Design of a Continuous Beam using Moment Redistribution

Determine the required reinforcement areas for the spandrel beam at an intermediate floor level as shown, using moment redistribution provisions in ACI 318, Chapter 8 to optimize and reduce total reinforcement required.

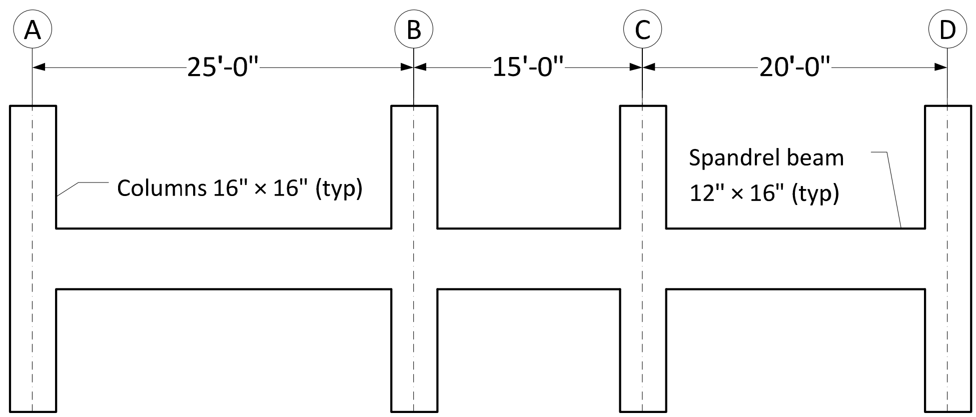


Figure 1 – Concrete Floor Continuous Beam Configuration

Code

Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary (ACI 318R-11)

Reference

PCA Notes on ACI 318-08, Tenth Edition, 2008 Portland Cement Association, Example 8.2

Design Data

Columns $= 16 \times 16$ in

Story height $= 10$ ft

Spandrel beam $= 12 \times 16$ in

$f'_c = 4000$ psi

$f_y = 60000$ psi

Dead Load, $DL = 1167$ lb/ft

Live Load, $LL = 450$ lb/ft

Solution

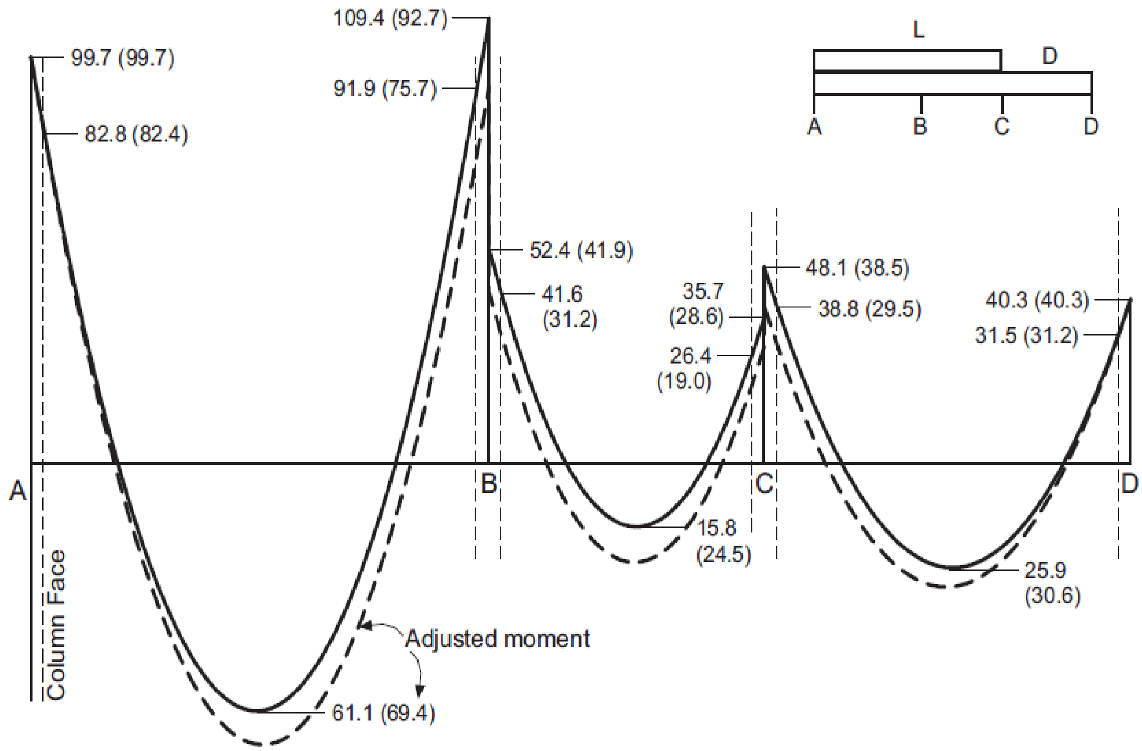
1. Determine the elastic bending moment diagrams for the load patterns per ACI 318-11, 8.11 as shown in Figure 2(a) to (e) and the maximum moment envelope values for all patterns. Factored Loads are:

$$U = 1.2D + 1.6L$$
$$w_d = 1.2 \times 1.167 = 1.4 \text{ kips/ft}$$

$w_\ell = 1.6 \times 0.45 = 0.72$ kips/ft

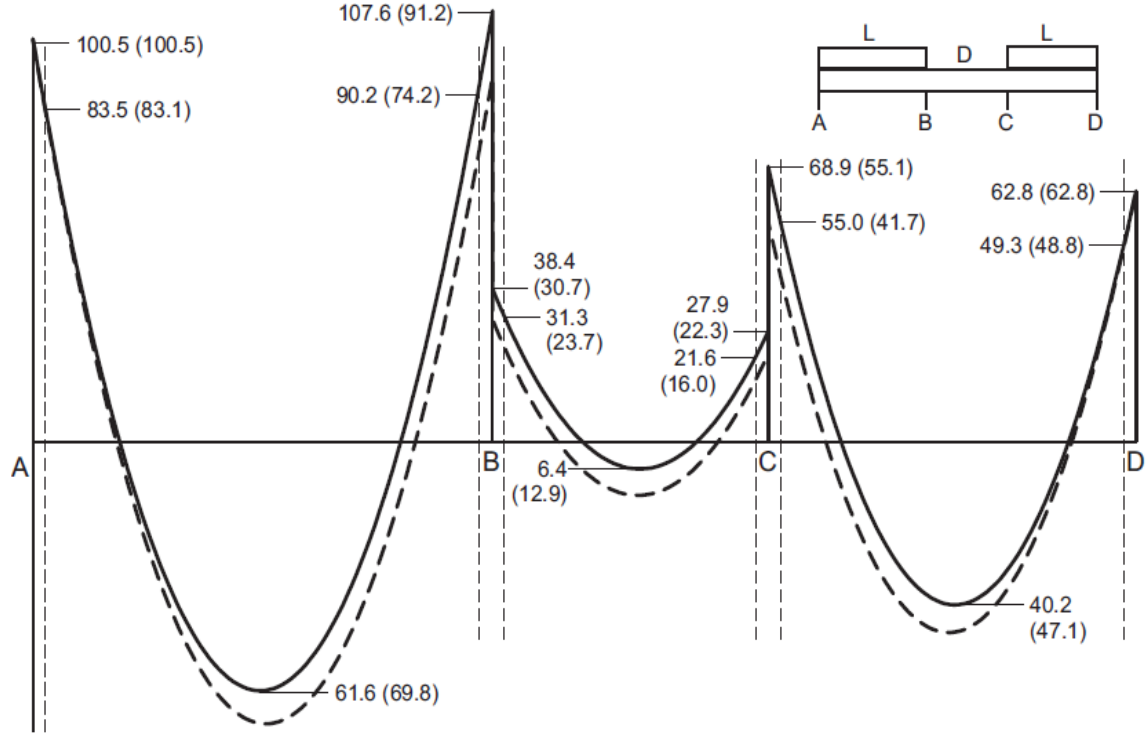
$w_u = 2.12$ kips/ft

Maximum negative moments at column centerlines and column faces, and positive mid-span moments were determined by computer analysis using spBeam program for each of the five loading configurations. Adjusted moments after redistribution are also shown by dashed lines. The values of the adjusted moments are given in parentheses.

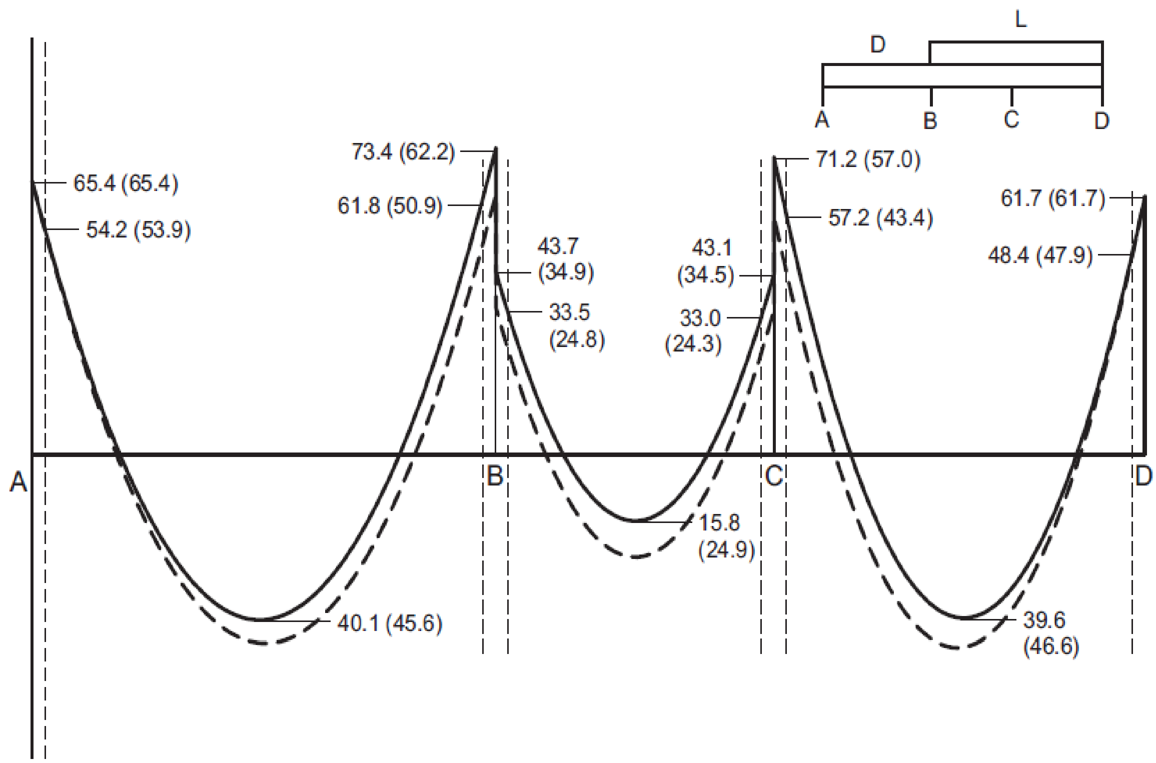


(a) Load Pattern I (moments in ft-kips)

<u>Figure 2 - Redistribution of Moments</u>

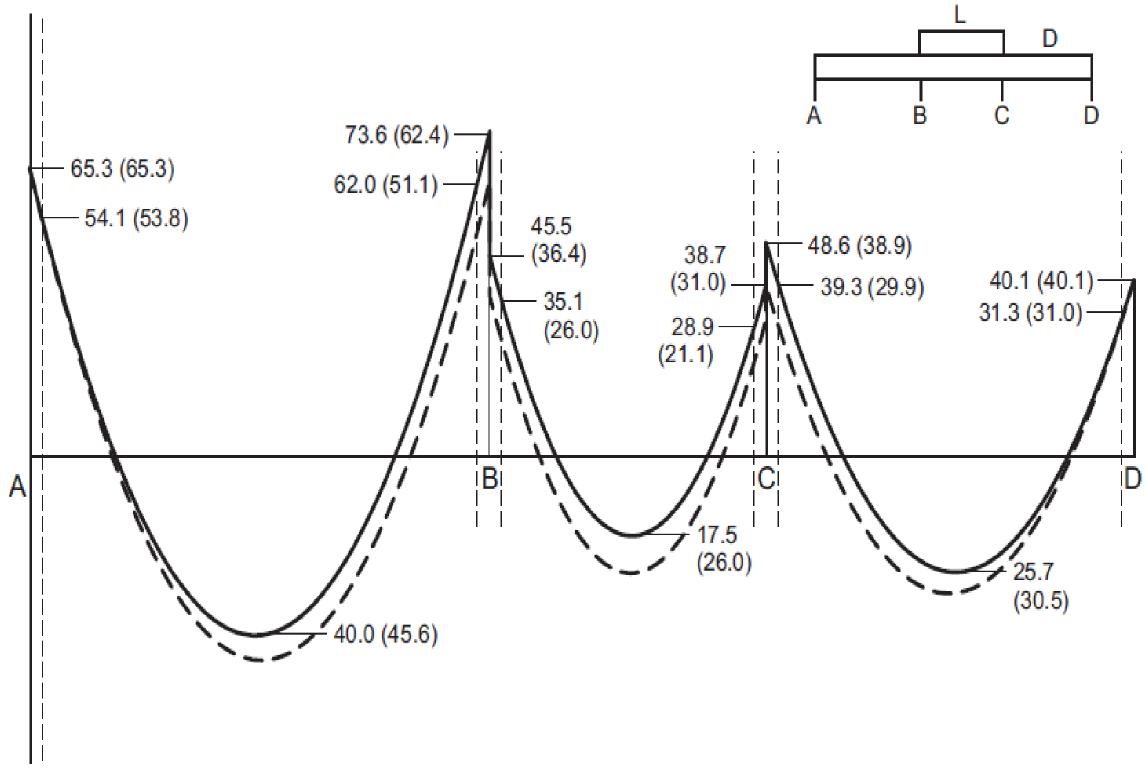


(b) Load Pattern II (moments in ft-kips)

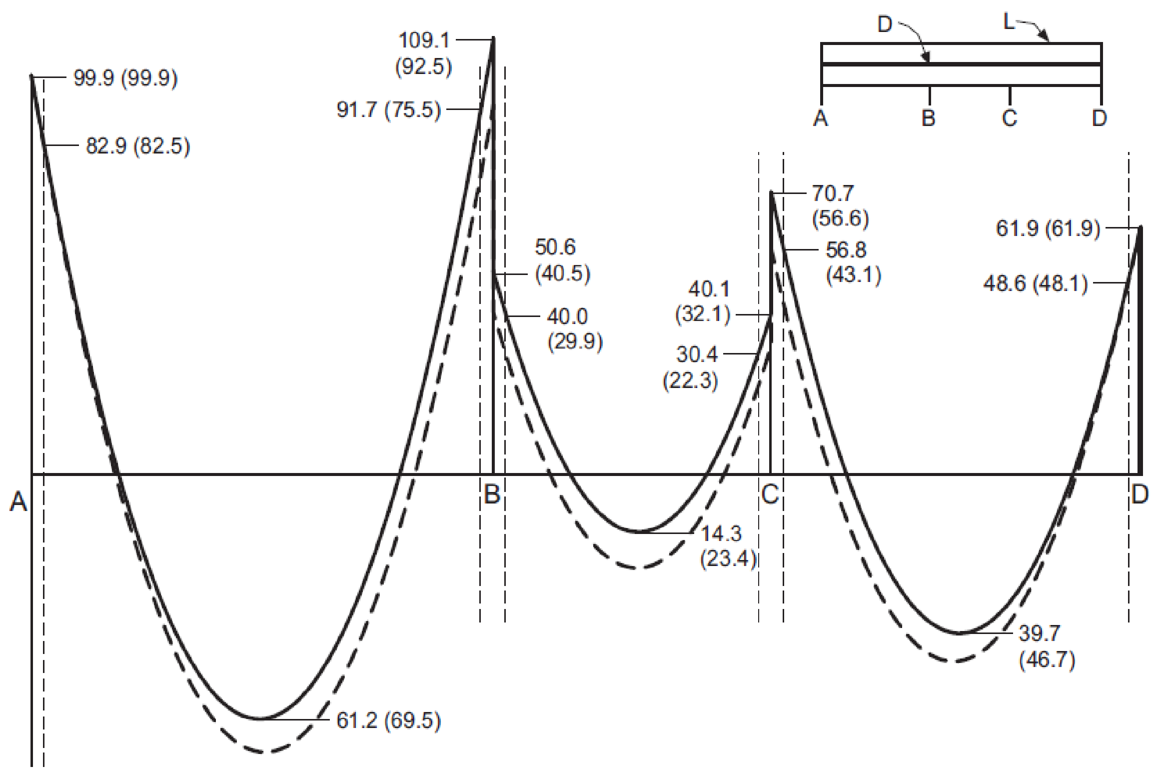


(c) Load Pattern III (moments in ft-kips)

Figure 2 (continued) - Redistribution of Moments

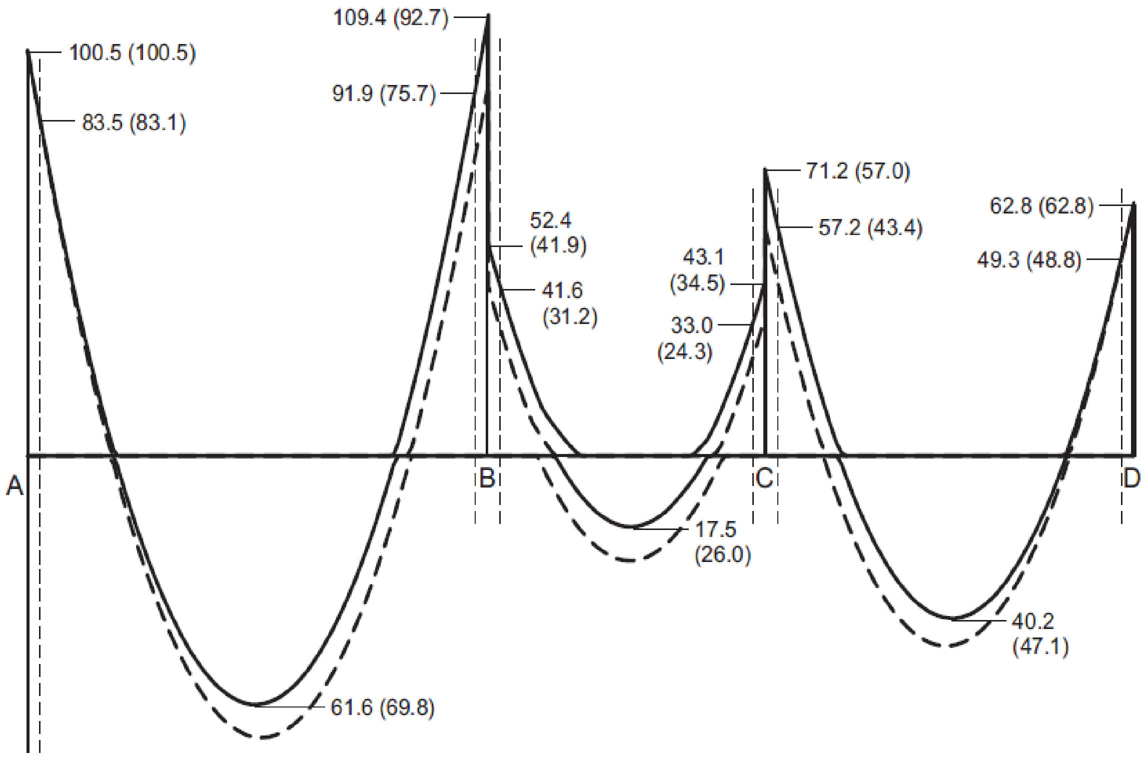


(d) Load Pattern IV (moments in ft-kips)



(e) Load Pattern V (moments in ft-kips)

Figure 2 (continued) - Redistribution of Moments



(f) Maximum Moment Envelopes for Pattern Loading (moments in ft-kips)

Figure 2 (continued) Redistribution of Moments

2. Determine maximum allowable percentage decrease in negative moments :

use $d = 14.0$ in ; $cover = 1.5$ in.

Calculate $\frac{R_n}{f'_c} = \frac{M_u}{\phi f'_c b d^2}$ and corresponding $\varepsilon_t = 0.003\left(\frac{\beta_1}{1-\sqrt{1-\frac{40}{17}\frac{R_n}{f'_c}}}-1\right)$.

For M_u , use envelope value at support face. Based on ε_t , calculate the adjustment. Iterate until the adjusted moments converge. (See Table 1 below)

Table 1 Moment Adjustments at Supports							
		Support					
		A	B		C		D
		Right	Left	Right	Left	Right	Left
Iteration 1	M_u (ft-kips)	83.5	91.9	41.6	33.0	57.2	49.3
	R_n / f'_c	0.1184	0.1303	0.0589	0.0467	0.0811	0.0699
	ε_t	0.0139	0.0122	0.0325	0.0421	0.0224	0.0267
	Adjustment (%)	13.9	12.2	20.0	20.0	20.0	20.0
Iteration 2	M_u (ft-kips)	71.9	80.7	33.3	26.4	45.8	39.4
	R_n / f'_c	0.1019	0.1143	0.0471	0.0374	0.0649	0.0559
	ε_t	0.0169	0.0146	0.0417	0.0537	0.0291	0.0345
	Adjustment (%)	16.9	14.6	20.0	20.0	20.0	20.0
Iteration 3	M_u (ft-kips)	69.4	78.5				
	R_n / f'_c	0.0984	0.1113				
	ε_t	0.0177	0.0151				
	Adjustment (%)	17.7	15.1				
Iteration 4	M_u (ft-kips)	68.8	78.0				
	R_n / f'_c	0.0975	0.1106				
	ε_t	0.0179	0.0152				
	Adjustment (%)	17.9	15.2				
Iteration 5	M_u (ft-kips)	68.6	77.9				
	R_n / f'_c	0.0972	0.1104				
	ε_t	0.0179	0.0153				
	Adjustment (%)	17.9	15.3				
Iteration 6	M_u (ft-kips)		77.9				
	R_n / f'_c		0.1104				
	ε_t		0.0153				
	Adjustment (%)		15.3				
Final Allowable Adjustment (%)		17.9	15.3	20.0	20.0	20.0	20.0

3. Adjustment of Moments and Reinforcement Ratios

Adjustments of moments, is a decision to be made by the engineer. Here, it is decided to reduce the negative moments on both sides of supports B and C and accept the increase in the corresponding positive moments, and not to adjust the negative moments at the exterior supports A and D.

Referring to Figs. 2 (a) through (e), the following adjustments in moments are made.

Load Pattern I – Fig. 2 (a)

$M_{B,Left} = 109.4$ ft-kips (adjustment $= 15.3\%$)

Reduction to $M_{B,Left} = -109.44 \times 0.153 = 16.7$ ft-kips

Adjusted $M_{B,Left} = -109.4 - (16.7) = -92.7$ ft-kips

Increase in positive moment in span A-B

$M_A = -99.7$ ft-kips

Adjusted $M_{B,Left} = -92.7$ ft-kips

Mid-span ordinate on line M_A to $M_{B,Left} = \dfrac{-99.7 + (-92.7)}{2} = -96.2$ ft-kips

Moment due to uniform load $= \dfrac{w_u \ell^2}{2} = \dfrac{2.12 \times 25^2}{8} = 165.6$ ft-kips

Adjusted positive moment at the mid-span $= -96.2 + 165.6 = 69.4$ ft-kips

Decrease in negative moment at the left face of support B

Ordinate on line M_A to $M_{B,Left} = -99.7 + \dfrac{-92.7 - (-99.7)}{25.0} \times 24.33 = 92.9$ ft-kips

Moment due to uniform load $= \dfrac{1}{2} w_u x(\ell - x) = \dfrac{1}{2} \times 2.12 \times 24.33 \times (25.0 - 24.33) = -17.2$ ft-kips

Adjusted negative moment at the left face of support B $= -92.9 + 17.2 = -75.7$ ft-kips

Similar calculations are made to determine the adjusted moment at other locations and for other load patterns. Results of the additional calculations are shown in Table 2.

3.1 After the adjusted moments have been determined analytically, the adjusted bending moment diagrams for each loading pattern can be determined. The adjusted moment curves were determined graphically and are indicated by the dashed lines in Figures 2 (a) through (e).

3.2 An Adjusted maximum moment envelope can now be obtained from the adjusted moment curves as shown in Fig. 2 (f) by dashed lines.

3.3 Final steel ratios ρ can now be obtained on the basis of the adjusted moments.

From the redistribution moment envelopes of Fig. 2 (f), the design factored moments and the required reinforcement area are obtained as shown in Table 3.

Table 2 Moments Before and After Redistribution (moments in ft-kips)

Location	Load Pattern I		Load Pattern II		Load Pattern III		Load Pattern IV		Load Pattern V	
	M_u	M_{adj}	M_u	M_{adj}	M_u	M_{adj}	M_u	M_{adj}	M_u	M_{adj}
A	-99.7	-99.7	-100.5	-100.5	-65.4	-65.4	-65.3	-65.3	-99.9	-99.9
A Right Face	-82.8	-82.4	-83.5	-83.1	-54.2	-53.9	-54.1	-53.8	-82.9	-82.5
Mid-Span A-B	+61.1	+69.4	+61.6	+69.8	+40.1	+45.6	+40.0	+45.6	+61.2	+69.5
B Left Face	-91.9	-75.7	-90.2	-74.2	-61.8	-50.9	-62.0	-51.1	-91.7	-75.5
B Left Center	-109.4	-92.7	-107.6	-91.2	-73.4	-62.2	-73.6	-62.4	-109.1	-92.5
B Right Center	-52.4	-41.9	-38.4	-30.7	-43.7	-34.9	-45.5	-36.4	-50.6	-40.5
B Right Face	-41.6	-31.2	-31.3	-23.7	-33.5	-24.8	-35.1	-26.0	-40.0	-29.9
Mid-Span B-C	+15.8	+24.5	+6.4	+12.9	+15.8	+24.9	+17.5	+26.0	+14.3	+23.4
C Left Face	-26.4	-19.0	-21.6	-16.0	-33.0	-24.3	-28.9	-21.1	-30.4	-22.3
C Left Center	-35.7	-28.6	-27.9	-22.3	-43.1	-34.5	-38.7	-31.0	-40.1	-32.1
C Right Center	-48.1	-38.5	-68.9	-55.1	-71.2	-57.0	-48.6	-38.9	-70.7	-56.6
C Right Face	-38.8	-29.5	-55.0	-41.7	-57.2	-43.4	-39.3	-29.9	-56.8	-43.1
Mid-Span C-D	+25.9	+30.6	+40.2	+47.1	+39.6	+46.6	+25.7	+30.5	+39.7	+46.7
D Left Face	-31.5	-31.2	-49.3	-48.8	-48.4	-47.9	-31.3	-31.0	-48.6	-48.1
D	-40.3	-40.3	-62.8	-62.8	-61.7	-61.7	-40.1	-40.1	-61.9	-61.9
		Final design moments after redistribution								

Table 3 Summary of Final Design

Location		Moment (ft-kips)	Load Case	Required	
				A_s (in^2)	ρ
Support A	Right Face	-83.1	II	1.43	0.0085
Midspan A-B		69.8	II	1.18	0.0070
Support B	Left Face	-75.7	I	1.29	0.0077
	Right Face	-31.2	I	0.51	0.0030
Midspan B-C		26	IV	0.42	0.0025
Support C	Left Face	-24.3	III	0.39	0.0023
	Right Face	-43.4	III	0.72	0.0043
Midspan C-D		47.1	II	0.78	0.0046
Support D	Left Face	-48.8	II	0.81	0.0048

$$\text{Use } A_{s,min} = 200\frac{b_w d}{f_y} = 200 \times \frac{12 \times 14}{60{,}000} = 0.56\,\text{in}^2$$

4. Conclusions & Observations

The moment redistribution is often utilized for the investigation of existing structures for conditions such as change of use, additional loading, or verifying adequacy for the latest design code. In these conditions, if there is a reserve capacity from existing reinforcement layout at mid-span (or support) of a span may be utilized to compensate for the inadequacy of the support (or mid-span) of the same span.

The moment redistribution can also be utilized in the design of a new structure. One such example of its application may help reduce the negative moment at an interior support and corresponding top reinforcement while increasing the positive moment at mid-span. The advantage of this may be the alleviation of the congestion of rebar at support top regions.

The calculation of moment redistribution is a tedious process especially while considering live load patterning as presented in this example. spBeam software program performs the moment redistribution calculations with speed and accuracy.