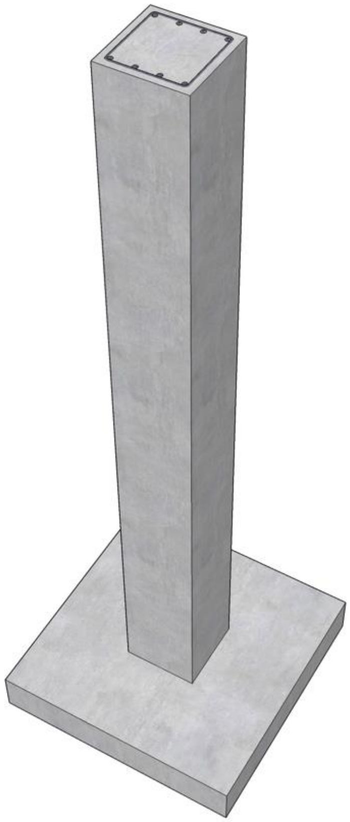
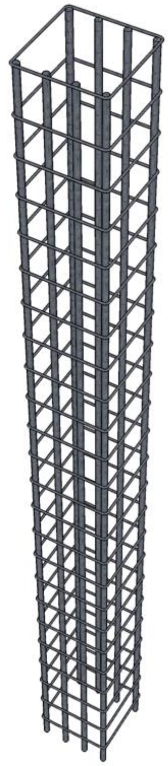
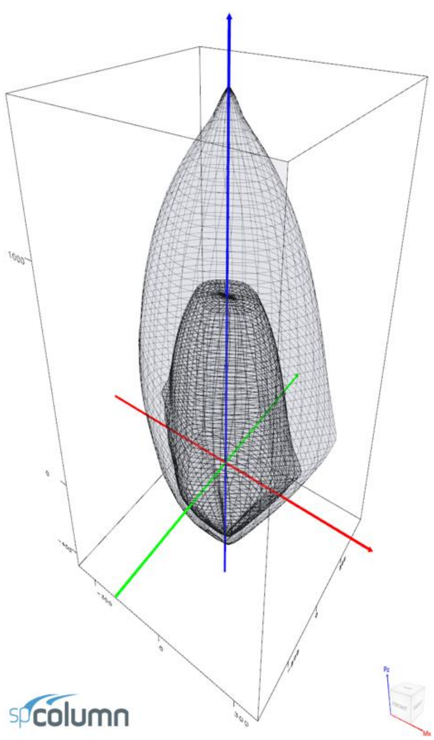
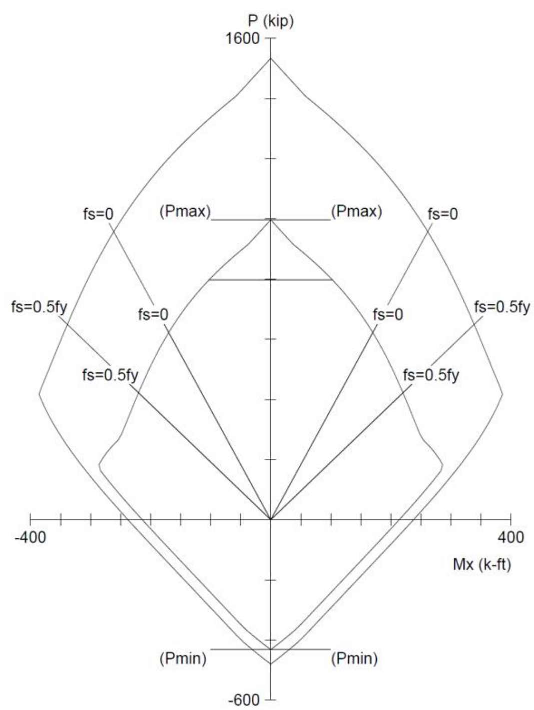
Interaction Diagram - Tied Reinforced Concrete Column









Interaction Diagram - Tied Reinforced Concrete Column

Develop an interaction diagram for the square tied concrete column shown in the figure below about the x-axis. Determine seven control points on the interaction diagram and compare the calculated values in the Reference and with exact values from the complete interaction diagram generated by spColumn engineering software program from StructurePoint.

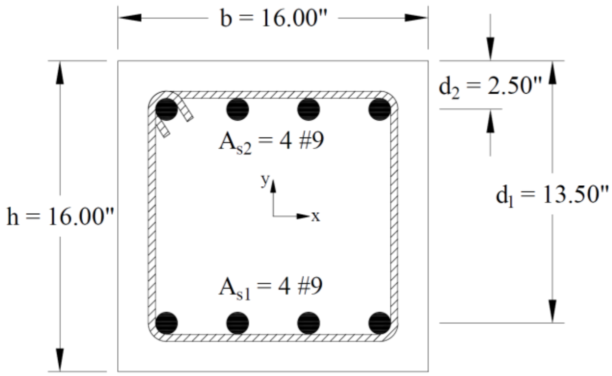


Figure 1 – Reinforced Concrete Column Cross-Section

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Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference

Reinforced Concrete Mechanics and Design, 6th Edition, 2011, James Wight and James MacGregor, Pearson

Design Data

f_c' = 5000 psi

f_y = 60,000 psi

Cover = 2.5 in. to the center of the reinforcement

Column 16 in. x 16 in.

Top reinforcement = 4 #9

Bottom reinforcement = 4 #9

Solution

Use the traditional hand calculations approach to generate the interaction diagram for the concrete column section shown above by determining the following seven control points:

Point 1: Pure compression

Point 2: Bar stress near tension face of member equal to zero, ($f_s = 0$)

Point 3: Bar stress near tension face of member equal to 0.5 f_y ($f_s = -0.5\, f_y$)

Point 4: Bar stress near tension face of member equal to f_y ($f_s = -f_y$)

Point 5: Bar strain near tension face of member equal to 0.005

Point 6: Pure bending

Point 7: Pure tension

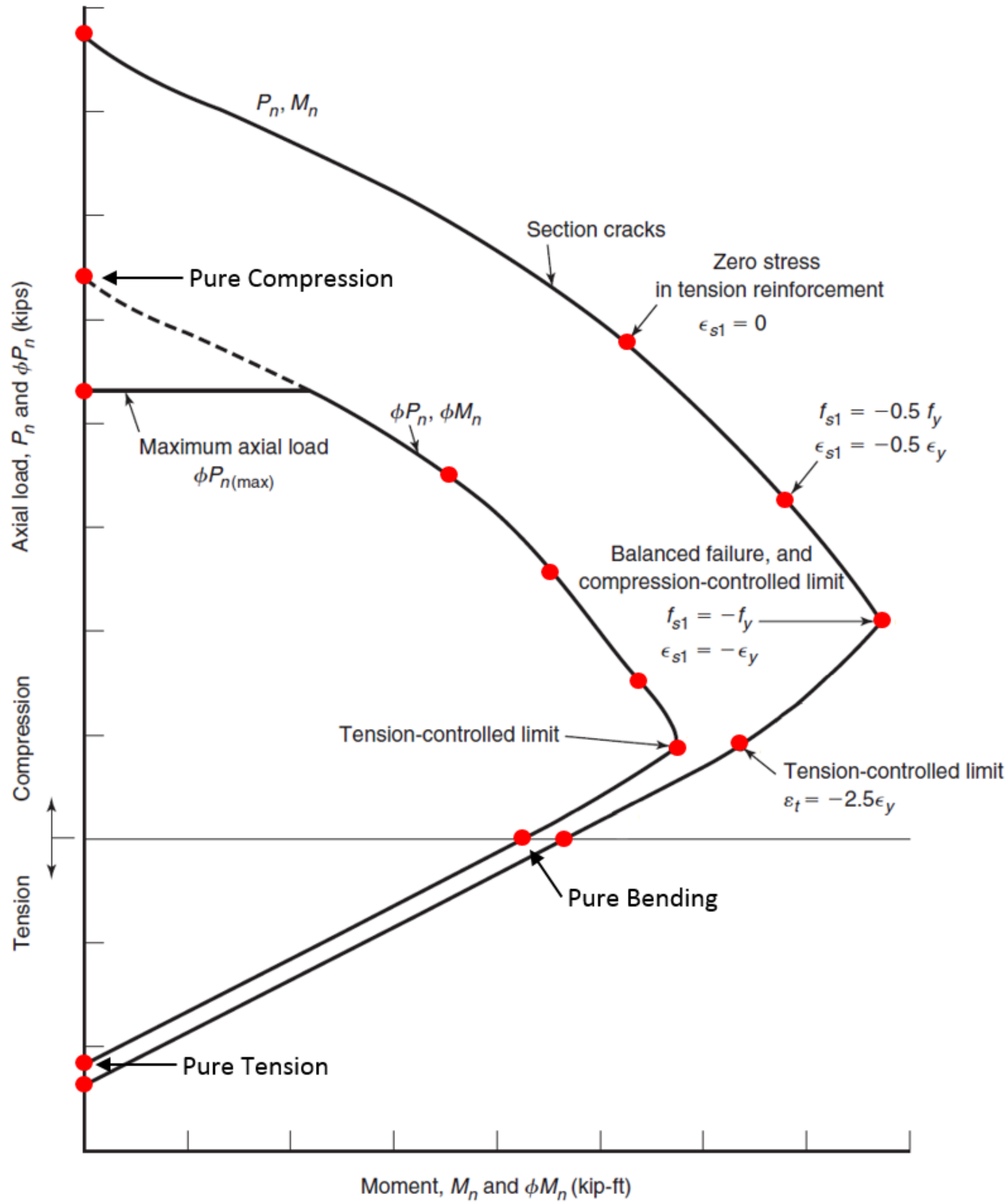

Figure 2 – Control Points

1. Pure Compression

1.1. Nominal axial compressive strength at zero eccentricity

$$P_o = 0.85 f'_c (A_g - A_{st}) + f_y A_{st}$$ ***ACI 318-14 (22.4.2.2)***

$$P_o = 0.85 \times 5000 \times (16 \times 16 - 8 \times 1.00) + 60000 \times 8 \times 1.00 = 1,530 \text{ kips}$$

1.2. Factored axial compressive strength at zero eccentricity

Since this column is a tied column with steel strain in compression:

$$\phi = 0.65$$ ***ACI 318-14 (Table 21.2.2)***

$$\phi P_o = 0.65 \times 1,530 = 997 \text{ kips}$$

1.3. Maximum (allowable) factored axial compressive strength

$$\phi P_{n,max} = 0.80 \times \phi P_o = 0.80 \times 997 = 798 \text{ kips}$$ ***ACI 318-14 (Table 22.4.2.1)***

2. Bar Stress Near Tension Face of Member Equal to Zero, ($\varepsilon_s = f_s = 0$)

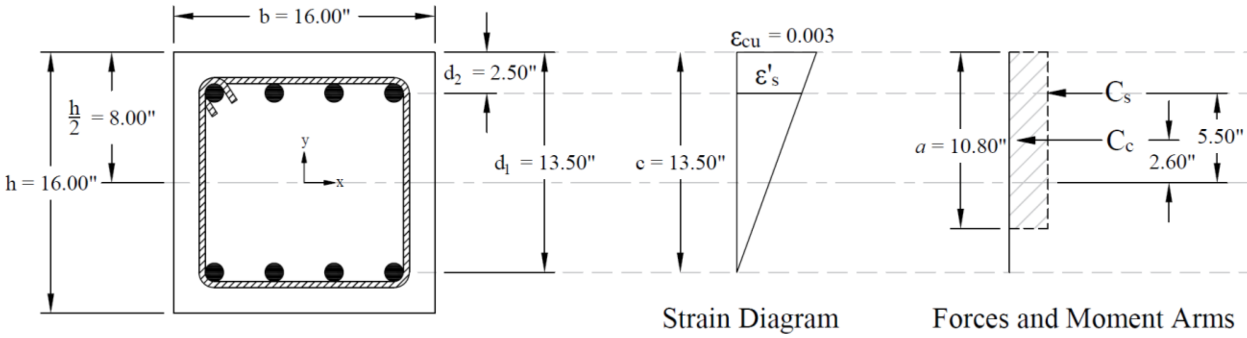

Figure 3 – Strains, Forces, and Moment Arms ($\varepsilon_t = f_s = 0$)

Strain ε_s is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices. ***ACI 318-14 (10.7.5.2.1 and 2)***

2.1. c, *a*, and strains in the reinforcement

$$c = d_1 = 13.5 \text{ in.}$$

Where *c* is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-14 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 13.5 = 10.80 \text{ in.}$$ ***ACI 318-14 (22.2.2.4.1)***

Where:

a = Depth of equivalent rectangular stress block

$$\beta_1 = 0.85 - \frac{0.05 \times \left(f_c^{'} \times 4000\right)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-14 (Table 22.2.2.4.3)***

$$\varepsilon_s = 0$$

$$\therefore \phi = 0.65$$ ***ACI 318-14 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-14 (22.2.2.1)***

$$\varepsilon_s^{'} = (c - d_2) \times \frac{\varepsilon_{cu}}{c} = (13.50 - 2.5) \times \frac{0.003}{13.50} = 0.00244 \text{ (Compression)} > \varepsilon_y = \frac{F_y}{E_s} = \frac{60}{29{,}000} = 0.00207$$

2.2. Forces in the concrete and steel

$$C_c = 0.85 \times f_c^{'} \times a \times b = 0.85 \times 5{,}000 \times 10.80 \times 16 = 734.4 \text{ kip}$$ ***ACI 318-14 (22.2.2.4.1)***

$f_s = 0 \text{ psi} \rightarrow \mathrm{T}_s = f_s \times A_{s1} = 0 \text{ kip}$

Since $\varepsilon_s^{'} > \varepsilon_y \rightarrow$ compression reinforcement has yielded

$\therefore f_s^{'} = f_y = 60{,}000 \text{ psi}$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract $0.85f_c{}'$ from $f_s{}'$ before computing C_s:

$$\mathrm{C}_s = \left(f_s^{'} - 0.85 f_c^{'}\right) \times A_{s2} = \left(60{,}000 - 0.85 \times 5{,}000\right) \times 4 = 223 \text{ kip}$$

2.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 734.4 + 223 - 0 = 957 \text{ kip}$$

$$\phi P_n = 0.65 \times 957 = 622 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 734.4 \times \left(\frac{16}{2} - \frac{10.80}{2}\right) + 223 \times \left(\frac{16}{2} - 2.5\right) + 0 \times \left(13.50 - \frac{16}{2}\right) = 261 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 261 = 170 \text{ kip.ft}$$

3. Bar Stress Near Tension Face of Member Equal to 0.5 f_y, (f_s = - 0.5 f_y)

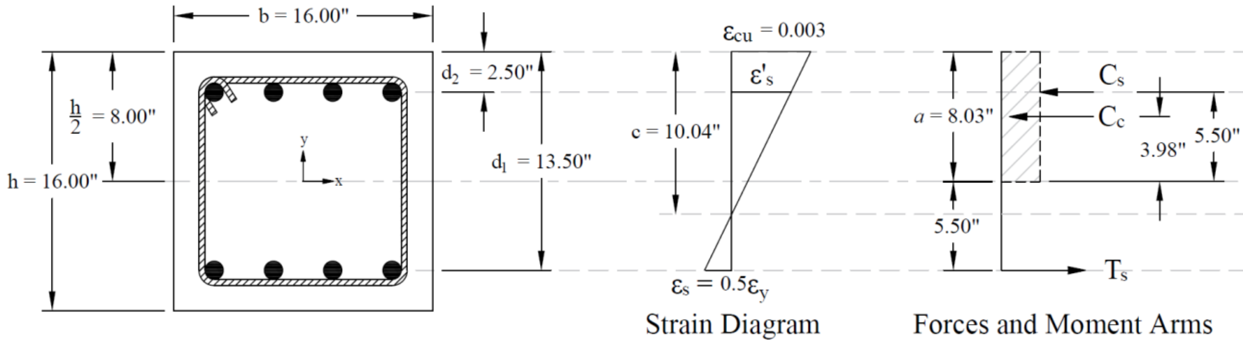

Figure 4 – Strains, Forces, and Moment Arms (f_s = - 0.5 f_y)

3.1. c, *a*, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = \frac{\varepsilon_y}{2} = \frac{0.00207}{2} = 0.00103 < \varepsilon_y \rightarrow \text{tension reinforcement has not yielded}$$

$$\therefore \phi = 0.65$$ ***ACI 318-14 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-14 (22.2.2.1)***

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00103 + 0.003} \times 0.003 = 10.04 \text{ in.}$$

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-14 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 10.04 = 8.03 \text{ in.}$$ ***ACI 318-14 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times \left(f_c^{'} \times 4000\right)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-14 (Table 22.2.2.4.3)***

$$\varepsilon_s^{'} = (c - d_2) \times \frac{0.003}{c} = (10.04 - 2.5) \times \frac{0.003}{10.04} = 0.00225 \text{ (Compression)} > \varepsilon_y$$

3.2. Forces in the concrete and steel

$$C_c = 0.85 \times f_c^{'} \times a \times b = 0.85 \times 5,000 \times 8.03 \times 16 = 546.1 \text{ kip}$$ ***ACI 318-14 (22.2.2.4.1)***

$$f_s = \varepsilon_s \times E_s = 0.00103 \times 29,000,000 = 30,000 \text{ psi}$$

$$\text{T}_s = f_s \times A_{s1} = 30,000 \times 4 = 120 \text{ kip}$$

Since $\varepsilon_s^{'} > \varepsilon_y \rightarrow$ compression reinforcement has yielded

$\therefore f_s^{'} = f_y = 60,000$ psi

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$$\mathrm{C}_s = \left(f_s^{'} - 0.85 f_c^{'}\right) \times A_{s2} = (60,000 - 0.85 \times 5,000) \times 4 = 223 \text{ kip}$$

3.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 546.1 + 223 - 120 = 649 \text{ kip}$$

$$\phi P_n = 0.65 \times 649 = 422 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 546.1 \times \left(\frac{16}{2} - \frac{8.03}{2}\right) + 223 \times \left(\frac{16}{2} - 2.5\right) + 120 \times \left(13.50 - \frac{16}{2}\right) = 339 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 339 = 220 \text{ kip.ft}$$

4. Bar Stress Near Tension Face of Member Equal to f_y, (f_s = - f_y)

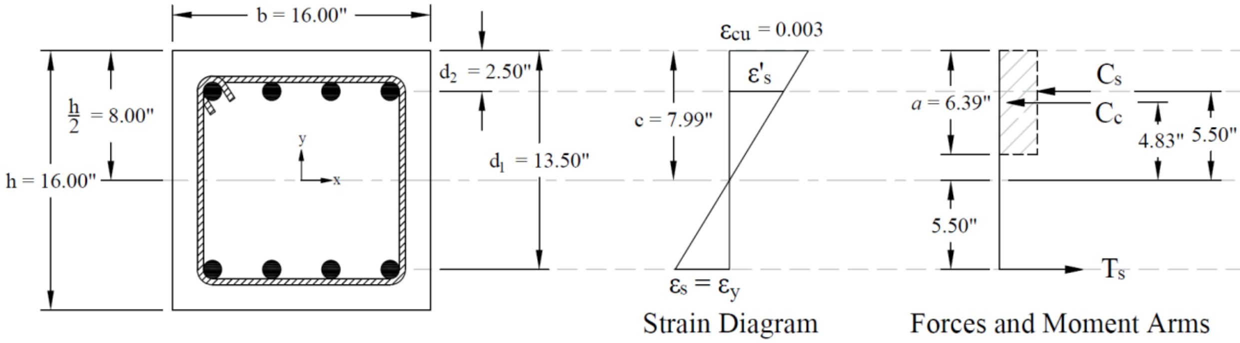

Figure 5 – Strains, Forces, and Moment Arms (f_s = - f_y)

This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement. It also marks the start of the transition zone for ϕ for columns in which ϕ increases from 0.65 (or 0.75 for spiral columns) up to 0.90.

4.1. c, *a*, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = \varepsilon_y = 0.00207 \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.65$$ ***ACI 318-14 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-14 (22.2.2.1)***

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00207 + 0.003} \times 0.003 = 7.99 \text{ in.}$$

Where *c* is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-14 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 7.99 = 6.39 \text{ in.}$$ ***ACI 318-14 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times \left(f_c^{'} \times 4000\right)}{1000} = 0.85 - \frac{0.05 \times \left(5000 \times 4000\right)}{1000} = 0.80$$ ***ACI 318-14 (Table 22.2.2.4.3)***

$$\varepsilon_s^{'} = (c - d_2) \times \frac{0.003}{c} = (7.99 - 2.5) \times \frac{0.003}{7.99} = 0.00206 \text{ (Compression)} < \varepsilon_y$$

4.2. Forces in the concrete and steel

$$C_c = 0.85 \times f_c^{'} \times a \times b = 0.85 \times 5{,}000 \times 6.39 \times 16 = 434.6 \text{ kip}$$ ***ACI 318-14 (22.2.2.4.1)***

$$f_s = f_y = 60{,}000 \text{ psi}$$

$$\text{T}_s = f_y \times A_{s1} = 60{,}000 \times 4 = 240 \text{ kip}$$

Since $\varepsilon_s^{'} < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore\ f_s^{'} = \varepsilon_s^{'} \times E_s = 0.00206 \times 29{,}000{,}000 = 59{,}778 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract 0.85f_c' from f_s' before computing C_s:

$$\text{C}_s = \left(f_s^{'} - 0.85 f_c^{'}\right) \times A_{s2} = \left(59{,}778 - 0.85 \times 5{,}000\right) \times 4 = 222.1 \text{ kip}$$

4.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 434.6 + 222.1 - 240 = 417 \text{ kip}$$

$$\phi P_n = 0.65 \times 417 = 271 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 434.6 \times \left(\frac{16}{2} - \frac{6.39}{2}\right) + 222.1 \times \left(\frac{16}{2} - 2.5\right) + 240 \times \left(13.50 - \frac{16}{2}\right) = 386 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 386 = 251 \text{ kip.ft}$$

5. Bar Strain Near Tension Face of Member Equal to 0.005 in./in., (ε_s = - 0.005 *in./in.*)

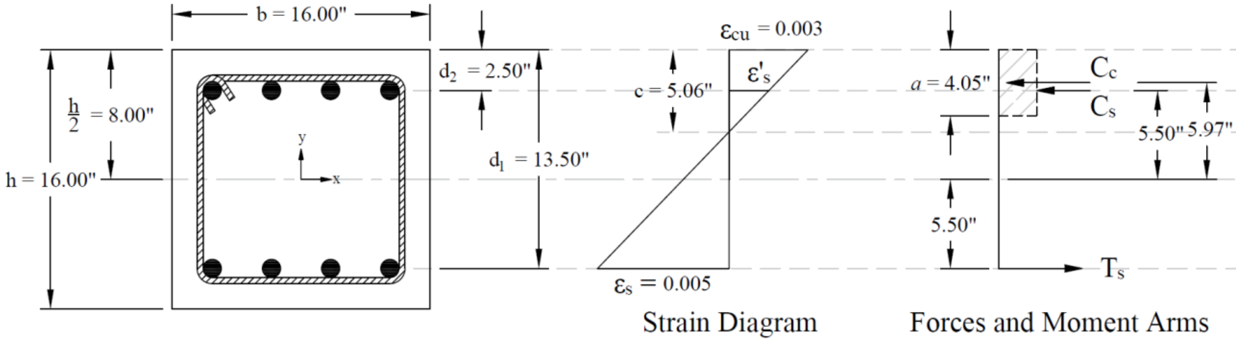

Figure 6 – Strains, Forces, and Moment Arms (ε_s = - 0.005 in./in.)

This corresponds to the tension-controlled strain limit of 0.005. It is the strain at the tensile limit of the transition zone for ϕ, used to define a tension-controlled section.

5.1. c, *a*, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = 0.005 > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.9$$ ***ACI 318-14 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-14 (22.2.2.1)***

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.005 + 0.003} \times 0.003 = 5.06 \text{ in.}$$

Where *c* is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-14 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 5.06 = 4.05 \text{ in.}$$ ***ACI 318-14 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times \left(f_c^{'} \times 4000 \right)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-14 (Table 22.2.2.4.3)***

$$\varepsilon_s^{'} = (c - d_2) \times \frac{0.003}{c} = (5.06 - 2.5) \times \frac{0.003}{5.06} = 0.00152 \text{ (Compression)} < \varepsilon_y$$

5.2. Forces in the concrete and steel

$$C_c = 0.85 \times f_c^{'} \times a \times b = 0.85 \times 5,000 \times 4.05 \times 16 = 275.4 \text{ kip}$$ ***ACI 318-14 (22.2.2.4.1)***

$$f_s = f_y = 60,000 \text{ psi}$$

$$\text{T}_s = f_y \times A_{s1} = 60,000 \times 4 = 240 \text{ kip}$$

Since $\varepsilon_s^{'} < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore\ f_s^{'} = \varepsilon_s^{'} \times E_s = 0.00152 \times 29,000,000 = 44,037 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (*ab*) used to compute C_c. As a result, it is necessary to subtract 0.85f_c' from f_s' before computing C_s:

$$\text{C}_s = \left(f_s^{'} - 0.85 f_c^{'} \right) \times A_{s2} = \left(44,037 - 0.85 \times 5,000\right) \times 4 = 159.1 \text{ kip}$$

5.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 275.4 + 159.1 - 240 = 195 \text{kip}$$

$$\phi P_n = 0.90 \times 195 = 175 \text{kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 275.4 \times \left(\frac{16}{2} - \frac{4.05}{2}\right) + 159.1 \times \left(\frac{16}{2} - 2.5\right) + 240 \times \left(13.50 - \frac{16}{2}\right) = 320 \text{kip.ft}$$

$$\phi M_n = 0.90 \times 320 = 288 \text{kip.ft}$$

6. Pure Bending

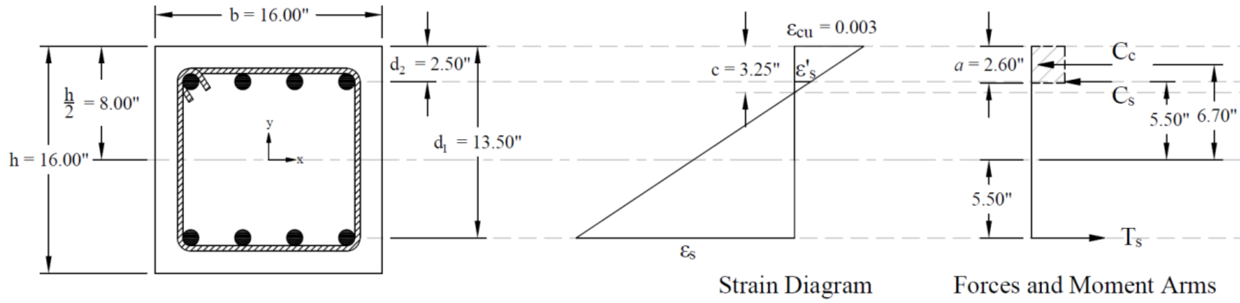

Figure 7 – Strains, Forces, and Moment Arms (Pure Moment)

This corresponds to the case where the nominal axial load capacity, P_n, is equal to zero. Iterative procedure is used to determine the nominal moment capacity as follows:

6.1. c, *a*, and strains in the reinforcement

Try $c = 3.25$ in.

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-14 (22.2.2.4.2)

$a = \beta_1 \times c = 0.80 \times 3.25 = 2.60$ in. ***ACI 318-14 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times \left(f_c^{'} \times 4000\right)}{1000} = 0.85 - \frac{0.05 \times (5000 \times 4000)}{1000} = 0.80$$ ***ACI 318-14 (Table 22.2.2.4.3)***

$\varepsilon_{cu} = 0.003$ ***ACI 318-14 (22.2.2.1)***

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29{,}000} = 0.00207$$

$$\varepsilon_s = (d_1 - c) \times \frac{0.003}{c} = (13.50 - 3.25) \times \frac{0.003}{3.25} = 0.00946 \text{ (Tension)} > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$\therefore \phi = 0.9$ ***ACI 318-14 (Table 21.2.2)***

$$\varepsilon_s^{'} = (c - d_2) \times \frac{0.003}{c} = (3.25 - 2.5) \times \frac{0.003}{3.25} = 0.00069 \text{ (Compression)} < \varepsilon_y$$

6.2. Forces in the concrete and steel

$C_c = 0.85 \times f_c^{'} \times a \times b = 0.85 \times 5{,}000 \times 2.6 \times 16 = 176.8$ kip ***ACI 318-14 (22.2.2.4.1)***

$f_s = f_y = 60{,}000$ psi

$$\mathrm{T}_s = f_y \times A_{s1} = 60,000 \times 4 = 240 \text{ kip}$$

Since $\varepsilon_s^{'} < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore\ f_s^{'} = \varepsilon_s^{'} \times E_s = 0.00069 \times 29,000,000 = 20,077 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (*ab*) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$$\mathrm{C}_s = \left(f_s^{'} - 0.85 f_c^{'}\right) \times A_{s2} = \left(20,077 - 0.85 \times 5,000\right) \times 4 = 63.3 \text{ kip}$$

6.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 176.8 + 63.3 - 240 \approx 0 \,\text{kip} \rightarrow \phi P_n \approx 0 \,\text{kip}$$

The assumption that c = 3.25 in. is correct

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 176.8 \times \left(\frac{16}{2} - \frac{2.60}{2}\right) + 63.3 \times \left(\frac{16}{2} - 2.5\right) + 240 \times \left(13.50 - \frac{16}{2}\right) = 238 \,\text{kip.ft}$$

$$\phi M_n = 0.90 \times 238 = 214 \,\text{kip.ft}$$

7. Pure Tension

The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.

7.1. P_{nt} and ϕP_{nt}

$$P_{nt} = f_y \times (A_{s1} + A_{s2}) = 60{,}000 \times (4+4) = 480 \text{ kip}$$ ***ACI 318-14 (22.4.3.1)***

$$\phi = 0.9$$ ***ACI 318-14 (Table 21.2.2)***

$$\phi P_{nt} = 0.90 \times 480 = 432 \text{ kip}$$

7.2. M_n and ϕM_n

Since the section is symmetrical

$$M_n = \phi M_n = 0 \text{ kip.ft}$$

8. Column Interaction Diagram - spColumn Software

spColumn program performs the analysis of the reinforced concrete section conforming to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility. For this column section, we ran in investigation mode with control points using the 318-14. In lieu of using program shortcuts, spSection (Figure 9) was used to place the reinforcement and define the cover to illustrate handling of irregular shapes and unusual bar arrangement.

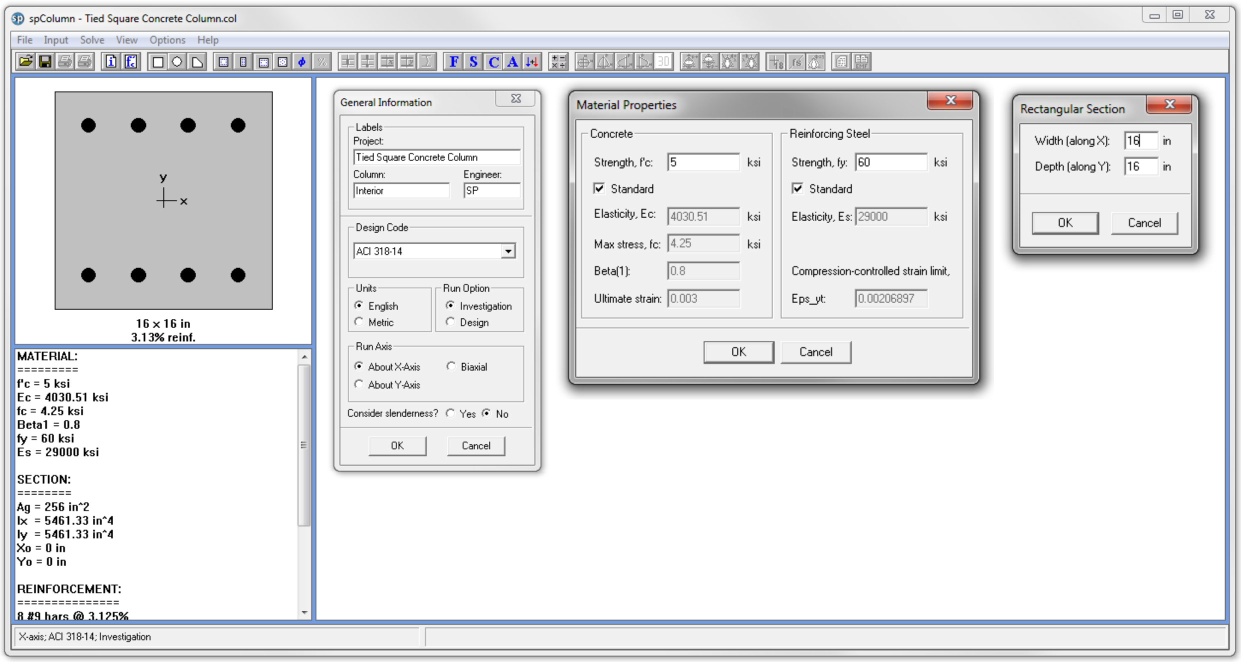

Figure 8 – Generating spColumn Model

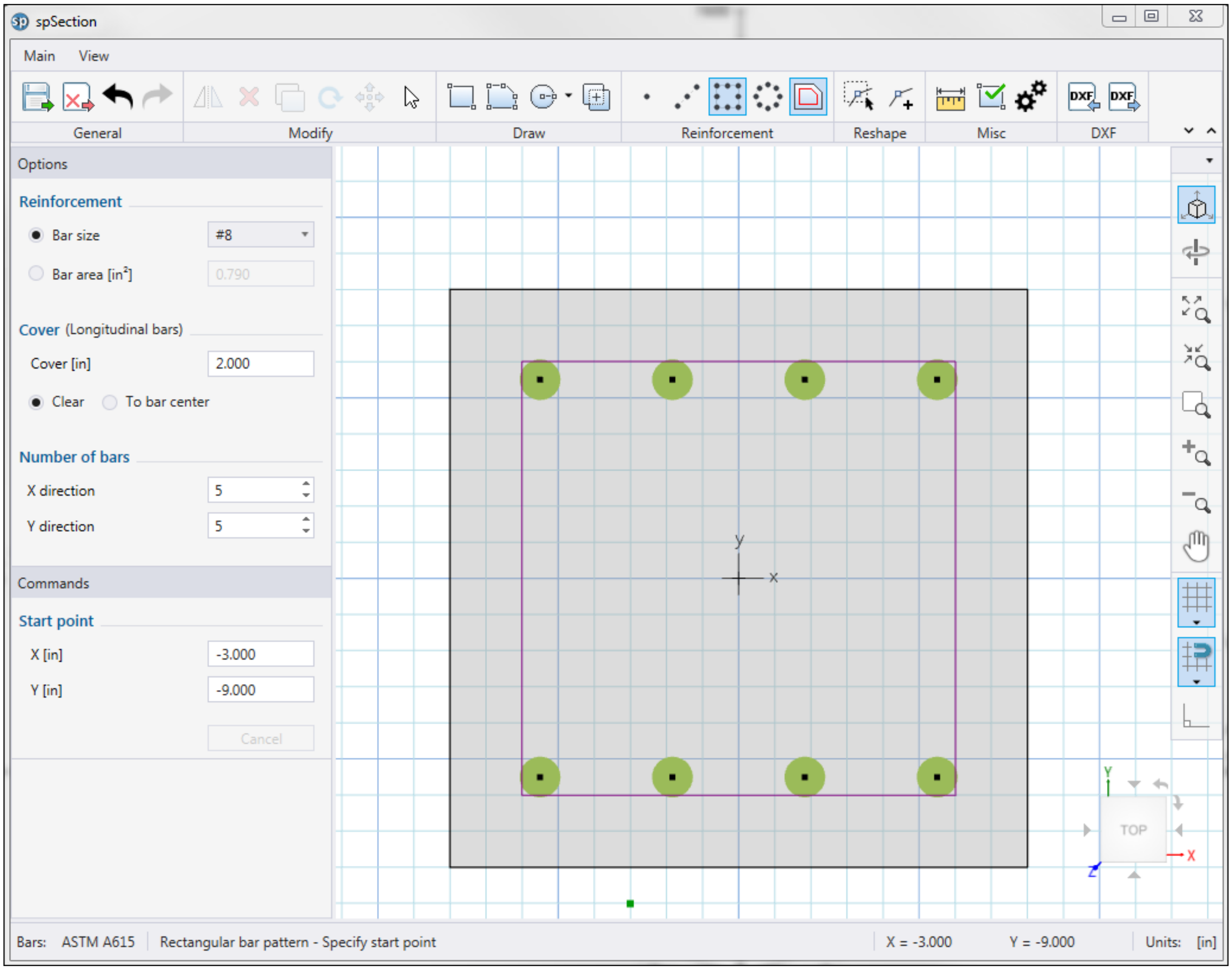

Figure 9 – spColumn Model Editor (spSection)

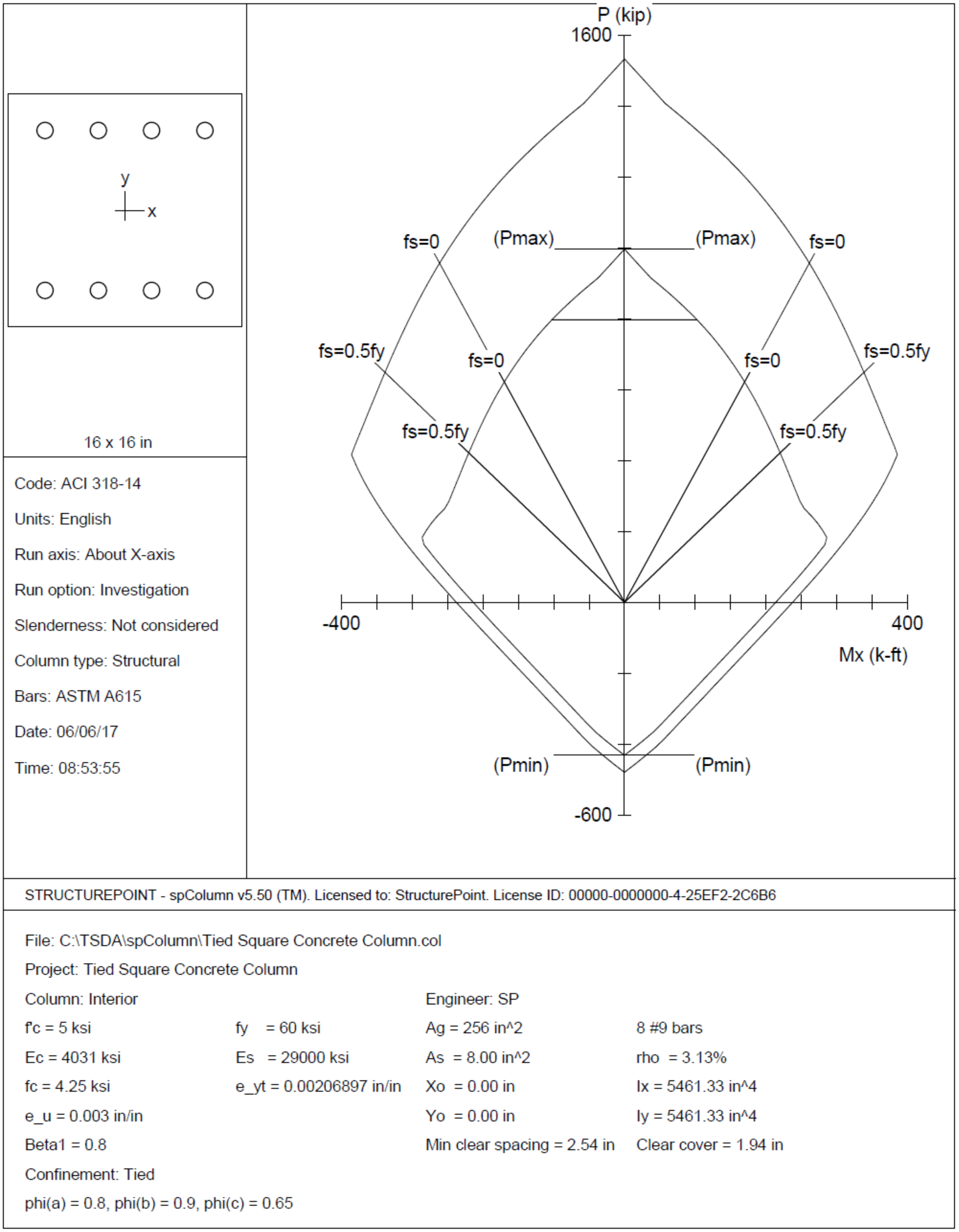

Figure 10 – Column Section Interaction Diagram about the X-Axis (spColumn)

```
STRUCTUREPOINT - spColumn v5.50 (TM)                                                                Page    1
Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-2C6B6                                06/02/17
C:\TSDA\spColumn\Tied Square Concrete Column.col                                                    10:45 AM



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=========================================================================================================
                                          spColumn v5.50 (TM)
                      Computer program for the Strength Design of Reinforced Concrete Sections
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=========================================================================================================


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STRUCTUREPOINT - spColumn v5.50 (TM)                                                                        Page   2
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C:\TSDA\spColumn\Tied Square Concrete Column.col                                                            10:45 AM


General Information:
====================
  File Name: C:\TSDA\spColumn\Tied Square Concrete Column.col
  Project:  Tied Square Concrete Column
  Column:   Interior                                Engineer: SP
  Code:     ACI 318-14                              Units: English

  Run Option: Investigation                         Slenderness: Not considered
  Run Axis:   X-axis                                Column Type: Structural

Material Properties:
====================
  Concrete: Standard                                Steel: Standard
  f'c    = 5 ksi                                    fy     = 60 ksi
  Ec     = 4030.51 ksi                              Es     = 29000 ksi
  fc     = 4.25 ksi                                 Eps_yt = 0.00206897 in/in
  Eps_u = 0.003 in/in
  Beta1 = 0.8

Section:
========
  Rectangular: Width = 16 in                        Depth = 16 in

  Gross section area, Ag =  256 in^2
  Ix =  5461.33 in^4                                Iy =  5461.33 in^4
  rx =  4.6188 in                                   ry =  4.6188 in
  Xo =  0 in                                        Yo =  0 in

Reinforcement:
==============
  Bar Set: ASTM A615
  Size Diam (in) Area (in^2)   Size Diam (in) Area (in^2)   Size Diam (in) Area (in^2)
  ---- --------- -----------   ---- --------- -----------   ---- --------- -----------
  #  3      0.38        0.11   #  4      0.50        0.20   #  5      0.63        0.31
  #  6      0.75        0.44   #  7      0.88        0.60   #  8      1.00        0.79
  #  9      1.13        1.00   # 10      1.27        1.27   # 11      1.41        1.56
  # 14      1.69        2.25   # 18      2.26        4.00

  Confinement: Tied; #3 ties with #10 bars,  #4 with larger bars.
  phi(a) = 0.8,  phi(b) = 0.9,  phi(c) = 0.65

  Layout: Rectangular
  Pattern: Sides Different  (Cover to longitudinal reinforcement)
  Total steel area: As = 8.00 in^2 at rho = 3.13%
  Minimum clear spacing = 2.54 in

                    Top       Bottom        Left       Right
                 --------    --------    --------    --------
    Bars          4  # 9      4  # 9      0  # 9      0  # 9
    Cover(in)      1.936       1.936       1.936       1.936

Control Points:
===============
                          Axial Load P     X-Moment     Y-Moment NA depth Dt depth    eps_t    Phi
  Bending about                    kip         k-ft         k-ft       in       in
  -------------------- ------------ ------------ ------------ -------- -------- -------- ------
   X @ Max compression        997.1         0.00         0.00    43.50    13.50 -0.00207  0.650
     @ Allowable comp.        797.7       102.64         0.00    17.35    13.50 -0.00067  0.650
     @ fs = 0.0               622.3       169.86         0.00    13.50    13.50 -0.00000  0.650
     @ fs = 0.5*fy            421.9       220.05         0.00    10.04    13.50  0.00103  0.650
     @ Balanced point         270.9       250.77         0.00     7.99    13.50  0.00207  0.650
     @ Tension control        175.1       288.06         0.00     5.06    13.50  0.00500  0.900
     @ Pure bending             0.0       213.91         0.00     3.25    13.50  0.00946  0.900
     @ Max tension           -432.0         0.00         0.00     0.00    13.50  9.99999  0.900
```

```
STRUCTUREPOINT - spColumn v5.50 (TM)                                                                Page   3
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  -X @ Max compression         997.1         0.00         0.00     43.50     13.50 -0.00207  0.650
     @ Allowable comp.         797.7      -102.64        -0.00     17.35     13.50 -0.00067  0.650
     @ fs = 0.0                622.3      -169.86        -0.00     13.50     13.50 -0.00000  0.650
     @ fs = 0.5*fy             421.9      -220.05        -0.00     10.04     13.50  0.00103  0.650
     @ Balanced point          270.9      -250.77        -0.00      7.99     13.50  0.00207  0.650
     @ Tension control         175.1      -288.06        -0.00      5.06     13.50  0.00500  0.900
     @ Pure bending              0.0      -213.91        -0.00      3.25     13.50  0.00946  0.900
     @ Max tension            -432.0         0.00         0.00      0.00     13.50  9.99999  0.900

  *** End of output ***
```

9. Summary and Comparison of Design Results

Table 1 - Comparison of Results						
Support	**ϕP_n, kip**			**ϕM_n, kip.ft**		
	Hand	**Reference***	**spColumn**	**Hand**	**Reference***	**spColumn**
Max compression	997	997	997	0	0	0
Allowable compression	798	798	798	---	---	---
f_s = 0.0	622	622	622	170	170	170
f_s = 0.5 f_y	422	422	422	220	220	220
Balanced point	271	271	271	251	251	251
Tension control	175	175	175	288	288	288
Pure bending	0	0	0	214	214	214
Max tension	432	432	432	0	0	0
* Reinforced Concrete Mechanic and Design, 6th Edition, James Wight & MacGregor – Example 11-1						

In all of the hand calculations and the reference used illustrated above, the results are in precise agreement with the automated exact results obtained from the spColumn program.

10. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In the calculation shown above a P-M interaction diagram was generated with moments about the X-Axis (Uniaxial bending). Since the reinforcement in the section is not symmetrical, a different P-M interaction diagram is needed for the other orthogonal direction about the Y-Axis (See the following Figure for the case where f_s = f_y).

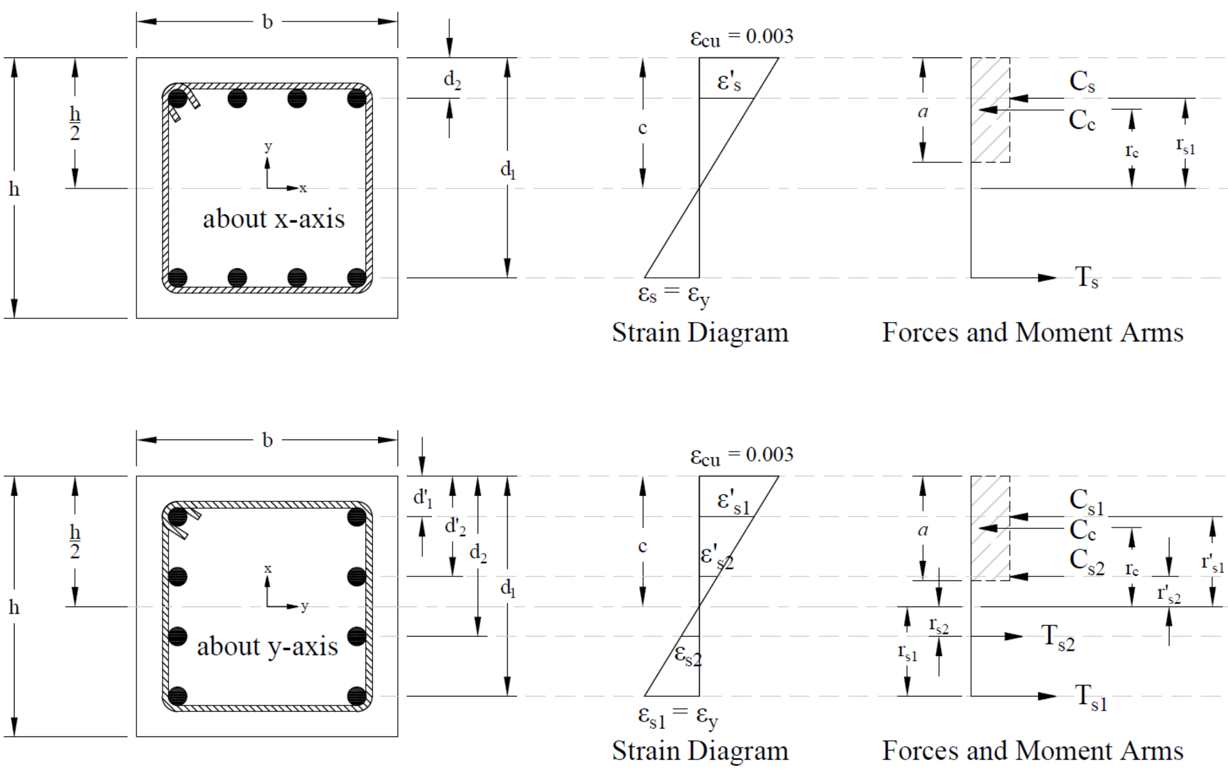

Figure 11 – Strains, Forces, and Moment Arms (f_s = - f_y Moments About x- and y-axis)

When running about the Y-Axis, we have 2 bars in 4 layers instead of 4 bars in just 2 layers (about X-Axis) resulting in a completely different interaction diagram as shown in the following Figure.

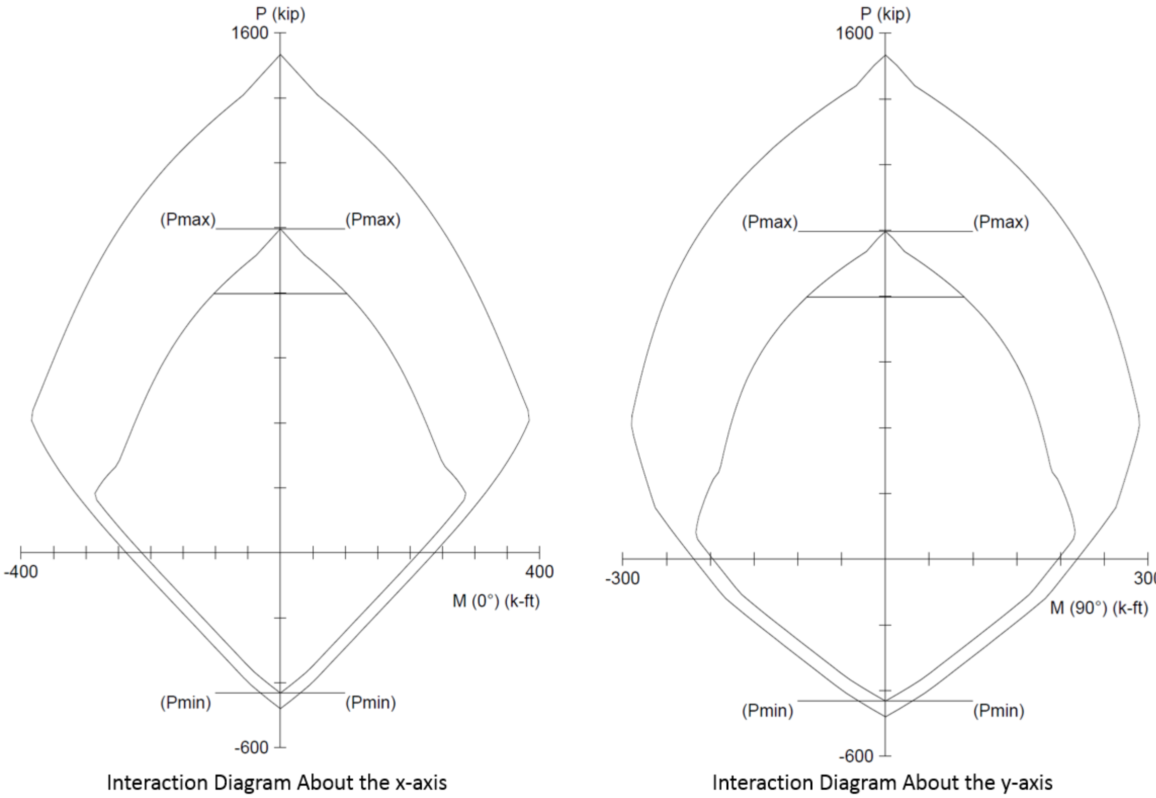

Figure 12 – Comparison of Column Interaction Diagrams about X-Axis and Y-Axis (spColumn)

Further differences in the interaction diagram in both directions can result if the column cross section geometry is irregular.

In most building design calculations, such as the examples shown for flat plate or flat slab concrete floor systems, all building columns are subjected to M_x and M_y due to lateral forces and unbalanced moments from both directions of analysis. This requires an evaluation of the column P-M interaction diagram in two directions simultaneously (biaxial bending).

StucturePoint's spColumn program can also evaluate column sections in biaxial mode to produce the results shown in the following Figure for the column section in this example.

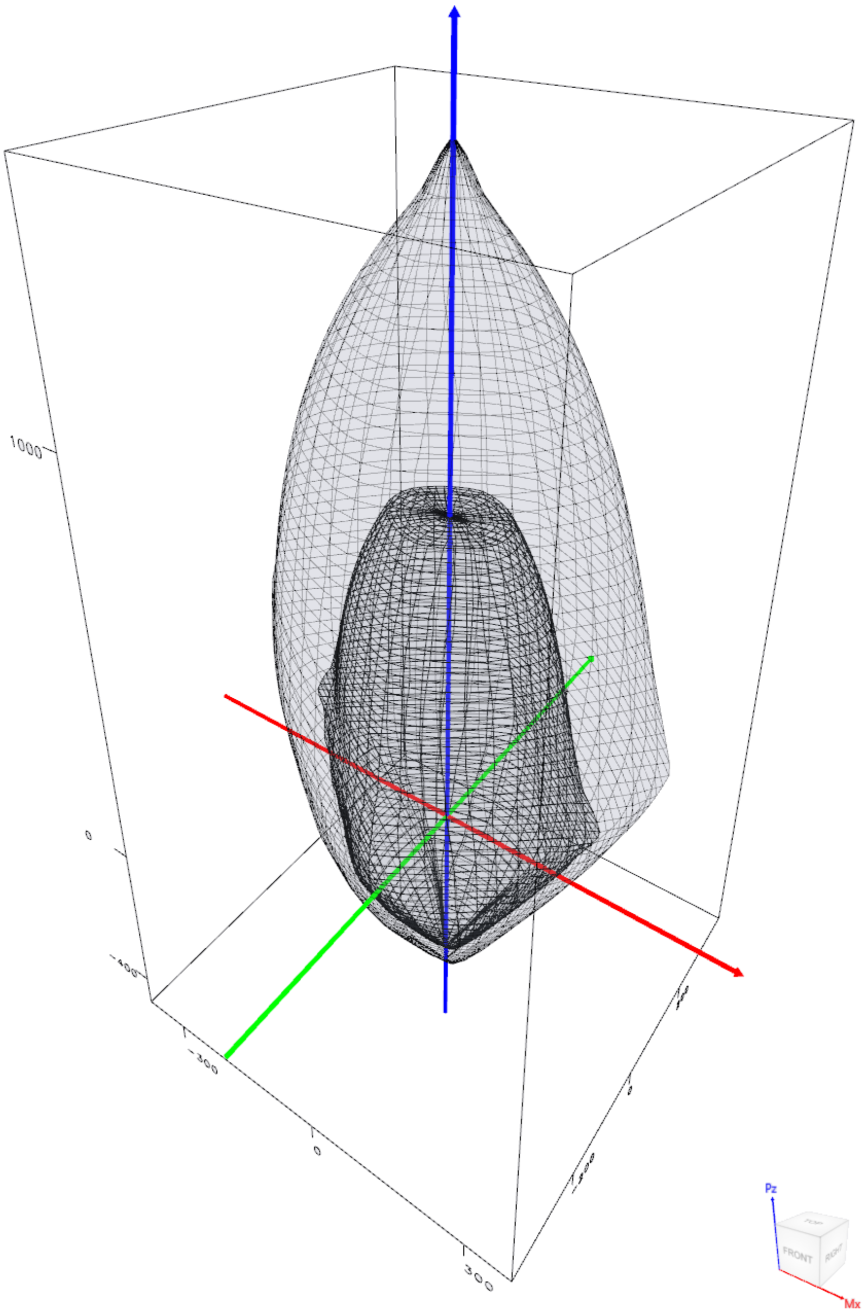

Figure 13 – Nominal & Design Interaction Diagram in Two Directions (Biaxial) (spColumn)