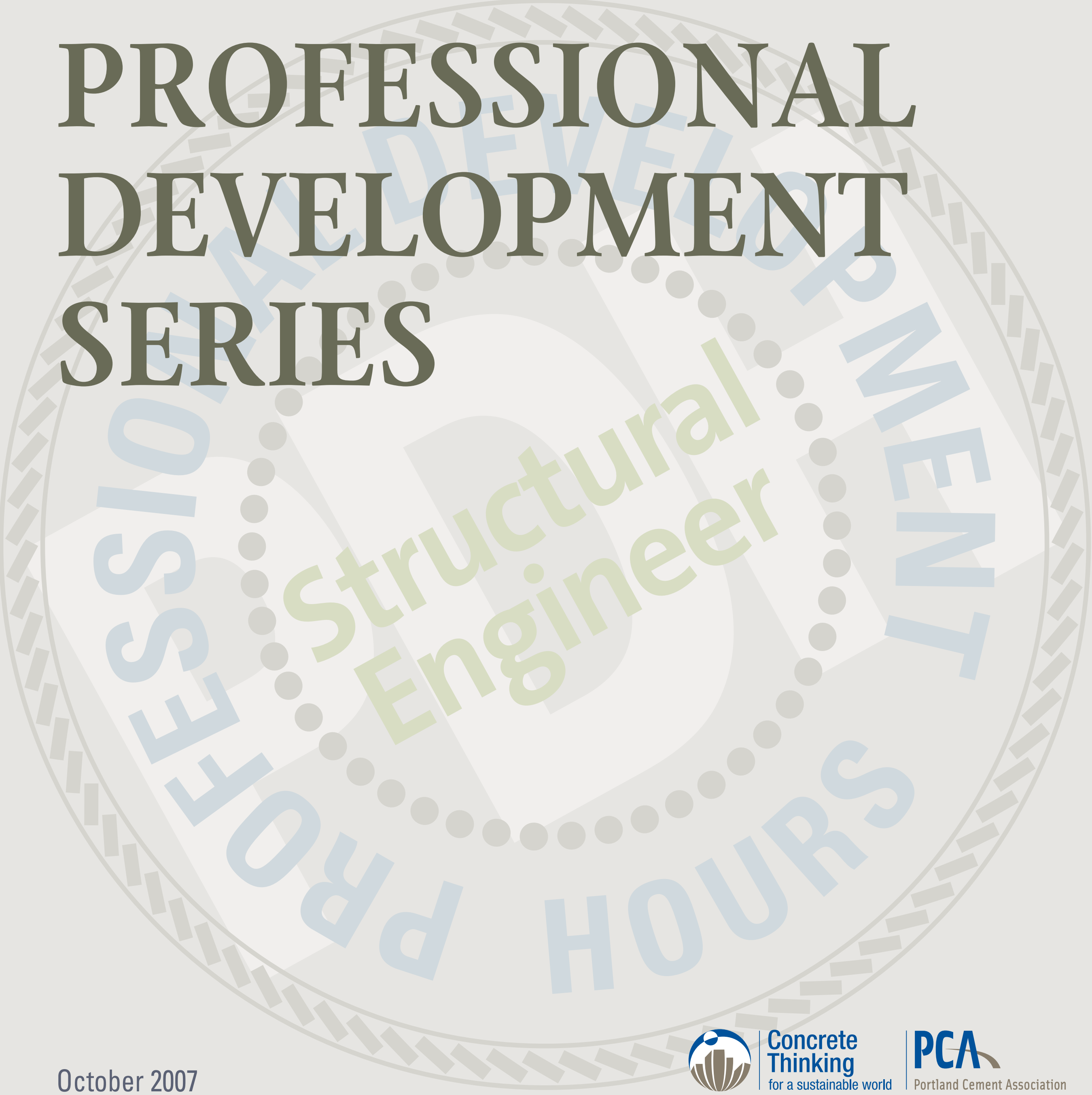
Strut-and-Tie Model for Structural Concrete Design

By Attila B. Beres, Ph.D., P.E., and Basile G. Rabbat, Ph.D., S.E.

PROFESSIONAL DEVELOPMENT SERIES



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Strut-and-tie model (STM) is a tool for analysis, design, and detailing of structural concrete members. It is essentially a truss analogy. It is based on the fact that concrete is strong in compression and steel is strong in tension. Truss members that are in compression are made up of concrete, while truss members that are in tension consist of steel reinforcement.

STM in the ACI Code

For the Building Code, Appendix A, Strut-and-Tie Models, was introduced in ACI 318-02. No changes were made for the 2005 edition (Reference 1; specific sections of this document are referenced below). Minor editorial clarifications have been proposed to the 2008 version of this appendix. STM provides a design approach applicable to an array of design problems that do not have an explicit design solution in the body of the code. This method requires the designer consciously to select a realistic load path within the structural member in the form of an idealized truss.

Rational detailing of the truss elements and compliance with equilibrium ensures the safe transfer of loads to the supports or to other regions designed by conventional procedures. While solutions provided with this powerful design and analysis tool are not unique, they represent a conservative lower bound approach. As opposed to some of the prescriptive formulations in the body of ACI 318, the very visual, rational STM of Appendix A gives insight into detailing needs of irregular (load or geometric discontinuity) regions of concrete structures and promotes ductility at the strength limit state. The only serviceability provision in the present form of Appendix A is the crack control reinforcement for the struts.

The design methodology presented in Appendix A is largely based on the seminal articles on the subject by Schlaich et al., Collins and Mitchell, and Marti (see Reference 2). Since publication of these documents, STM has received increased attention by researchers and textbook writers. MacGregor described the background of provisions incorporated in Appendix A in ACI Special Publication SP208 (Reference 3). In addition to SP208, examples of applications have been published in Reference 2 for buildings and in Reference 4 for bridges. Note that the AASHTO LRFD Bridge Design Specifications treatment of STM is slightly different than that of the ACI 318 Building Code.

STM calls for distinction of two types of zones in a concrete component depending on the characteristics

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Learning Objectives

This article discusses the strut-and-tie model in structural concrete structures. Upon reading the article and completing the quiz, the reader should be able to understand the basic principles and the related code requirements. The article presents the corresponding provisions and requirements of the American Concrete Institute's Building Code (ACI 318-05). All notation and definitions in the article are in accordance with those of ACI 318-05 Appendix A.

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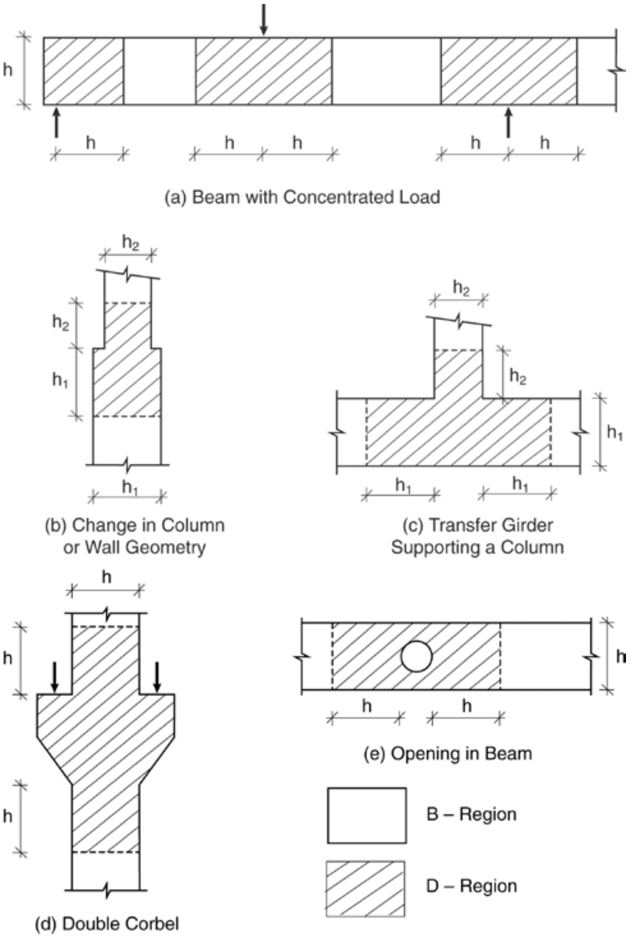

(a) Beam with Concentrated Load

(b) Change in Column or Wall Geometry

(c) Transfer Girder Supporting a Column

(d) Double Corbel

(e) Opening in Beam

Figure 1: Load and geometric discontinuities

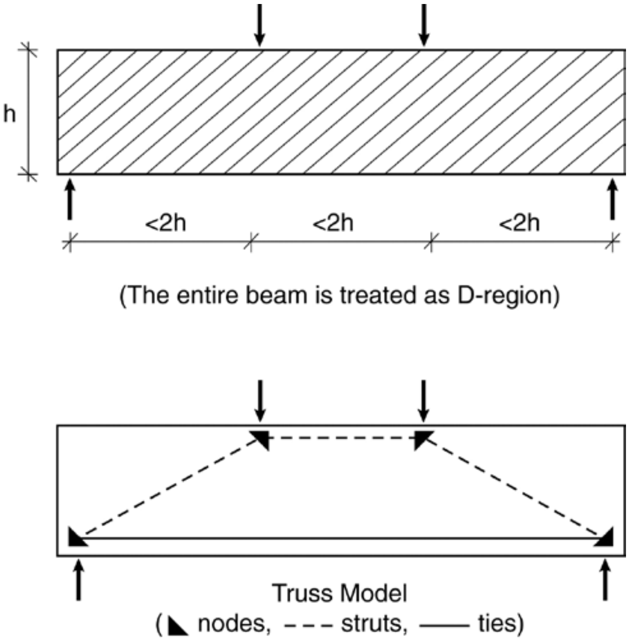

Figure 2: Strut-and-tie model

of stress fields at each location. Thus, structural members are divided into B-regions and D-regions.

B-regions represent portions of a member in which the "plane section" assumption of the classical beam theory can be applied with a sectional design approach.

D-regions are all the zones outside the B-regions where cross-sectional planes do not remain plane upon loading. D-regions are typically assumed at portions of a member where discontinuities (or disturbances) of stress distribution occur because of concentrated forces (loads or reactions) or abrupt changes of geometry. Based on St. Venant's Principle, the normal stresses (due to axial load and bending) approach quasi-linear distribution at a distance approximately equal to the larger of the overall height (*h*) and width of the member, away from the location of the concentrated force or geometric irregularity. Figure 1 illustrates typical discontinuities, D-Regions (cross-hatched areas), and B-Regions.

While B-regions can be designed with the traditional methods (ACI 318 Chapters 10 and 11), the STM was primarily introduced to facilitate design of D-regions, and can be extended to the B-regions as well. STM depicts the D-region of the structural member with a truss system consisting of compression struts and tension ties connected at nodes as shown in Figure. 2. This truss system is designed to transfer the factored loads to the supports or to adjacent B-regions. At the same time, forces in the truss members should maintain equilibrium with the applied loads and reactions.

Struts are the compression elements of the STM representing the resultants of compression field. Both parallel and fan-shaped compression fields can be modeled by their resultant compression struts as shown in Figure 3.

Typically, compression struts would take a bottle-shape wherever the strut can spread laterally at mid-length. As a design simplification, prismatic compression members

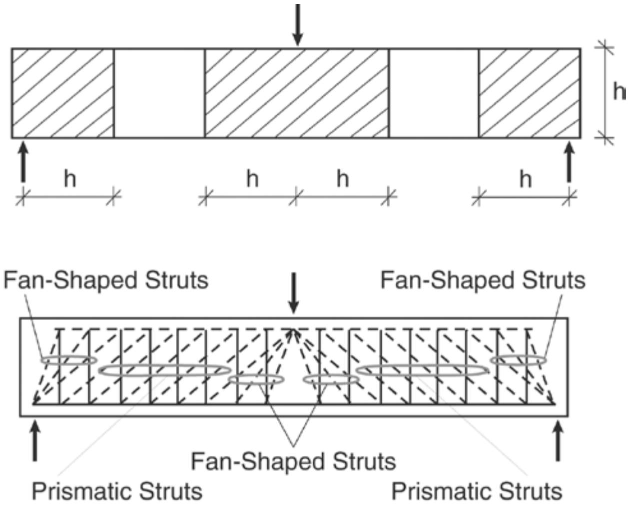

Figure 3: Prismatic and fan-shaped struts

commonly idealize struts, however, other shapes are also possible. Should the compression stress be high in the strut, reinforcement may be necessary to prevent splitting caused by transverse tension (similar to the splitting crack that develops in a cylinder supported on edge and loaded in compression).

Ties consist of conventional reinforcing steel, or prestressing steel, or both, plus a portion of the surrounding concrete that is concentric with the axis of the tie. The surrounding concrete is not considered to resist axial tension in the model. However, it reduces elongation of the tie (tension stiffening), in particular, under service loads. It also defines the zone in which the forces in the struts and ties are to be anchored.

A nodal zone is the volume of concrete that is assumed to transfer strut-and tie forces through the node. The early STMs used hydrostatic nodal zones, which were lately superseded by extended nodal zones. The faces of a hydrostatic nodal zone are perpendicular to the axes of the struts and ties acting on the node, as depicted in Figure 4. The term hydrostatic refers to the fact that the in-plane stresses are the same in all directions. (Note that in a true hydrostatic stress state, the out-of-plane stresses should also be equal.)

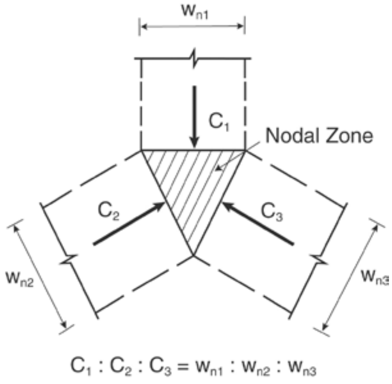

Figure 4: Hydrostatic nodal zone

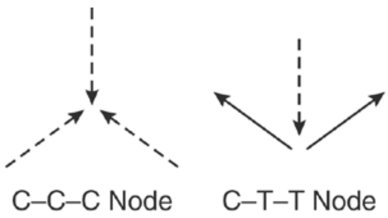

Figure 5: Classification of nodes

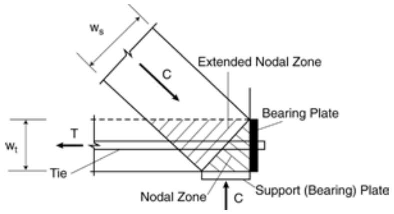

Figure 6: Extended nodal zone

Assuming identical stresses on all faces of a C-C-C nodal zone with three struts implies that the ratios of the lengths of the sides of the nodal zones ($w_{n1} : w_{n2} : w_{n3}$) are proportional to the magnitude of the strut forces ($C_1 : C_2 : C_3$). Note that C denotes compression and T denotes tension.

Nodes are the intersection points of the axes of the struts, ties, and concentrated forces, representing the joints of an STM. To maintain equilibrium, at least three forces should act on a given node of the model. Nodes are classified depending on the sign of the forces acting upon them (e.g., a C-C-C node resists three compression forces; a C-T-T node resists one compression force and two tensile forces; etc.) as shown in Figure 5.

The extended nodal zone is a portion of a member bounded by the intersection of the effective strut width, w_s, and the effective tie width, w_t. This is shown in Figure 6.

STM design procedure

A design with STM typically involves the following steps:

1) Define and isolate D-regions.
2) Compute resultant forces on each D-region boundary.
3) Devise a truss model to transfer the resultant forces across the D-region. The axes of the struts and ties are oriented to coincide approximately with the axes of the compression and tension stress fields, respectively.
4) Calculate forces in the truss members.
5) Determine the effective widths of the struts and nodal zones, considering the forces from the previous steps and the effective concrete strengths (defined in A.3.2 and A.5.2). Strength checks are based on $\phi F_n \geq F_u$, where F_u is the largest factored force obtained from the applicable load combinations; F_n is the nominal strength of the strut, tie, or node; and the strength reduction factor, ϕ, is listed in 9.3.2.6 as 0.75 for ties, strut, nodal zones, and bearing areas of STM.
6) Provide reinforcement for the ties considering the steel strengths defined in A.4.1. The reinforcement must be detailed to provide proper anchorage on either side of the critical sections.

In addition to the strength limit states represented by the STM, check structural members for serviceability requirements. Traditional elastic analysis can be used for deflection checks. Crack control can be verified using provisions of 10.6.4, assuming that the tie is encased in a prism of concrete corresponding to the area of tie (RA.4.2).

There are usually several STMs that can be devised for a given structural member and loading condition. Models that satisfy the serviceability requirements the best have struts and ties that follow the compressive and tensile stress trajectories, respectively. Certain construction rules of STMs are imposed to mitigate potential cracking problems and to avoid incompatibilities due to shortening of the struts and lengthening of the ties in almost the same direction. For example, the angle, θ, between the axes of any strut and any tie entering a single node shall not be taken as less than 25 degrees (A.2.5).

Strength of struts

The nominal compressive strength of a strut without longitudinal reinforcement shall be taken as

$$F_{ns} = f_{ce} A_{cs}$$

to be calculated at the weaker end of the compression member. A_{cs} is the cross-sectional area at the end of the strut. In typical, two-dimensional members, the width of the strut (w_s) can be taken as the width of the member. The effective compressive strength of the concrete (f_{ce}) for this purpose shall be taken as the lesser of the concrete strengths at the two sides of the nodal zone/strut interface. Section A.3.2 specifies the calculation of f_{ce} for the strut (detailed below), while A.5.2 provides for the same in the nodal zone (discussed later). The effective compressive strength of the concrete in a strut is calculated, similarly to basic strength equations, as:

$$f_{ce} = 0.85\beta_s f_c'$$

The β_s factor accounts for the effect of cracking and possible presence of transverse reinforcement. The strength of the concrete in a strut can be computed with $\beta_s = 1.0$ for struts that have uniform cross sectional area over their length. This is quasi-equivalent to the rectangular stress block in the compression zone of a beam or column. For bottle-shaped struts (Figure 7) with reinforcement placed to resist the splitting forces (satisfying A.3.3), $\beta_s = 0.75$, or without adequate confinement to resist splitting forces, $\beta_s = 0.6\,\lambda$ (where λ is a correction factor (11.7.4.3) for lightweight concrete).

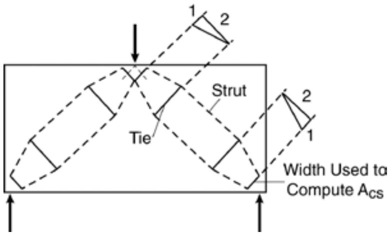

Figure 7: Bottle-shaped compression strut

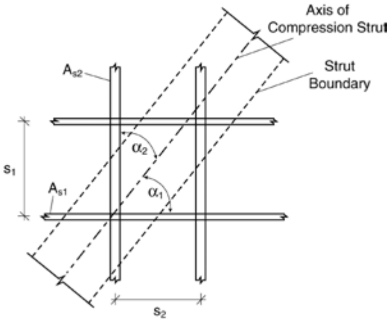

Figure 8: Layers of reinforcement to restrain splitting cracks of struts

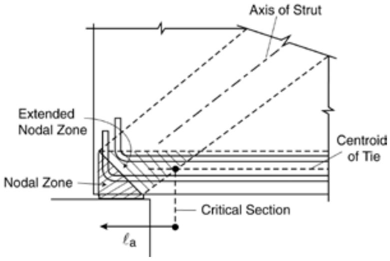

Figure 9: Anchorage of tie reinforcement

For struts intersecting cracks in a tensile zone, β_s is reduced to 0.4. Examples include STMs used to design the longitudinal and transverse reinforcement of the tension flanges of beams, box-girders, and walls. For all other cases (e.g., in beam webs where struts are likely to be crossed by inclined cracks), the β_s factor can be conservatively taken as 0.6.

Section A.3.3 addresses cases where transverse reinforcement is provided to cross the bottle-shaped struts. The compression forces in the strut may be assumed to spread at a 2:1 slope. The rebars are intended to resist the transverse tensile forces resulting from the compression force spreading in the strut. They may be placed in one direction (when the angle, α, between the rebar and the axis of the strut is at least 40 degrees) or in two orthogonal directions.

To allow for $\beta_s = 0.75$, for concrete strength not exceeding 6,000 pounds per square inch (psi), the reinforcement ratio needed to cross the strut is computed from:

$$\Sigma \frac{A_{si}}{b_s s_i} \sin\alpha_i \geq 0.003$$

where A_{si} is the total area of reinforcement at spacing s_i in a layer of reinforcement with bars at an angle γ_i to the axis of the strut (shown in Figure 8), and b_s is the width of the strut. In those cases, $\beta_s = 0.6\,\lambda$ shall be used.

If substantiated by test and analyses, increased effective compressive strength of a strut due to confining reinforcement may be used (e.g., at anchorage zones of prestressing tendons). Additional strength can be provided to the struts by including compression reinforcement parallel to the axis of the strut. These bars must be properly anchored and enclosed by ties or spirals per 7.10. The compressive strength of these longitudinally reinforced struts can be calculated as:

$$F_{ns} = f_{ce}\,A_{cs} + A_s'\,f_s'$$

where f_s' is the stress in the longitudinal strut reinforcement at nominal strength. It can be either obtained from strain analyses at the time the strut crushes or taken as $f_s' = f_y$ for Grade 40 and 60 rebars.

Strength of Ties

The nominal strength of a tie is calculated as the sum of yield strength of the conventional reinforcement plus the force in the prestressing steel:

$$F_{nt} = A_{ts} f_y + A_{tp}\,(f_{se} + \Delta f_p)$$

Note, that A_{tp} is zero if there is no prestressing present in the tie. The actual prestressing stress ($f_{se} + \Delta f_p$) should not exceed the yield stress f_{py} of the prestressing steel. Also, if not calculated, the code allows estimating the increase in prestressing steel stress due to factored loads, Δf_p, as 60,000 psi for bonded prestressed reinforcement, or 10,000 psi for unbonded prestressed reinforcement. Since the intent of having a tie is to provide for a tension element in a truss, the axis of the reinforcement centroid shall coincide with the axis of the tie assumed in the model. Depending on the distribution of the tie reinforcement, the effective tie width (w_t) may vary between the following limits:

- The minimum width for configurations where only one layer of reinforcement is provided in a tie, w_t, can be taken as the diameter of the bars in the tie plus twice the concrete cover to the surface of the ties. Should the tie be wider than this, the reinforcement shall be distributed evenly over the width.

- The upper limit is established as the width corresponding to the width in a hydrostatic nodal zone, calculated as $w_{t,max} = F_{nt}/(f_{ce} b_s)$ where f_{ce} is the applicable effective compression strength of a nodal zone discussed below and b_s is the width of the tie.

Nodes shall be able to develop the difference between the forces of truss members connecting to them. Thus, besides providing adequate amount of tie reinforcement, special attention shall be paid to proper anchorage. Anchorage can be achieved using mechanical devices, post-tensioning anchorage devices, standard hooks, headed bars, or straight bar embedment. The reinforcement in a tie should be anchored before it leaves the extended nodal zone, i.e., at the point defined by the intersection of the centroid of the bars in the tie and the extensions of the outlines of either the strut or the bearing area as shown in Figure 9. For truss layouts where more than one tie intersects at a node, each tie force shall be developed at the point where the centroid of the reinforcement in the tie leaves the extended nodal zone. (Note that transverse reinforcement required by A.3.3 shall be anchored according to the provisions of 12.13.)

In many cases, the structural configuration does not provide for the straight development length for a tie. For such cases, anchorage is provided through mechanical devices, hooks, or splicing with several layers of smaller bars. These options often require a wider structural member and/or additional confinement reinforcement (e.g., to avoid cracking along the outside of the hooks).

Strength of nodal zones

The nominal compression strength at the face of a nodal zone or at any section through the nodal zone shall be:

$$F_{nn} = f_{ce} A_{nz}$$

where A_{nz} is taken as the area of the face of the nodal zone that the strut force F_u acts on, if the face is perpendicular to the line of action of F_u. If the nodal zone is limited by some other criteria, the node-to-strut interface may not be perpendicular to the axis of the strut. Therefore, the axial stresses in the compression-only strut will generate both shear and normal stresses acting on the interface. In those cases, the A_{nz} parameter shall be the area of a section taken through the nodal zone perpendicular to the strut axis. The STM is applicable to three-dimensional situations as well. To keep calculations simple, A.5.3 allows the area of the nodal faces to be less than that described above. The shape of each face of the nodal zones must be similar to the shape of the projection of the end of the struts onto the corresponding faces of the nodal zones.

The effective compressive strength of the concrete in the nodal zone (f_{ce}) is calculated as:

$$f_{ce} = 0.85\beta_n f_c'$$

and must not exceed the effective concrete compressive strength on the face of a nodal zone due to STM forces, unless confining reinforcement is provided within the nodal zone and its effect is evidenced by tests and analysis. The sign of forces acting on the node influences the capacity at the nodal zones as reflected by the β_n value. The presence of tensile stresses due to ties decreases the nodal zone concrete strength:

- $\beta_n = 1.0$ in nodal zones bounded by struts or bearing areas (e.g., C-C-C nodes);
- $\beta_n = 0.8$ in nodal zones anchoring one tie (e.g., C-C-T nodes); and
- $\beta_n = 0.6$ in nodal zones anchoring two or more ties (e.g., C-T-T or T-T-T nodes).

Conclusion

STM is gaining acceptance in designing concrete structures for conditions that are unique and difficult to address efficiently with conventional methods. This article provided a comprehensive summary of the calculation and detailing requirements following the provisions of Appendix A in the American Concrete Institute's Building Code (ACI 318-05). The reader is referred to detailed, worked-out numerical examples that illustrate the material presented in this paper on the Portland Cement Association's website (www.cement.org/buildings/design_aids.asp), as well as to several resource documents (References 2 and 4). ▼

Selected References

1) ACI Committee 318, Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (ACI 318R-05), American Concrete Institute, Farmington Hills, Mich., 443 pages.

2) Notes on ACI 318-05 Building Code Requirements for Structural Concrete, EB705, Portland Cement Association, Skokie, Ill., 2005.

3) Examples for the Design of Structural Concrete with Strut-and-Tie Models, SP208, American Concrete Institute, Farmington Hills, Mich., 2002, 242 pages.

4) Mitchell, D.; Collins, M.P.; Bhide, S.B.; and Rabbat, B.G., AASHTO LRFD Strut-and Tie Model Design Examples, EB 231, Portland Cement Association, Skokie, Ill., 2004.

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Strut-and-Tie Model for Structural Concrete Design

1. STM requirements can be found in the ACI code provision addressing:
- a) shear issues in Section 11
- b) not in ACI 318, only in AASTHO
- c) Appendix A
- d) deep beam design-related code sections

2. What is the value of the strength reduction factor, ϕ, used in the STM?
a) 0.90 b) 0.85 c) 0.75 d) 0.60

3. B-regions are:
- a) bearing areas in compression
- b) areas of a concrete structure where the classical flexural beam theory is applicable
- c) boundary elements of shearwalls
- d) bent-rebar encapsulated beam sections

4. D-regions are:
- a) portions of members with geometrical discontinuity
- b) portions of members with force discontinuity
- c) deep beams
- d) all of the above

5. Whose principle is used to determine the extent of D-regions?
a) St. Venant b) Navier c) Bernouilli d) Timoshenko

6. Are there any extensive code text changes for the STM-related provision in the ACI 318 documents succeeding the 2002 version?
a) yes b) no

7. What is the minimum angle between the axes of strut and ties framing into the same node?
- a) 45 degrees
- b) 30 degrees
- c) 25 degrees
- d) 20 degrees

8. Bottle-shaped compression struts are considered to be stronger than prismatic struts.
- a) always true
- b) true if at least two compression struts frame into the same node
- c) true only if transverse reinforcement is provided
- d) false

9. Can prestressing tendons be considered tie elements?
- a) only unbonded post-tensioned tendons
- b) only bonded pre-tensioned tendons
- c) yes
- d) no

10. Would the presence of the tie framing into the node increase the strength of the node?
- a) no
- b) yes
- c) yes, if the node is triangular in shape
- d) yes, if the angle of the tie is more than 30 degrees to any of the compression struts framing into the same node

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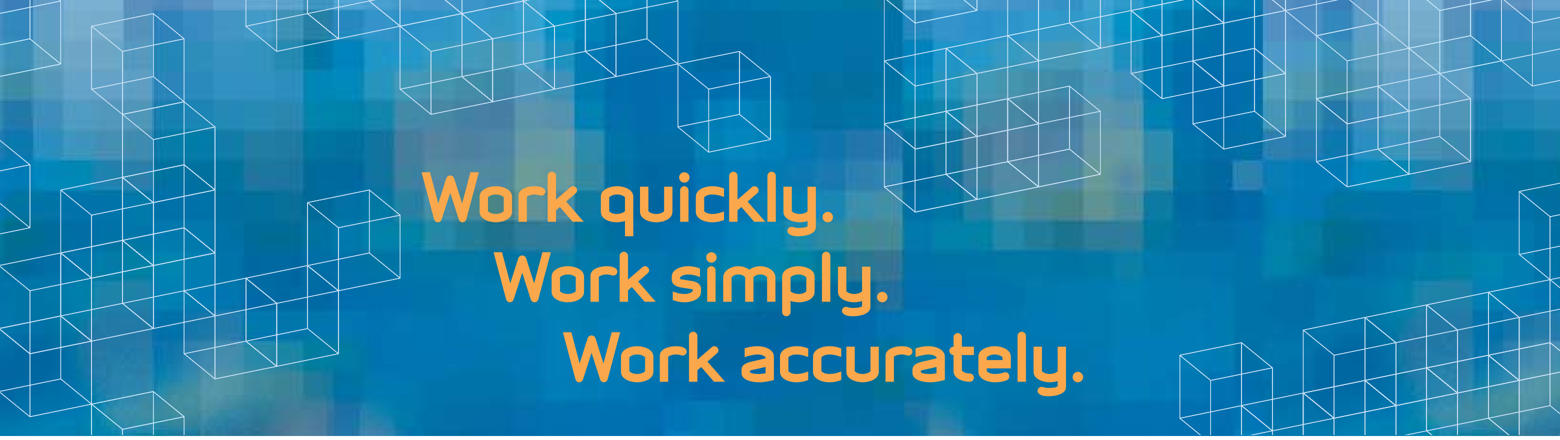

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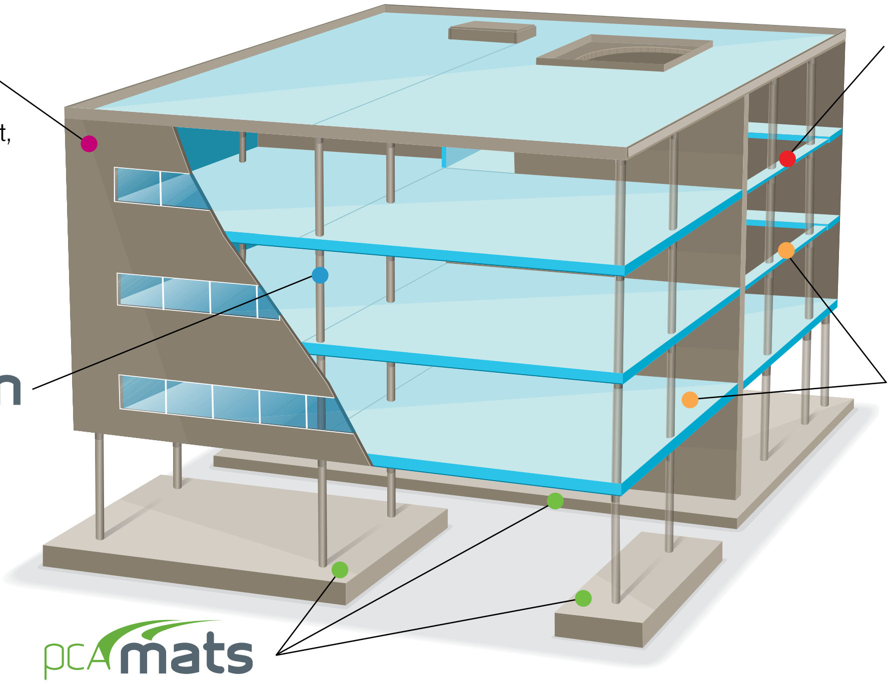

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