StructurePoint
CONCRETE SOFTWARE SOLUTIONS

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ACI 318-08 Slender Column Requirements in Sway Frames

1. Design information

Units: kips := 1000lbf

Geometry:

Width along X: b := 24·in Width along Y: h := 24·in $A_g := b \cdot h$ $A_g = 576 \cdot in^2$

Reinforcement: 8 - #8, All Sides Equal $I_{se} := 439.09 in^4$

$I_x := \frac{b \cdot h^3}{12}$ $I_x = 27648 \cdot in^4$ $r := \sqrt{\frac{I_x}{A_g}}$ $r = 6.9282 \cdot in$

Clear height: $l_u := 13.33 \cdot ft$ $l_u = 159.96 \cdot in$

Concrete:

$f'_c := 6 \cdot ksi$ $w_c := 150 \frac{lbf}{ft^3}$ $E_c := 57000 \cdot \sqrt{\frac{f'_c}{psi}} \cdot psi$ $E_c = 4415 \cdot ksi$ (ACI 318. 8.5.1)

Reinforcing Steel:

$f_y := 60ksi$ $E_s := 29000ksi$ $0.2 \cdot E_c \cdot I_x + E_s \cdot I_{se} = 3.715 \times 10^7 \cdot kip \cdot in^2$

2. Loading Information

Load Type	Load Factors	Axial Load	Story Shear	Bending Moment (Top & Bottom)	Sustained Load Ratio
Dead Load, D	$f_D := 1.2$	$P_D := 1087.6 \cdot kip$	$\Sigma V_D := 0 \cdot kips$	$M_{D.top} := -2.0 \cdot ft \cdot kips$ $M_{D.bot} := 1.0 \cdot ft \cdot kips$	$St_D := 1$
Live Load, L	$f_L := 0.5$	$P_L := 134.5 \cdot kips$	$\Sigma V_L := 0 \cdot kips$	$M_{L.top} := -15.6 \cdot ft \cdot kips$ $M_{L.bot} := 7.8 \cdot ft \cdot kips$	$St_L := 0$
Snow Load, S	$f_S := 0.5$	$P_S := 17.3 \cdot kips$	$\Sigma V_S := 0 \cdot kips$	$M_{S.top} := 0 \cdot ft \cdot kips$ $M_{S.bot} := 0 \cdot ft \cdot kips$	$St_S := 0$
Wind Load, W	$f_W := 1.6$	$P_W := -0.3 \cdot kips$	$\Sigma V_W := 0 \cdot kips$	$M_{W.top} := 43.5 \cdot ft \cdot kips$ $M_{W.bot} := -205.0 \cdot ft \cdot kips$	$St_W := 0$
Seismic Load, EQ	$f_{EQ} := 0.0$	$P_{EQ} := 0 \cdot kips$	$\Sigma V_{EQ} := 0 \cdot kips$	$M_{EQ.top} := 0 \cdot ft \cdot kips$ $M_{EQ.bot} := 0 \cdot ft \cdot kips$	$St_{EQ} := 0$

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Column loads:

$P_u := f_D \cdot P_D + f_L \cdot P_L + f_S \cdot P_S + f_W \cdot P_W + f_{EQ} \cdot P_{EQ}$ $P_u = 1380.54 \cdot kips$

$P_{u.st} := f_D \cdot P_D \, St_D + f_L \cdot P_L \, St_L + f_S \cdot P_S \, St_S + f_W \cdot P_W \cdot St_W + f_{EQ} \cdot P_{EQ} \cdot St_{EQ}$ $P_{u.st} = 1305.12 \cdot kips$

Factored end moments due to loads that cause no appreciable sidesway

$M_{ns.top} := f_D \cdot M_{D.top} + f_L \cdot M_{L.top} + f_S \cdot M_{S.top}$ $M_{ns.top} = -10.2 \cdot ft \cdot kips$

$M_{ns.bot} := f_D \cdot M_{D.bot} + f_L \cdot M_{L.bot} + f_S \cdot M_{S.bot}$ $M_{ns.bot} = 5.1 \cdot ft \cdot kips$

Factored end moments due to loads that cause appreciable sidesway

$M_{s.top} := f_W \cdot M_{W.top} + f_{EQ} \cdot M_{EQ.top}$ $M_{s.top} = 69.6 \cdot ft \cdot kips$

$M_{s.bot} := f_W \cdot M_{W.bot} + f_{EQ} \cdot M_{EQ.bot}$ $M_{s.bot} = -328 \cdot ft \cdot kips$

Factored total end moments

$M_{u.top} := M_{ns.top} + M_{s.top}$ $M_{u.top} = 59.4 \cdot ft \cdot kips$

$M_{u.bot} := M_{ns.bot} + M_{s.bot}$ $M_{u.bot} = -322.9 \cdot ft \cdot kips$

Story loads for all columns:

The total factored vertical story load: $\Sigma P_u := 16.37 \cdot P_u$ $\frac{\Sigma P_u}{P_u} = 16.37$ $\Sigma P_u = 22599.44 \cdot kips$

$\Sigma V_u := f_D \cdot \Sigma V_D + f_L \cdot \Sigma V_L + f_S \cdot \Sigma V_S + f_W \cdot \Sigma V_W + f_{EQ} \cdot \Sigma V_{EQ}$

$\Sigma V_{u.st} := f_D \cdot \Sigma V_D \, St_D + f_L \cdot \Sigma V_L \, St_L + f_S \cdot \Sigma V_S \, St_S + f_W \cdot \Sigma V_W \cdot St_W + f_{EQ} \cdot \Sigma V_{EQ} \cdot St_{EQ}$

$\Sigma V_u = 0 \cdot kips$ $\Sigma V_{u.st} = 0 \cdot kips$

3. Determine column designation as sway/nonsway and slenderness considerations

Columns and stories within a structure are designated sway or nonsway in accordance with the stability index for a story, Q, per ACI 318 10.10.5. Consider a column in a sway frame/story.

Determine k_{ns} (nonsway frames) and k_s (sway frames) from the alignment charts [ACI 318 Fig. 10.10.1.1. (a) and (b)] respectively or alternate simplified equations used in spColumn Software.

$k_{ns} := 1.00$ $k_s := 1.82$ $\frac{k_s \cdot l_u}{r} = 42.021$

Slenderness effects must be considered in sway frames if $k_s l_u / r > 22$ (ACI 318-08 10.10.1.a)

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4. Moment magnification at ends of compression member (Sway) ACI 318-08 10.10.7

Magnify column moments per 10.10.7 for second order effects at ends of member as required by 10.10.5

$\beta_{ds} := \frac{\Sigma V_{u.st}}{\Sigma V_u}$ $\beta_{ds} = 0.000$ (ACI Cl. 10.10.4.2)

$EI_s := \left[\frac{\left(0.2 \cdot E_c \cdot I_x + E_s \cdot I_{se}\right)}{\left(1 + \beta_{ds}\right)}\right]$ $EI_s = 3.71 \times 10^7 \cdot kip \cdot in^2$ (ACI Eq. 10-14)

Critical Load: $P_{c.s} := \frac{\pi^2 \cdot EI_s}{\left(k_s \cdot l_u\right)^2}$ $P_{c.s} = 4325.82 \cdot kip$ (ACI Eq. 10-13)

The sum of critical load in all columns in the story considered: $\Sigma P_{c.s} := 24.75 \cdot P_{c.s}$ $\frac{\Sigma P_{c.s}}{P_{c.s}} = 24.75$ $\Sigma P_{c.s} = 107064.06 \cdot kips$

$\delta_s := \frac{1.0}{1 - \frac{\Sigma P_u}{0.75 \cdot \Sigma P_{c.s}}}$ $\delta_s = 1.392$ (ACI Eq. 10-21)

Magnify column moments M_1 and M_2 due to second order effects at the ends:

$M_{u.top.second} := M_{ns.top} + \delta_s \cdot M_{s.top}$ $M_{u.top.second} = 86.661 \cdot ft \cdot kips$

$M_{u.bot.second} := M_{ns.bot} + \delta_s \cdot M_{s.bot}$ $M_{u.bot.second} = -451.371 \cdot ft \cdot kips$

Set M2 to the larger of the absolute value of top and bot second order magnified moments.

$M_{2.second} := \max\left(\left|M_{u.top.second}\right|, \left|M_{u.bot.second}\right|\right)$ $M_{2.second} = 451.371 \cdot ft \cdot kips$

$M_{1.second} := \min\left(\left|M_{u.top.second}\right|, \left|M_{u.bot.second}\right|\right)$ $M_{1.second} = 86.661 \cdot ft \cdot kips$

Determine the first order factored moment, $M_{2.first}$ at the end at which the larger second order end moment, M_2.second acts.

$M_{2.first} := \text{if}\left(\left|M_{u.top.second}\right| \geq \left|M_{u.bot.second}\right|, \left|M_{u.top}\right|, \left|M_{u.bot}\right|\right)$ $M_{2.first} = 322.9 \cdot ft \cdot kips$

$M_{1.first} := \text{if}\left(\left|M_{u.top.second}\right| \geq \left|M_{u.bot.second}\right|, \left|M_{u.bot}\right|, \left|M_{u.top}\right|\right)$ $M_{1.first} = 59.4 \cdot ft \cdot kips$

The ratio of second order moments to first order moments after moment magnification at column ends (sway effect) is:

$\frac{M_{2.second}}{M_{2.first}} = 1.398$ $\frac{M_{1.second}}{M_{1.first}} = 1.459$

The next step in this calculation was not required in earlier codes unless the parameter $k"l_u/r$ exceeded 35 as previously given in 318-05 10.13.5. This condition amplified the design moment but no stability check was required except for ACI 318-05 10.13.6.

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5. Moment magnification along length of compression member (nonsway) ACI 318-08 10.10.6

Continue to magnify column moments per 10.10.6 for second order effects along the length of member as required by 10.10.2.2.

Determine stiffness reduction factor for sustained axial load $\beta_{dns} := \min\left(\frac{P_{u.st}}{P_u}, 1.0\right)$ $\beta_{dns} = 0.945$ (ACI 10.10.6.2)

$EI_{ns} := \left[\frac{\left(0.2 \cdot E_c \cdot I_x + E_s \cdot I_{se}\right)}{\left(1 + \beta_{dns}\right)}\right]$ $EI_{ns} = 1.91 \times 10^7 \cdot kip \cdot in^2$ (ACI Eq. 10-14)

$P_{c.ns} := \frac{\pi^2 \cdot EI_{ns}}{\left(k_{ns} \cdot l_u\right)^2}$ $P_{c.ns} = 7365.62 \cdot kips$ (ACI Eq. 10-13)

Determine and select whether the column is bent in single (1) or double (2) curvature due to second order moments. $Curve := 2$

If column is bent in single curvature, M_1, the smaller of top and bot second order magnified moment is to be taken as positive. Otherwise, M_1 is to be taken as negative.

$M_{1.second} := if\left(Curve = 1, M_{1.second}, -M_{1.second}\right)$ $M_{1.second} = -86.661\ ft \cdot kips$

$C_m := 0.6 + 0.4\frac{M_{1.second}}{M_{2.second}}$ $C_m = 0.523$ (ACI Eq. 10-16)

Set C_m to 1.0 if M_2, or M_1 and M_2 are equal to zero. $C_m := if\left(M_{2.second} = 0, 1, 0.6 + 0.4 \cdot \frac{M_{1.second}}{M_{2.second}}\right)$ $C_m = 0.523$

$C_m := \max\left(C_m, 0.4\right)$ $C_m = 0.523$ $C_m := \min\left(C_m, 1\right)$ $C_m = 0.523$

$\delta := \max\left[1, \frac{C_m}{1 - \left(\frac{P_u}{0.75 \cdot P_{c.ns}}\right)}\right]$ $\delta = 1.000$ (ACI Eq. 10-12)

Check factored moment M_2 is not less than required by 10.10.6.5 equation 10-17 for minimum moment:

$M_{2.second} = 451.371\ ft \cdot kips$ $M_{2.first} = 322.9\ ft \cdot kips$

$M_{min} := P_u \cdot (0.6in + 0.03 \cdot h)$ $M_{min} = 151.86 \cdot ft \cdot kips$

Determine final moment value amplified per equation 10-11:

$M_2 := \max\left(M_{2.second}, M_{min}\right)$ $M_2 = 451.37 \cdot ft \cdot kips$

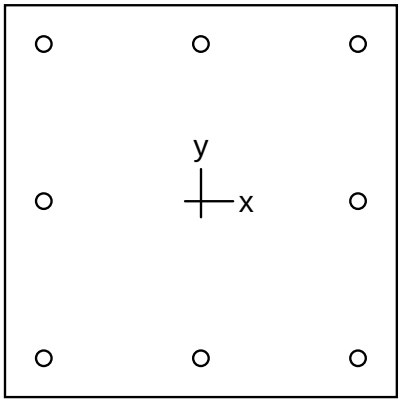
$M_c := \delta \cdot M_2$ $M_c = 451.37 \cdot ft \cdot kips$

Check the ratio of second order moment including further moment magnification effects along the length of compression member (ACI 318-08 10.10.2.2) to first order moments. ACI 318-08 10.10.2.1. requires the ratio not exceed 1.40.

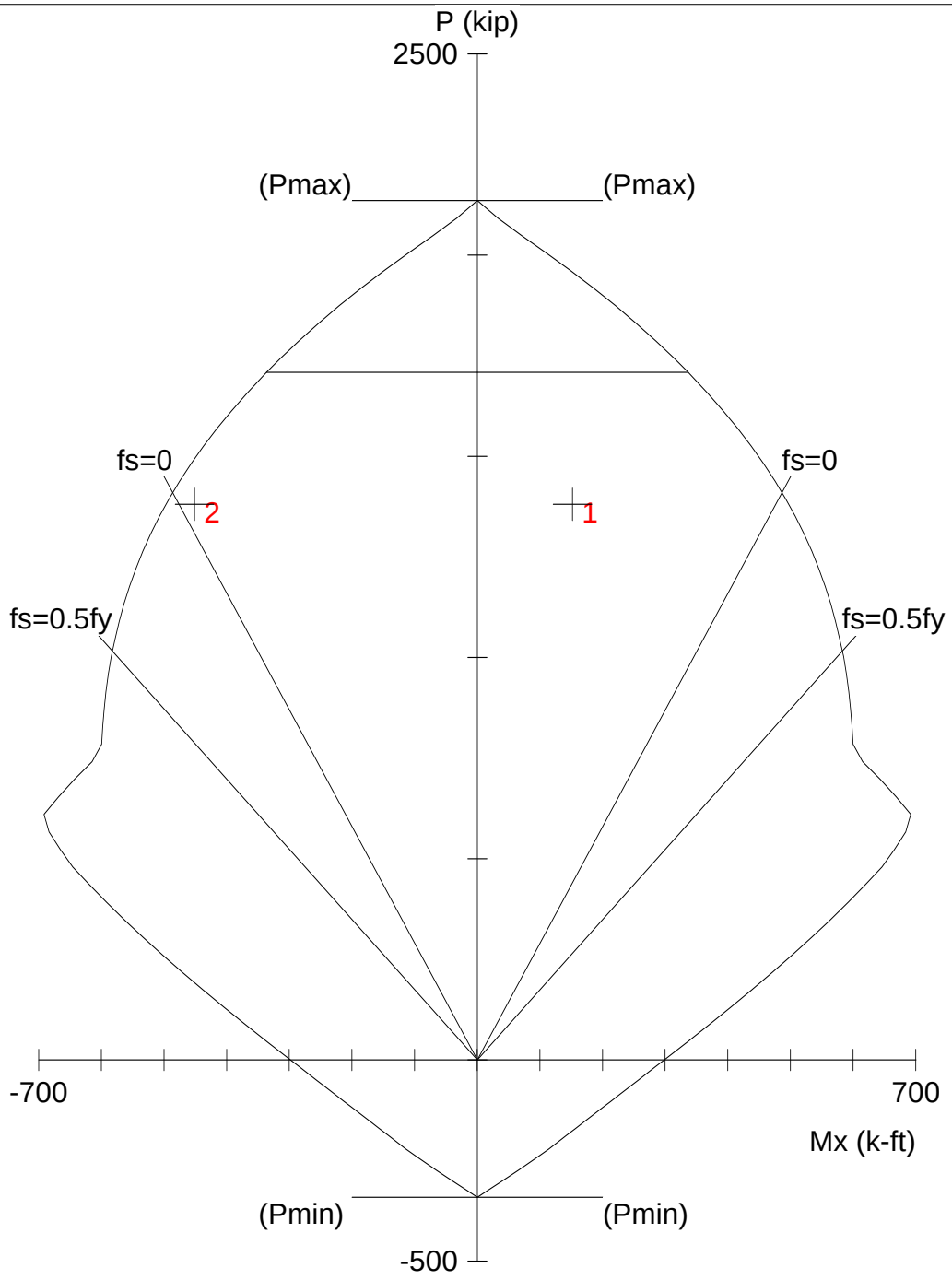
$\frac{M_c}{\max\left(M_{2.first}, M_{min}\right)} = 1.398$

6. Check column strength based on spColumn output

$\phi P_{n.max} := 1708.00 \cdot kips$ $\frac{P_u}{\phi P_{n.max}} = 0.81$ OK $\phi M_{nx} := 496.87 \cdot ft \cdot kips$ $\frac{M_c}{\phi M_{nx}} = 0.91$ OK



24 x 24 in



Code: ACI 318-08

Units: English

Run axis: About X-axis

Run option: Investigation

Slenderness: Considered

Column type: Structural

Bars: ASTM A615

Date: 06/24/10

Time: 13:55:43

File: C:\Users\mdeger\Documents\spColumn v4.60 - ACI 318-08 Cl 10.10.2.2 Implementation\Original MathCad...\CASE A.col

Project: PCA Notes 08 Ex 11.2-L Combo 6

Column: C2

Engineer: SP

f'c = 6 ksi	fy = 60 ksi	Ag = 576 in^2	8 #8 bars
Ec = 4415 ksi	Es = 29000 ksi	As = 6.32 in^2	rho = 1.10%
fc = 5.1 ksi		Xo = 0.00 in	Ix = 27648 in^4
e_u = 0.003 in/in		Yo = 0.00 in	Iy = 27648 in^4
Beta1 = 0.75		Min clear spacing = 8.63 in	Clear cover = 1.88 in
Confinement: Tied		kx(nonsway) = 1	kx(sway) = 1.82

phi(a) = 0.8, phi(b) = 0.9, phi(c) = 0.65

General Information:
==========
File Name: C:\Users\mdeger\Documents\spColumn v4.60 - ACI 318-08 Cl 10.10.2.2 Implemen...\CASE A.col
Project: PCA Notes 08 Ex 11.2-L Combo 6
Column: C2 Engineer: SP
Code: ACI 318-08 Units: English

Run Option: Investigation Slenderness: Considered
Run Axis: X-axis Column Type: Structural

Material Properties:
==========
f'c = 6 ksi fy = 60 ksi
Ec = 4415.21 ksi Es = 29000 ksi
Ultimate strain = 0.003 in/in
Beta1 = 0.75

Section:
========
Rectangular: Width = 24 in Depth = 24 in

Gross section area, Ag = 576 in^2
Ix = 27648 in^4 Iy = 27648 in^4
rx = 6.9282 in ry = 6.9282 in
Xo = 0 in Yo = 0 in

Reinforcement:
==========
Bar Set: ASTM A615

Size	Diam (in)	Area (in^2)	Size	Diam (in)	Area (in^2)	Size	Diam (in)	Area (in^2)
# 3	0.38	0.11	# 4	0.50	0.20	# 5	0.63	0.31
# 6	0.75	0.44	# 7	0.88	0.60	# 8	1.00	0.79
# 9	1.13	1.00	# 10	1.27	1.27	# 11	1.41	1.56
# 14	1.69	2.25	# 18	2.26	4.00			

Confinement: Tied; #3 ties with #11 bars, #4 with larger bars.
phi(a) = 0.8, phi(b) = 0.9, phi(c) = 0.65

Layout: Rectangular
Pattern: All Sides Equal (Cover to transverse reinforcement)
Total steel area: As = 6.32 in^2 at rho = 1.10%
Minimum clear spacing = 8.63 in

8 #8 Cover = 1.5 in

Service Loads:
==========

No.	Load Case	Axial Load kip	Mx @ Top k-ft	Mx @ Bot k-ft	My @ Top k-ft	My @ Bot k-ft
1	Dead	1087.60	-2.00	-1.00	0.00	0.00
	Live	134.50	-15.60	-7.80	0.00	0.00
	Wind	-0.30	43.50	205.00	0.00	0.00
	EQ	0.00	0.00	0.00	0.00	0.00
	Snow	17.30	0.00	0.00	0.00	0.00

Sustained Load Factors:
==========

Load Case	Factor (%)
Dead	100
Live	0
Wind	0
EQ	0
Snow	0

Load Combinations:
==========
U1 = 1.200*Dead + 0.500*Live + 1.600*Wind + 0.000*EarthQuake + 0.500*Snow

Slenderness:
==========
Sway Criteria:

X-axis: Sway column. SumPc = 24.75 * Pc SumPu = 16.37 * Pu
Second-order effects along length considered

Column	Axis	Height ft	Width in	Depth in	I in^4	f'c ksi	Ec ksi
Design	X	13.33	24	24	27648	6	4415.21
Above	X	(no column specified...)					
Below	X	(no column specified...)					

X-Beams Location	Axis	Length ft	Width in	Depth in	I in^4	f'c ksi	Ec ksi
Above Left		(no beam specified...)					
Above Right		(no beam specified...)					
Below Left		(no beam specified...)					
Below Right		(no beam specified...)					

Effective Length Factors:

Axis	Psi(top)	Psi(bot)	k(Nonsway)	k(Sway)	klu/r
X	0.000	0.000	1.000	1.820	42.02

Moment Magnification Factors:

Stiffness reduction factor, phi(K) = 0.75
Cracked-section coefficients: cI(beams) = 0.35; cI(columns) = 0.7

0.2*Ec*Ig + Es*Ise (X-axis) = 3.71e+007 kip-in^2

X-axis Ld/Comb	At Ends SumPu(kip)	Pc(kip)	SumPc(kip)	Betads	Deltas	Along Length Pu(kip)	k'lu/r	Pc(kip)	Betad	Cm	Delta
1 U1	22599.44	4325.92	107066.45	0.000	1.392	1380.54	(N/A)	7365.78	0.945	0.523	1.000

Factored Moments due to First-Order and Second-Order Effects:

Minimum eccentricity, Ex,min = 1.32 in

NOTE: Each loading combination includes the following cases:
First line - at column top
Second line - at column bottom

X-axis Load Combo	1st Order Mns k-ft	Ms k-ft	Mu k-ft	Mmin k-ft		2nd Order Mi k-ft	Mc k-ft	Ratio - 2nd/1st
1 U1	-10.20	69.60	59.40	151.86	M1=	86.66	151.86	1.000
	5.10	-328.00	-322.90	-151.86	M2=	-451.37	-451.37	1.398

Factored Loads and Moments with Corresponding Capacities:

NOTE: Each loading combination includes the following cases:
First line - at column top
Second line - at column bottom

No.	Load Combo	Pu kip	Mux k-ft	PhiMnx k-ft	PhiMn/Mu	NA depth in	Dt depth in	eps_t	Phi
1	1 U1	1380.54	151.86	496.87	3.272	21.20	21.63	0.00006	0.650
2		1380.54	-451.37	-496.87	1.101	21.20	21.63	0.00006	0.650

*** End of output ***

General Information:
==========
File Name: C:\Users\mdeger\Documents\spColumn v4.60 - ACI 318-08 Cl 10....\CASE A-Control Points.col
Project: PCA Notes 08 Ex 11.2-L Combo 6
Column: C2 Engineer: SP
Code: ACI 318-08 Units: English

Run Option: Investigation Slenderness: Not considered
Run Axis: X-axis Column Type: Structural

Material Properties:
==========
f'c = 6 ksi fy = 60 ksi
Ec = 4415.21 ksi Es = 29000 ksi
Ultimate strain = 0.003 in/in
Beta1 = 0.75

Section:
==========
Rectangular: Width = 24 in Depth = 24 in

Gross section area, Ag = 576 in^2
Ix = 27648 in^4 Iy = 27648 in^4
rx = 6.9282 in ry = 6.9282 in
Xo = 0 in Yo = 0 in

Reinforcement:
==========
Bar Set: ASTM A615

Size	Diam (in)	Area (in^2)	Size	Diam (in)	Area (in^2)	Size	Diam (in)	Area (in^2)
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# 6	0.75	0.44	# 7	0.88	0.60	# 8	1.00	0.79
# 9	1.13	1.00	# 10	1.27	1.27	# 11	1.41	1.56
# 14	1.69	2.25	# 18	2.26	4.00			

Confinement: Tied; #3 ties with #11 bars, #4 with larger bars.
phi(a) = 0.8, phi(b) = 0.9, phi(c) = 0.65

Layout: Rectangular
Pattern: All Sides Equal (Cover to transverse reinforcement)
Total steel area: As = 6.32 in^2 at rho = 1.10%
Minimum clear spacing = 8.63 in

8 #8 Cover = 1.5 in

Control Points:
==========

Bending about	Axial Load P kip	X-Moment k-ft	Y-Moment k-ft	NA depth in	Dt depth in	eps_t	Phi
X @ Max compression	2135.0	0.00	0.00	69.68	21.63	-0.00207	0.650
@ Allowable comp.	1708.0	336.54	-0.00	26.10	21.63	-0.00051	0.650
@ fs = 0.0	1409.5	486.20	0.00	21.63	21.63	0.00000	0.650
@ fs = 0.5*fy	1015.3	582.25	0.00	16.08	21.63	0.00103	0.650
@ Balanced point	761.4	600.22	0.00	12.80	21.63	0.00207	0.650
@ Tension control	599.8	696.78	0.00	8.11	21.63	0.00500	0.900
@ Pure bending	-0.0	298.42	0.00	2.48	21.63	0.02312	0.900
@ Max tension	-341.3	-0.00	-0.00	0.00	21.63	9.99999	0.900
-X @ Max compression	2135.0	-0.00	-0.00	69.68	21.63	-0.00207	0.650
@ Allowable comp.	1708.0	-336.54	-0.00	26.10	21.63	-0.00051	0.650
@ fs = 0.0	1409.5	-486.20	-0.00	21.63	21.63	0.00000	0.650
@ fs = 0.5*fy	1015.3	-582.25	0.00	16.08	21.63	0.00103	0.650
@ Balanced point	761.4	-600.22	0.00	12.80	21.63	0.00207	0.650
@ Tension control	599.8	-696.78	0.00	8.11	21.63	0.00500	0.900
@ Pure bending	-0.0	-298.42	-0.00	2.48	21.63	0.02312	0.900
@ Max tension	-341.3	-0.00	-0.00	0.00	21.63	9.99999	0.900

*** End of output ***