021
		Loc	ft	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	in	0.120	---	0.098	0.098	0.120	0.218
		Loc	ft	13.000	---	13.250	13.250	13.000	13.250
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
3	Down	Def	in	0.030	---	0.040	0.040	0.030	0.071
		Loc	ft	15.000	---	15.000	15.000	15.000	15.000
	Up	Def	in	-0.003	---	-0.001	-0.001	-0.003	-0.004
		Loc	ft	1.814	---	1.324	1.324	1.814	1.569
4	Down	Def	in	0.120	---	0.098	0.098	0.120	0.218
		Loc	ft	17.000	---	16.750	16.750	17.000	16.750
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
5	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.012	---	-0.008	-0.008	-0.012	-0.021
		Loc	ft	0.833	---	0.833	0.833	0.833	0.833

3.3. Long-term Deflections

3.3.1. Long-term Column Strip Deflection Factors

Notes:
Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

		M_{+ve}					M_{-ve}				
Span	Zone	$A_{s,top}$	b	d	Rho'	Lambda	$A_{s,bot}$	b	d	Rho'	Lambda
		in^2	in	in	%		in^2	in	in	%	
1	Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

3.3.2. Long-term Middle Strip Deflection Factors

Notes:
Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

		M_{+ve}					M_{-ve}				
Span	Zone	$A_{s,top}$	b	d	Rho'	Lambda	$A_{s,bot}$	b	d	Rho'	Lambda
		in^2	in	in	%		in^2	in	in	%	
1	Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

3.3.3. Extreme Long-term Column Strip Deflections and Corresponding Locations

Notes:
Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
1	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Up	Def	in	-0.025	-0.033	-0.033	-0.045
		Loc	ft	0.000	0.000	0.000	0.000
2	Down	Def	in	0.414	0.601	0.601	0.808
		Loc	ft	14.000	14.000	14.000	14.000
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
3	Down	Def	in	0.178	0.274	0.274	0.363
		Loc	ft	15.000	15.000	15.000	15.000
	Up	Def	in	-0.003	-0.004	-0.004	-0.006
		Loc	ft	1.078	1.078	1.078	1.078
4	Down	Def	in	0.414	0.601	0.601	0.808
		Loc	ft	16.000	16.000	16.000	16.000
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
5	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Up	Def	in	-0.025	-0.033	-0.033	-0.045
		Loc	ft	0.833	0.833	0.833	0.833

3.3.4. Extreme Long-term Middle Strip Deflections and Corresponding Locations

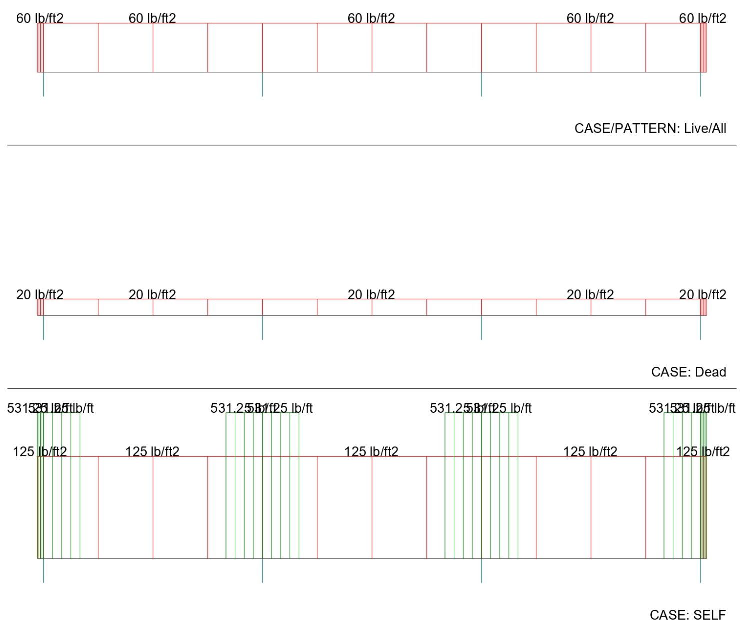
Notes:
Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.

Total deflections consist of dead, live, and creep and shrinkage deflections.

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
1	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Up	Def	in	-0.025	-0.033	-0.033	-0.045
		Loc	ft	0.000	0.000	0.000	0.000
2	Down	Def	in	0.241	0.339	0.339	0.459
		Loc	ft	13.000	13.000	13.000	13.000
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
3	Down	Def	in	0.060	0.101	0.101	0.131
		Loc	ft	15.000	15.000	15.000	15.000
	Up	Def	in	-0.006	-0.007	-0.007	-0.010
		Loc	ft	1.814	1.814	1.814	1.814
4	Down	Def	in	0.241	0.339	0.339	0.459
		Loc	ft	17.000	17.000	17.000	17.000
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
5	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Up	Def	in	-0.025	-0.033	-0.033	-0.045
		Loc	ft	0.833	0.833	0.833	0.833

4. Diagrams

4.1. Loads



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File: C:\StructurePoint\spSlab\Two-Way Flat Slab with Drop Panel ACI 318-14.slb

Project: Two-Way Flat Slab with Drop Panels ACI 318-14

Frame: Interior Frame

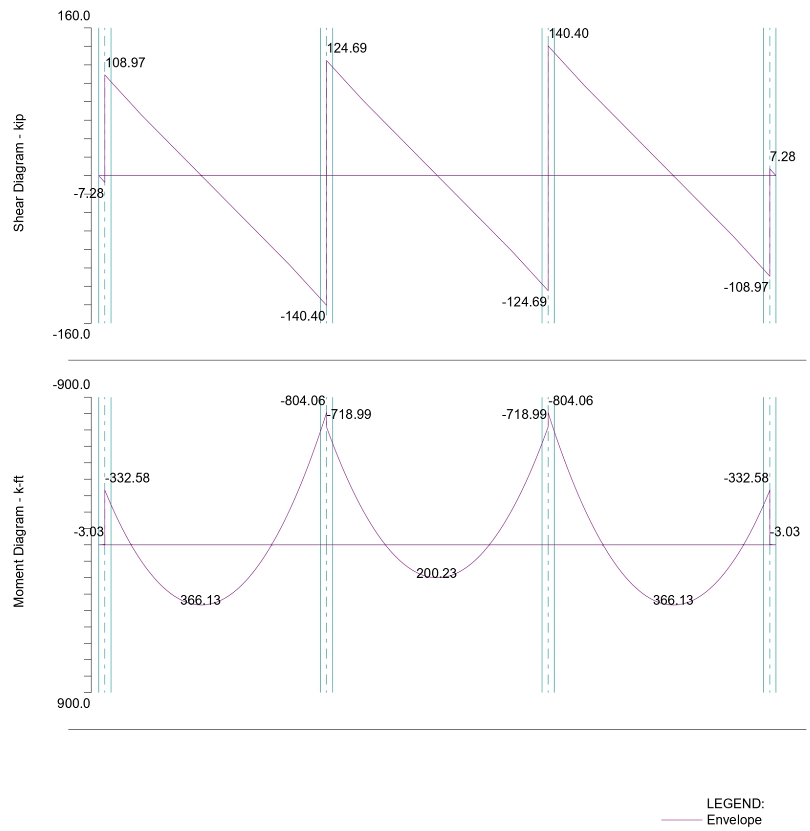
Engineer: SP

Code: ACI 318-14

Date: 05/04/23

Time: 10:06:38

4.2. Internal Forces



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Frame: Interior Frame

Engineer: SP

Code: ACI 318-14

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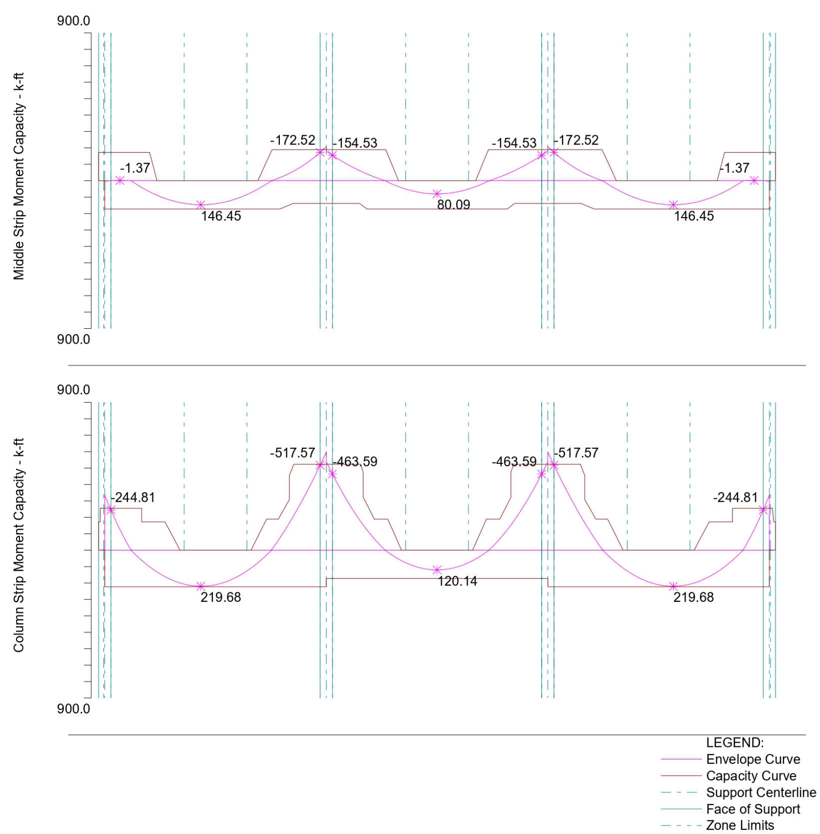
4.3. Moment Capacity



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Project: Two-Way Flat Slab with Drop Panels ACI 318-14

Frame: Interior Frame

Engineer: SP

Code: ACI 318-14

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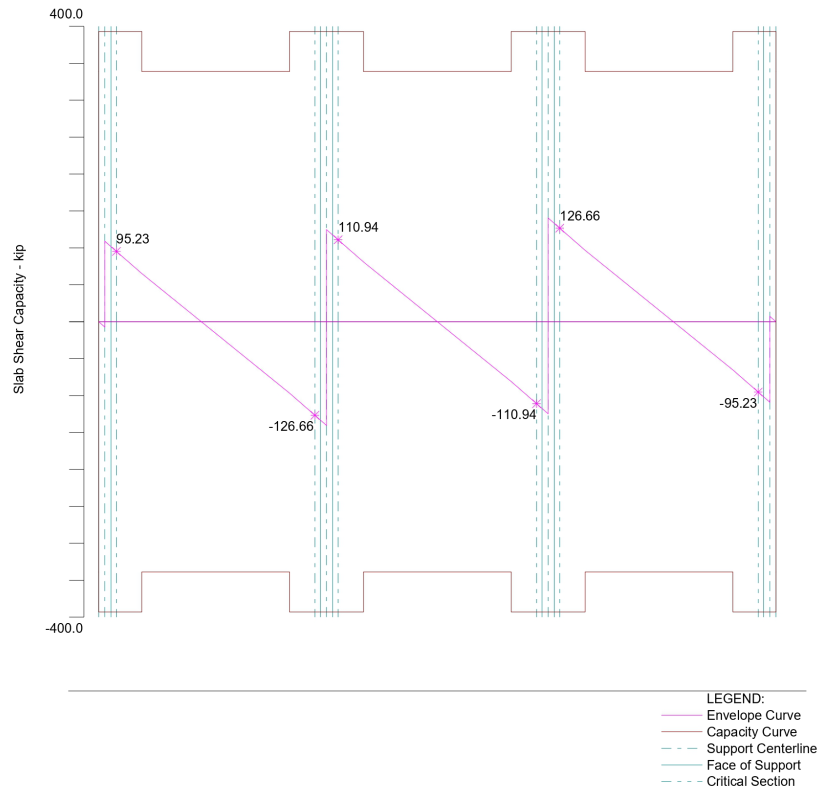
4.4. Shear Capacity



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Engineer: SP
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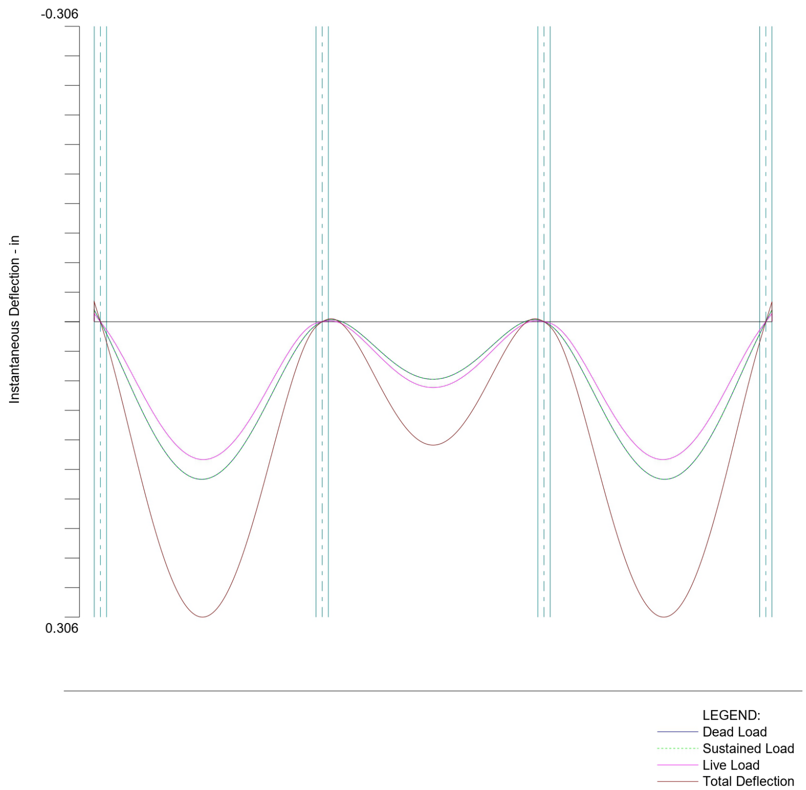
4.5. Deflection



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Project: Two-Way Flat Slab with Drop Panels ACI 318-14
Frame: Interior Frame
Engineer: SP
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4.6. Reinforcement

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Project: Two-Way Flat Slab with Drop Panels ACI 318-14

Frame: Interior Frame

Engineer: SP

Code: ACI 318-14

Date: 05/04/23

Time: 10:06:38

7. Summary and Comparison of Design Results

Table 9 - Comparison of Moments obtained from Hand (EFM) and spSlab Solution (ft-kips)

		Hand (EFM)	spSlab
Exterior Span			
Column Strip	Exterior Negative*	244.05	244.81
	Positive	219.08	219.68
	Interior Negative*	520.07	517.57
Middle Strip	Exterior Negative*	0.00	0.00
	Positive	146.05	146.45
	Interior Negative*	173.36	172.52
Interior Span			
Column Strip	Interior Negative*	465.73	463.59
	Positive	118.42	120.14
Middle Strip	Interior Negative*	155.24	154.53
	Positive	78.95	80.09

* Negative moments are taken at the faces of supports

Table 10 - Comparison of Reinforcement Results

Span Location		Reinforcement Provided for Flexure		Additional Reinforcement Provided for Unbalanced Moment Transfer*		Total Reinforcement Provided	
		Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior Span							
Column Strip	Exterior Negative	10-#6	10-#6	5-#6	5-#6	15-#6	15-#6
	Positive	13-#6	13-#6	n/a	n/a	13-#6	13-#6
	Interior Negative	21-#6	21-#6	---	---	21-#6	21-#6
Middle Strip	Exterior Negative	10-#6	10-#6	n/a	n/a	10-#6	10-#6
	Positive	10-#6	10-#6	n/a	n/a	10-#6	10-#6
	Interior Negative	11-#6	11-#6	n/a	n/a	11-#6	11-#6
Interior Span							
Column Strip	Positive	10-#6	10-#6	n/a	n/a	10-#6	10-#6
Middle Strip	Positive	10-#6	10-#6	n/a	n/a	10-#6	10-#6

Table 11 - Comparison of One-Way (Beam Action) Shear Check Results								
Span	V_u @ d (kips)		V_u @ drop panel (kips)		ϕV_c @ d (kips)		ϕV_c @ drop panel (kips)	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	125.77	126.66	96.86	96.72	392.97	392.97	338.88	338.88
Interior	109.91	110.94	81.00	81.00	392.97	392.97	338.88	338.88

Table 12 - Comparison of Two-Way (Punching) Shear Check Results (around Columns Faces)										
Support	b_1 (in.)		b_2 (in.)		b_o (in.)		V_u (kips)		c_{AB} (in.)	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	26.56	26.56	33.13	33.13	86.26	86.25	106.79	114.58	8.18	8.18
Interior	33.13	33.13	33.13	33.13	132.50	132.50	262.70	262.99	16.56	16.56
Corner	26.56	26.56	26.56	26.56	53.13	53.12	60.29	60.60	6.64	6.64
Support	J_c (in.^{4})		γ_v		M_{unb} (ft-kips)		v_u (psi)		ϕv_c (psi)	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	98,243	98,239	0.374	0.374	257.12	249.52	190.38	194.40	212.13	212.10
Interior	330,518	330,520	0.400	0.400	85.67	85.07	171.66	171.70	212.13	212.10
Corner	56,251	56,249	0.400	0.400	137.66	137.40	164.48	164.80	212.13	212.10

Table 13 - Comparison of Two-Way (Punching) Shear Check Results (around Drop Panels)										
Support	**b_1 (in.)**		**b_2 (in.)**		**b_o (in.)**		**V_u (kips)**		**c_{AB} (in.)**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	74.44	74.44	128.88	128.88	277.75	277.75	90.84	98.24	19.95	19.95
Interior	128.88	128.88	128.88	128.88	515.50	515.50	234.10	233.91	64.44	64.44
Corner	74.44	74.44	74.44	74.44	148.88	148.87	51.54	51.53	18.61	18.61
Support	**J_c (in.^{4})**		**γ_v**		**M_{unb} (ft-kips)**		**v_u (psi)**		**ϕv_c (psi)**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	1,467,996	1,468,000	N.A.	N.A.	N.A.	N.A.	36.85	39.90	156.90	156.90
Interior	12,679,372	12,679,000	N.A.	N.A.	N.A.	N.A.	51.17	51.10	142.59	142.60
Corner	766,946	766,930	N.A.	N.A.	N.A.	N.A.	39.01	39.00	169.30	169.30

Note: Shear stresses from spSlab are higher than hand calculations since it considers the load effects beyond the column centerline known in the model as right/left cantilevers. This small increase is often neglected in simplified hand calculations like the one used here.

Table 14 - Comparison of Immediate Deflection Results (in.)								
Column Strip								
Span	**D**		**D+LL$_{sus}$**		**D+LL$_{full}$**		**LL**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	0.207	0.207	0.207	0.207	0.405	0.395	0.198	0.188
Interior	0.097	0.089	0.097	0.089	0.200	0.185	0.103	0.096
Middle Strip								
Span	**D**		**D+LL$_{sus}$**		**D+LL$_{full}$**		**LL**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	0.112	0.120	0.112	0.120	0.227	0.218	0.114	0.098
Interior	0.035	0.030	0.035	0.030	0.073	0.071	0.037	0.040

Table 15 - Comparison of Time-Dependent Deflection Results						
Column Strip						
Span	**λ_Δ**		**Δ_{cs} (in.)**		**Δ_{total} (in.)**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	2.0	2.0	0.414	0.414	0.820	0.808
Interior	2.0	2.0	0.196	0.178	0.396	0.363
Middle Strip						
Span	**λ_Δ**		**Δ_{cs} (in.)**		**Δ_{total} (in.)**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	2.0	2.0	0.227	0.241	0.452	0.459
Interior	2.0	2.0	0.073	0.060	0.145	0.131

In all of the hand calculations illustrated above, the results are in close or exact agreement with the automated analysis and design results obtained from the spSlab model.

8. Conclusions & Observations

8.1. One-Way Shear Distribution to Slab Strips

In one-way shear checks above, shear is distributed uniformly along the width of the design strip (30 ft.). StructurePoint finds it necessary sometimes to allocate the one-way shears with the same proportion moments are distributed to column and middle strips.

spSlab allows the one-way shear check using two approaches: 1) calculating the one-way shear capacity using the average slab thickness and comparing it with the total factored one-shear load as shown in the hand calculations above; 2) distributing the factored one-way shear forces to the column and middle strips and comparing it with the shear capacity of each strip as illustrated in the following figures. An engineering judgment is needed to decide which approach to be used.

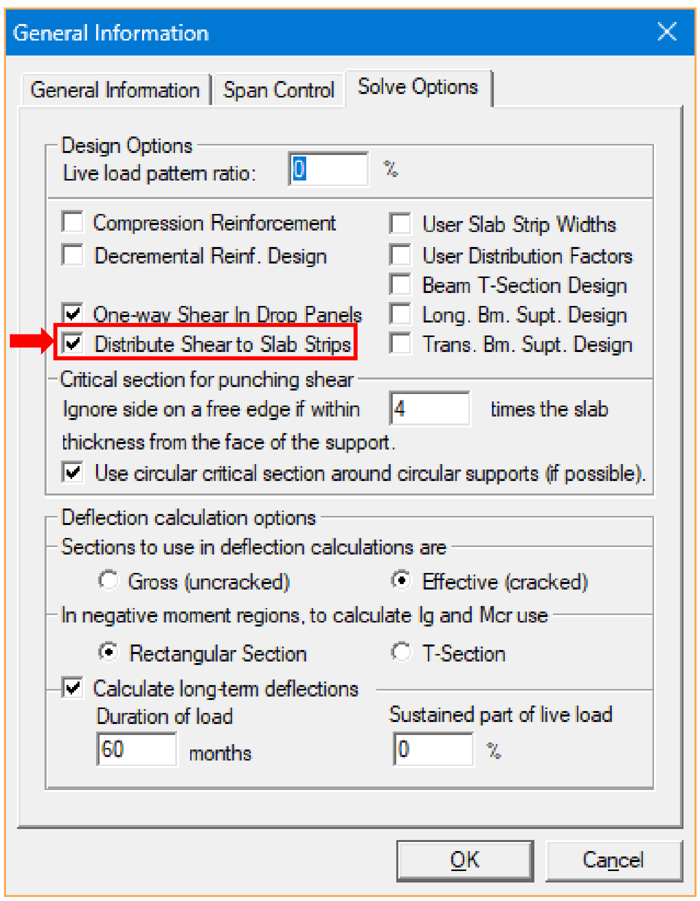
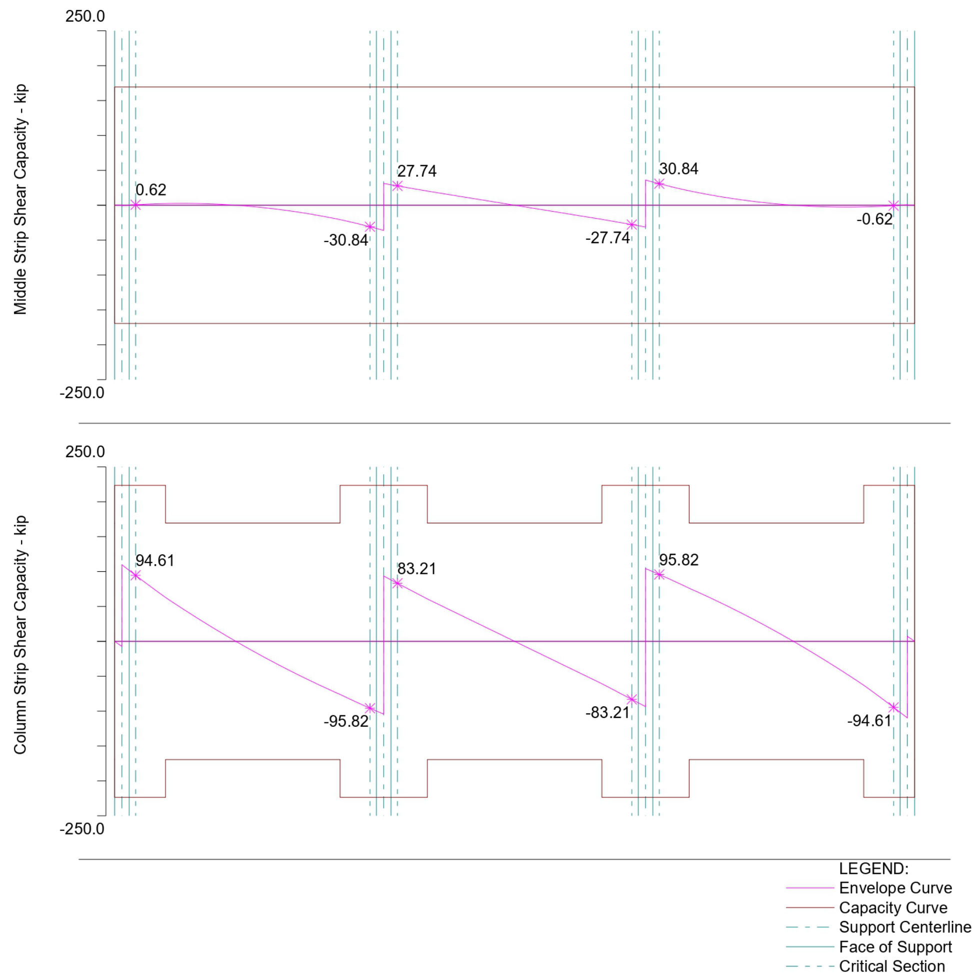


Figure 29 – Distributing Shear to Column and Middle Strips (spSlab Input)



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Project: Two-Way Flat Slab with Drop Panels ACI 318-14

Frame: Interior Frame

Engineer: SP

Code: ACI 318-14

Date: 05/09/23

Time: 10:19:07

Figure 30 – Distributed Column and Middle Strip Shear Force Diagram (spSlab Output)

1.1. Slab Shear Capacity

Span	Strip	b	d	V_{ratio}	ΦV_c	V_u	X_u
		in	in		kip	kip	ft
1	Column	180.00	8.88	1.000	169.44	7.28	0.00
		180.00	11.71	1.000	223.53	7.28	0.00
	Middle	180.00	8.88	0.000	169.44	0.00	0.00
		180.00	8.88	0.000	169.44	0.00	0.00
2	Column	180.00	11.71	0.993	223.53	94.61	1.57
		180.00	8.88	0.787	169.44	76.09	25.00
		180.00	11.71	0.757	223.53	95.82	28.43
	Middle	180.00	8.88	0.037	169.44	2.40	5.00
		180.00	8.88	0.213	169.44	20.62	25.00
		180.00	8.88	0.243	169.44	30.84	28.43
3	Column	180.00	11.71	0.750	223.53	83.21	1.57
		180.00	8.88	0.750	169.44	60.75	25.00
		180.00	11.71	0.750	223.53	83.21	28.43
	Middle	180.00	8.88	0.250	169.44	27.74	1.57
		180.00	8.88	0.250	169.44	20.25	25.00
		180.00	8.88	0.250	169.44	27.74	28.43
4	Column	180.00	11.71	0.757	223.53	95.82	1.57
		180.00	8.88	0.787	169.44	76.09	5.00
		180.00	11.71	0.993	223.53	94.61	28.43
	Middle	180.00	8.88	0.243	169.44	30.84	1.57
		180.00	8.88	0.213	169.44	20.62	5.00
		180.00	8.88	0.037	169.44	2.40	25.00
5	Column	180.00	11.71	1.000	223.53	0.00	0.83
		180.00	8.88	1.000	169.44	0.00	0.83
	Middle	180.00	8.88	0.000	169.44	0.00	0.00
		180.00	8.88	0.000	169.44	0.00	0.00

Figure 31 – Tabulated Shear Force & Capacity at Critical Sections (spSlab Output)

8.2. Two-Way Concrete Slab Analysis Methods

A slab system can be analyzed and designed by any procedure satisfying equilibrium and geometric compatibility. Three established methods are widely used. The requirements for two of them are described in detail in ***ACI 318-14 Chapter 8 (8.2.1)***.

Direct Design Method (DDM) is an approximate method and is applicable to two-way slab concrete floor systems that meet the stringent requirements of ***ACI 318-14 (8.10.2)***. In many projects, however, these requirements limit the usability of the Direct Design Method significantly.

The Equivalent Frame Method (EFM) does not have the limitations of Direct Design Method. It requires more accurate analysis methods that, depending on the size and geometry can prove to be long, tedious, and time-consuming.

StucturePoint's spSlab software program solution utilizes the Equivalent Frame Method to automate the process providing considerable time-savings in the analysis and design of two-way slab systems as compared to hand solutions using DDM or EFM.

Finite Element Method (FEM) is another method for analyzing reinforced concrete slabs, particularly useful for irregular slab systems with variable thicknesses, openings, and other features not permissible in DDM or EFM. Many reputable commercial FEM analysis software packages are available on the market today such as spMats. Using FEM requires critical understanding of the relationship between the actual behavior of the structure and the numerical simulation since this method is an approximate numerical method. The method is based on several assumptions and the operator has a great deal of decisions to make while setting up the model and applying loads and boundary conditions. The results obtained from FEM models should be verified to confirm their suitability for design and detailing of concrete structures.

The following table shows a general comparison between the DDM, EFM and FEM. This table covers general limitations, drawbacks, advantages, and cost-time efficiency of each method where it helps the engineer in deciding which method to use based on the project complexity, schedule, and budget.

Applicable ACI 318-14 Provision	Limitations/Applicability	Concrete Slab Analysis Method		
		DDM (Hand)	EFM (Hand//spSlab)	FEM (spMats)
8.10.2.1	Minimum of three continuous spans in each direction	☑		
8.10.2.2	Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span	☑		
8.10.2.3	Panels shall be rectangular, with ratio of longer to shorter panel dimensions, measured center-to-center supports, not exceed 2.	☑	☑	
8.10.2.4	Column offset shall not exceed 10% of the span in direction of offset from either axis between centerlines of successive columns	☑		
8.10.2.5	All loads shall be due to gravity only	☑		
8.10.2.5	All loads shall be uniformly distributed over an entire panel (q_u)	☑		
8.10.2.6	Unfactored live load shall not exceed two times the unfactored dead load	☑		
8.10.2.7	For a panel with beams between supports on all sides, slab-to-beam stiffness ratio shall be satisfied for beams in the two perpendicular directions.	☑		
8.7.4.2	Structural integrity steel detailing	☑	☑	☑
8.5.4	Openings in slab systems	☑	☑	☑
8.2.2	Concentrated loads	Not permitted	☑	☑
8.11.1.2	Live load arrangement (Load Patterning)	Not required	Required	Engineering judgment required based on modeling technique
R8.10.4.5*	Reinforcement for unbalanced slab moment transfer to column (M_{sc})	Moments @ support face	Moments @ support centerline	Engineering judgment required based on modeling technique
Irregularities (i.e. variable thickness, non-prismatic, partial bands, mixed systems, support arrangement, etc.)		Not permitted	Engineering judgment required	Engineering judgment required
Complexity		Low	Average	Complex to very complex
Design time/costs		Fast	Limited	Unpredictable/Costly
Design Economy		Conservative (see detailed comparison with spSlab output)	Somewhat conservative	Unknown - highly dependent on modeling assumptions: 1. Linear vs. non-linear 2. Isotropic vs non-isotropic 3. Plate element choice 4. Mesh size and aspect ratio 5. Design & detailing features
General (Drawbacks)		Very limited applications	Limited geometry	Limited guidance non-standard application (user dependent). Required significant engineering judgment
General (Advantages)		Very limited analysis is required	Detailed analysis is required or via software (e.g. spSlab)	Unlimited applicability to handle complex situations permissible by the features of the software used (e.g. spMats)

* The unbalanced slab moment transferred to the column M_{sc} (M_{unb}) is the difference in slab moment on either side of a column at a specific joint. In DDM only moments at the face of the support are calculated and are also used to obtain M_{sc} (M_{unb}). In EFM where a frame analysis is used, moments at the column center line are used to obtain M_{sc} (M_{unb}).