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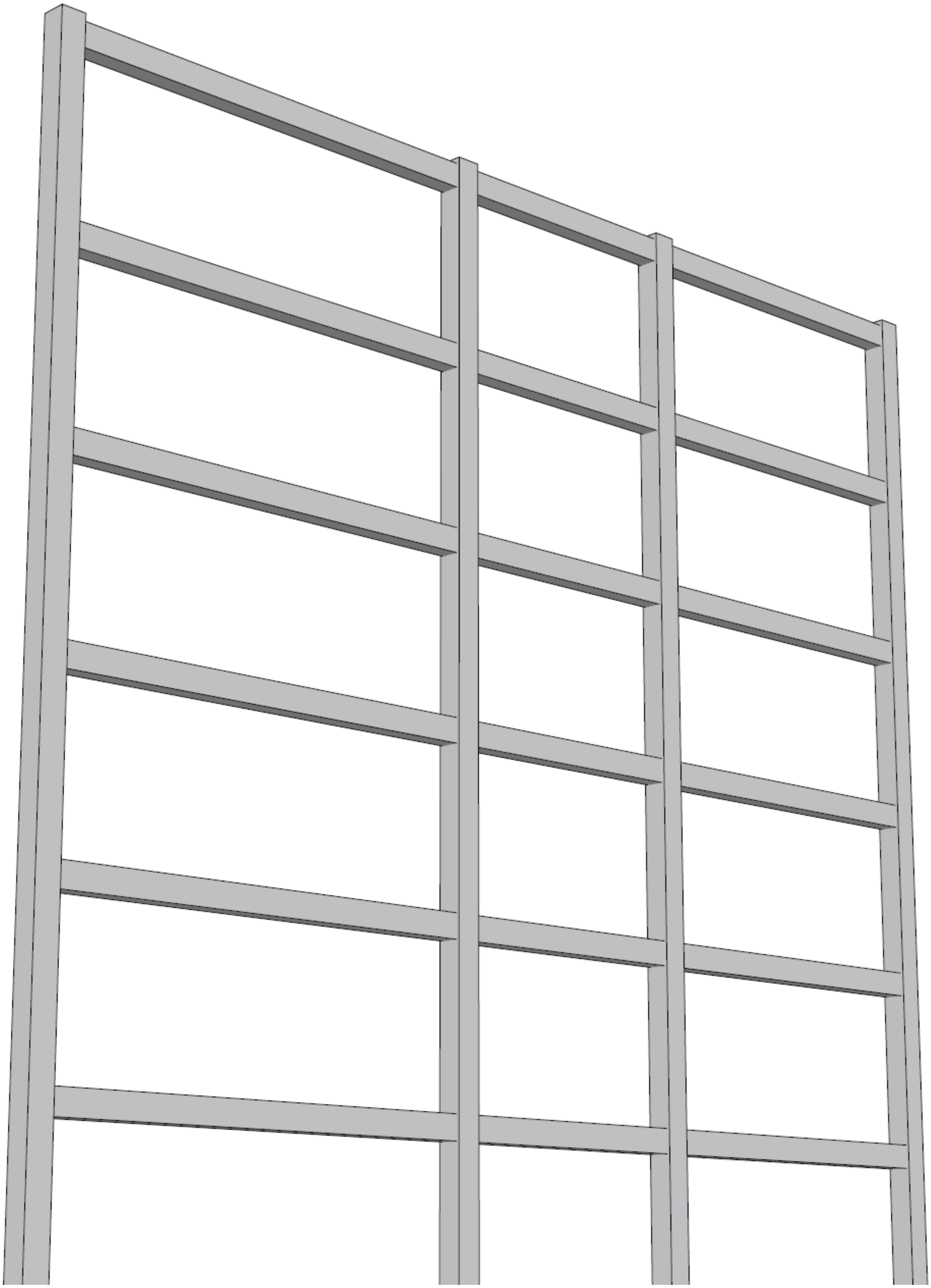
spBeam

Continuous Beam Design with Moment Redistribution (ACI 318-14)



Continuous Beam Design with Moment Redistribution (ACI 318-14)

A structural reinforced concrete continuous beam at an intermediate floor level in an exterior frame (spandrel Beam) provides gravity load resistance for the applied dead and live loads.

The required reinforcement areas are determined for this continuous beam after analysis are adjusted and optimized using moment redistribution provisions from ACI 318 standard. The results of hand calculations are then compared with numerical analysis results obtained from the spBeam engineering software program by StructurePoint.

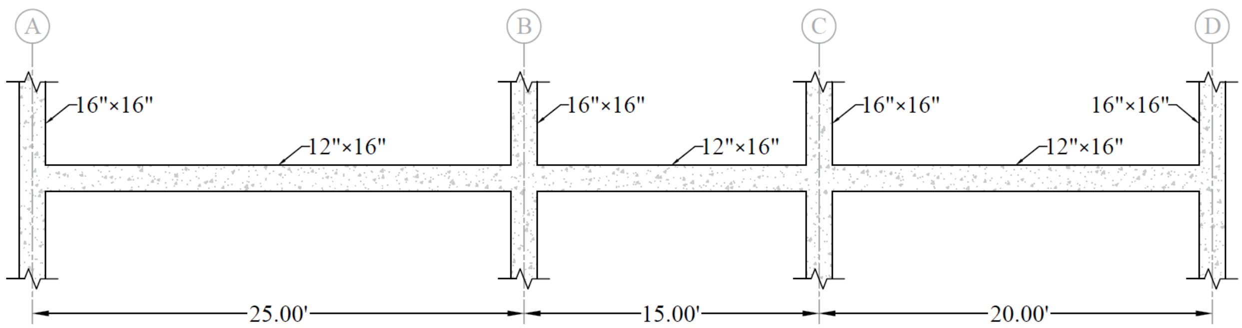


Figure 1 – Reinforced Concrete Continuous Beam at Intermediate Floor Level

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

References

- PCA Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association, Example 8.2 (In this example ACI 318-14 is used instead of ACI 318-11)
- spBeam Engineering Software Program Manual v10.00, STRUCTUREPOINT, 2024
- "One-Way Wide Module (Skip) Joist Concrete Floor System Design (ACI 318-14)" Design Example, STRUCTUREPOINT, 2023
- Contact Support@StructurePoint.org to obtain supplementary materials (spBeam model: DE-Moment-Redistribution-ACI-14.slbx)

Design Data

f_c' = 4,000 psi normal weight concrete (w_c = 150 lb/ft^3)

f_y = 60,000 psi

Story height = 10.00 ft

Columns = 16.00 in. × 16.00 in.

Spandrel beam = 12.00 in. × 16.00 in.

Dead Loads, DL = 1,167.00 lb/ft

Live Loads, LL = 450.00 lb/ft

Solution

Continuous beams are frequently analyzed and designed using simplified methods such as the approximate coefficients provided in ACI 318 to approximate the bending moments and shear forces. There are many important limitations to allow the use of coefficients. The factored moment and shear can be determined using the simplified method if the requirements are satisfied: ***ACI 318-14 (6.5.1)***

✓ Members are prismatic.

✓ Loads are uniformly distributed.

✓ $L \leq 3D$

✓ There are at least two spans.

X The longer of two adjacent spans does not exceed the shorter by more than 20 percent.

In this example the ratio of the two adjacent spans lengths exceeds 1.2 and coefficients can't be used. Therefore, the analysis of the continuous beam framing into columns must use traditional analysis methods and will be performed using the following steps:

1. Determine the factored loads.
2. Perform the structural analysis using the moment distribution method.
3. Repeat the analysis for each live load pattern to arrive at the enveloped maximum design moments.
4. Apply moment redistribution provisions to get adjusted (reduced) design moments.
5. Determine the required area of steel optimized to the adjusted design moments.

It is important in this example to distinguish between the two terms: **moment distribution** and **moment redistribution**. **Moment distribution** is a structural analysis method for statically indeterminate beams and frames, while **moment redistribution** refers to the behavior of statically indeterminate structures that are not completely elastic, but have some reserve plastic capacity. When one location first yields, further application of load to the structure causes the bending moment to redistribute differently from what a purely elastic analysis would suggest.

Moment redistribution is not commonly used primarily due to the extensive additional and tedious calculations required involving many live load patterns and the iterative nature of the procedure that lends itself to automation as is provided by spBeam engineering software program from StructurePoint.

When permitted, **moment redistribution** is used to reduce total reinforcement required and this example will illustrate the extent of redistribution of bending moments and the corresponding reduction of steel area achievable. Typically, negative moments over supports governs the design of reinforcement and any reduction in the required area of steel at the supports is favorable due to savings in materials, labor, and construction time and effort.

1. Continuous Beam Analysis – Moment Distribution Method

Determine moment distribution factors and fixed-end moments for the frame members. The moment distribution procedure will be used to analyze the frame. Stiffness factors, carry over factors, and fixed-end moment factors for the beams and columns are determined as follows:

Determine the elastic bending moment diagrams for each of the load patterns per ACI and the maximum moment envelope values for all patterns as shown in Table 1. ***ACI 318-14 (6.4)***

1.1. Load Combination

$U = 1.20 \times D + 1.60 \times L$ ***ACI 318-14 (Eq. 5.3.1b)***

$w_d = 1.20 \times 1.17 = 1.40 \text{ kips/ft}$

$w_l = 1.60 \times 0.45 = 0.72 \text{ kips/ft}$

$w_u = 1.40 + 0.72 = 2.12 \text{ kips/ft}$

1.2. Flexural Stiffness of Beams and Columns Ends, *K*

$$K = \frac{4 \times E_c \times I}{l}$$

Where K is referred to as stiffness factor at beam or column end and can be defined as the amount of moment required to rotate the end of the beam or column 1 rad.

$$I = \frac{b \times h^3}{12}$$

$$E = w_c^{1.5} \times 33 \times \sqrt{f_c'}$$ ***ACI 318-14 (Eq. 19.2.2.1.a)***

For Member AB:

$l = 25.00 \text{ ft}$

$$I = \frac{12.00 \times 16.00^3}{12} = 4{,}096.00 \text{ in.}^2$$

$$E = \frac{(150)^{1.5} \times 33 \times \sqrt{4{,}000}}{1{,}000} = 3{,}834.25 \text{ ksi}$$

$$K_{AB} = \frac{4 \times 3{,}834.25 \times 4{,}096.00}{25.00 \times 12.00} = 209.40 \times 10^6 \text{ in.-lb}$$

For Column Ends:

$$h = 10.00 \text{ ft}$$

$$I = \frac{16.00 \times 16.00^3}{12} = 5,461.33 \text{ in.}^2$$

$$K_{Col} = \frac{4 \times 3,834.25 \times 5,461.33}{10.00 \times 12.00} = 698.00 \times 10^6 \text{ in.-lb}$$

1.3. Distribution Factor, *DF*

$$DF = \frac{K}{\sum K}$$

The distribution factor for a member that is connected to a fixed joint is defined as the fraction of the total resisting moment supplied by this member.

For Member AB:

$$DF_{AB} = \frac{209.40 \times 10^6}{209.40 \times 10^6 + 698.00 \times 10^6 + 698.00 \times 10^6} = 0.130$$

1.4. Carry Over Factor, *COF*

$COF = 0.5$

Where *COF* is the Carry-Over Factor that represents the fraction of the moment that is "carried over" from the joint to the beam end when the beam far end is fixed.

1.5. Fixed-End Moments, *FEMs*

For a beam with uniformly distributed load and fixed ends, *FEM* can be found using the following equation:

$$FEM = \frac{w \times l^2}{12}$$

For Member AB for Load Pattern I:

$$FEM_{AB} = \frac{2.12 \times 25.00^2}{12} = 110.44 \text{ kip-ft}$$

1.6. Beam Analysis Using Moment Distribution Method

Repeat the previous steps to all frame members to obtain the parameters necessary for the analysis. Moment distribution for the five loading conditions is shown in Table 1. Counter-clockwise rotational moments acting on member ends are taken as positive. Maximum positive span moments are determined from the following equation:

$$M_{max}^{+}=\frac{w_u\times l_1^2}{8}-\frac{M_L^-+M_R^-}{2}+\frac{\left(M_L^--M_R^-\right)^2}{2\times w_u\times l_1^2}\text{ at distance }x_{max}=\frac{l_1}{2}+\frac{M_L^--M_R^-}{w_u\times l_1}$$

Where:

- M_{max}^{+} = Maximum positive moment in the span
- M_L^- = Negative moment in the left support
- M_R^- = Negative moment in the right support
- l_1 = The span length

For Load Pattern I:

Maximum positive moment in spans A-B:

$$M_{max}^{+}=\frac{(2.12)\times 25.00^2}{8}-\frac{99.75+109.38}{2}+\frac{(99.75-109.38)^2}{2\times(2.12)\times 25.00^2}=61.13\text{ kip-ft}$$

$$x_{max}=\frac{25.00}{2}+\frac{(99.75-109.38)}{(2.12)\times 25.00}=12.32\text{ ft}$$

Where:

$M_L^- = 99.75$ kip-ft $\quad\quad$ $M_R^- = 109.38$ kip-ft

Maximum positive moment in span B-C:

$$M_{max}^{+}=\frac{(2.12)\times 15.00^2}{8}-\frac{52.44+35.74}{2}+\frac{(52.44-35.74)^2}{2\times(2.12)\times 15.00^2}=15.84\text{ kip-ft}$$

$$x_{max}=\frac{15.00}{2}+\frac{(52.44+35.74)}{(2.12)\times 15.00}=8.03\text{ ft}$$

Where:

$M_L^- = 52.44$ kip-ft $\quad\quad$ $M_R^- = 35.74$ kip-ft

Maximum positive moment in span C-D:

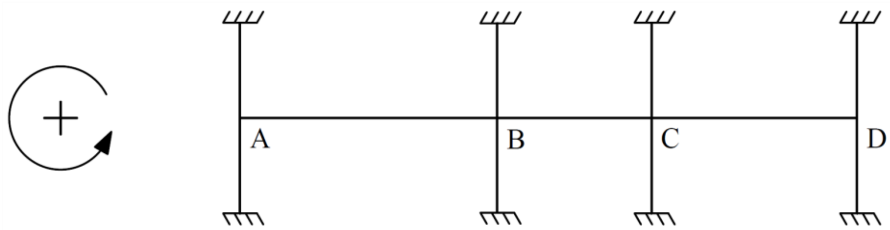
$$M^{+}_{max} = \frac{(1.40)\times 20.00^2}{8} - \frac{48.13+40.29}{2} + \frac{(48.13-40.29)^2}{2\times(1.40)\times 20.00^2} = 25.86 \text{ kip-ft}$$

$$x_{max} = \frac{20.00}{2} + \frac{(48.13-40.29)}{(1.40)\times 20.00} = 10.28 \text{ ft}$$

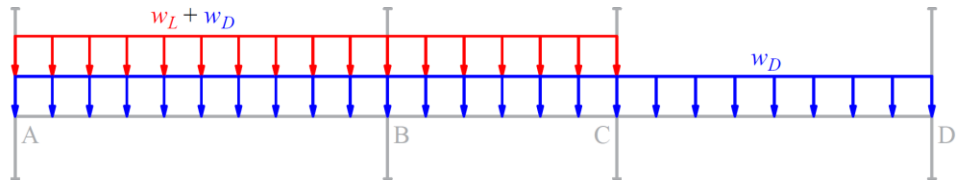
Where:

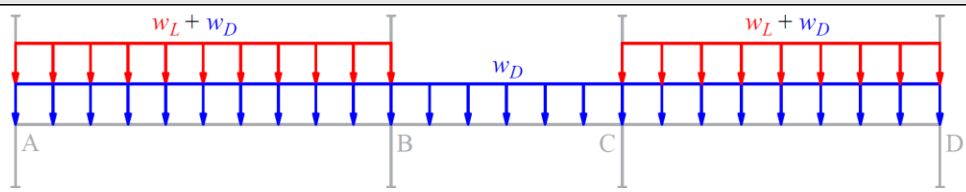
M_L^{-} = 48.13 kip-ft $\qquad$ M_R^{-} = 40.29 kip-ft

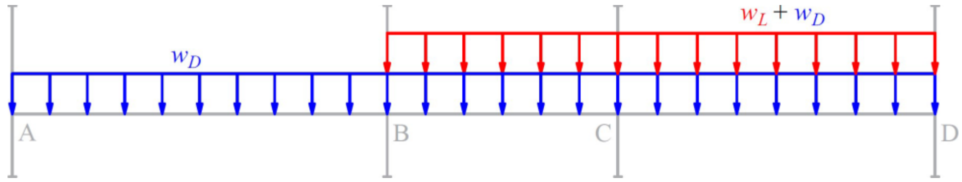
Table 1 – Moment Distribution‡

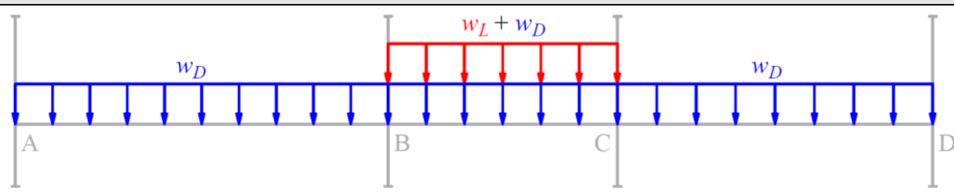


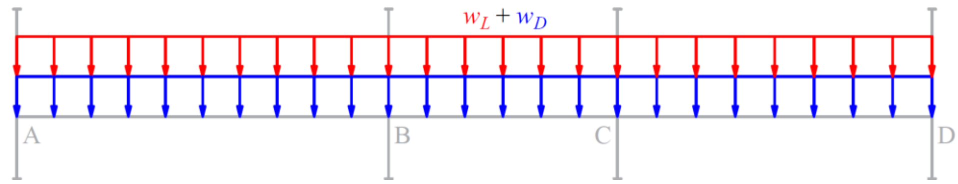
Joint	A	B		C		D
Member	**AB**	**BA**	**BC**	**CB**	**CD**	**DC**
DF	0.130	0.107	0.179	0.174	0.130	0.158
COF	0.500	0.500	0.500	0.500	0.500	0.500

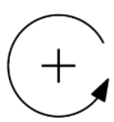
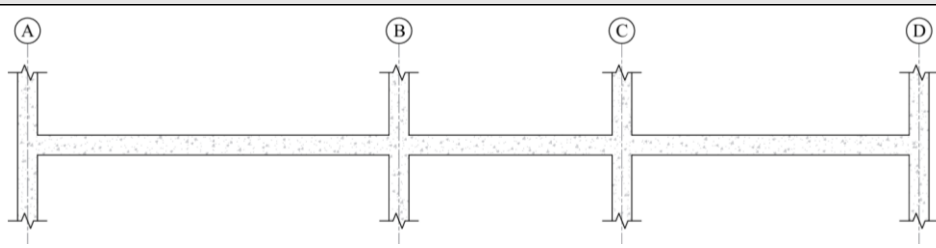
Load Pattern I (S2*)						
LP I						
Joint	**A**	**B**		**C**		**D**
Member	**AB**	**BA**	**BC**	**CB**	**CD**	**DC**
FEM	110.44	-110.44	39.76	-39.76	46.68	-46.68
Dist.	-14.40	7.57	12.62	-1.20	-0.90	7.37
CO	3.79	-7.20	-0.60	6.31	3.69	-0.45
Dist.	-0.49	0.84	1.39	-1.74	-1.30	0.07
CO	0.42	-0.25	-0.87	0.70	0.04	-0.65
Dist.	-0.05	0.12	0.20	-0.13	-0.10	0.10
CO	0.06	-0.03	-0.06	0.10	0.05	-0.05
Dist.	-0.01	0.01	0.02	-0.03	-0.02	0.01
CO	0.00	0.00	-0.01	0.01	0.00	-0.01
Dist.	0.00	0.00	0.00	0.00	0.00	0.00
M^- (kip-ft)	99.75	-109.38	52.44	-35.74	48.13	-40.29
M^+ (kip-ft)	61.13		15.84		25.86	

Load Pattern II (Odd*)						
LP II						
Joint	**A**	**B**		**C**		**D**
Member	**AB**	**BA**	**BC**	**CB**	**CD**	**DC**
FEM	110.44	-110.44	26.26	-26.26	70.68	-70.68
Dist.	-14.40	9.02	15.03	-7.73	-5.79	11.16
CO	4.51	-7.20	-3.86	7.52	5.58	-2.90
Dist.	-0.59	1.19	1.98	-2.28	-1.71	0.46
CO	0.59	-0.29	-1.14	0.99	0.23	-0.85
Dist.	-0.08	0.15	0.26	-0.21	-0.16	0.13
CO	0.08	-0.04	-0.11	0.13	0.07	-0.08
Dist.	-0.01	0.02	0.03	-0.03	-0.03	0.01
CO	0.01	-0.01	-0.02	0.01	0.01	-0.01
Dist.	0.00	0.00	0.00	0.00	0.00	0.00
M^- (kip-ft)	100.54	-107.60	38.43	-27.86	68.87	-62.76
M^+ (kip-ft)	61.60		6.42		40.23	

Load Pattern III (S3*)						
LP III						
Joint	**A**	**B**		**C**		**D**
Member	**AB**	**BA**	**BC**	**CB**	**CD**	**DC**
FEM	72.94	-72.94	39.76	-39.76	70.68	-70.68
Dist.	-9.51	3.56	5.93	-5.38	-4.03	11.16
CO	1.78	-4.76	-2.69	2.96	5.58	-2.02
Dist.	-0.23	0.80	1.33	-1.49	-1.11	0.32
CO	0.40	-0.12	-0.74	0.66	0.16	-0.56
Dist.	-0.05	0.09	0.15	-0.14	-0.11	0.09
CO	0.05	-0.03	-0.07	0.08	0.04	-0.05
Dist.	-0.01	0.01	0.02	-0.02	-0.02	0.01
CO	0.01	0.00	-0.01	0.01	0.00	-0.01
Dist.	0.00	0.00	0.00	0.00	0.00	0.00
M^- (kip-ft)	65.36	-73.38	43.67	-43.07	71.20	-61.74
M^+ (kip-ft)	40.07		16.26		39.60	

Load Pattern IV (Even*)						
LP IV						
Joint	**A**	**B**		**C**		**D**
Member	**AB**	**BA**	**BC**	**CB**	**CD**	**DC**
FEM	72.94	-72.94	39.76	-39.76	46.68	-46.68
Dist.	-9.51	3.56	5.93	-1.20	-0.90	7.37
CO	1.78	-4.76	-0.60	2.96	3.69	-0.45
Dist.	-0.23	0.57	0.96	-1.16	-0.87	0.07
CO	0.29	-0.12	-0.58	0.48	0.04	-0.43
Dist.	-0.04	0.07	0.12	-0.09	-0.07	0.07
CO	0.04	-0.02	-0.04	0.06	0.03	-0.03
Dist.	0.00	0.01	0.01	-0.02	-0.01	0.01
CO	0.00	0.00	-0.01	0.01	0.00	-0.01
Dist.	0.00	0.00	0.00	0.00	0.00	0.00
M^- (kip-ft)	65.26	-73.62	45.54	-38.72	48.59	-40.09
M^+ (kip-ft)	40.01		17.56		25.75	

Load Pattern V (All*)						
LP V						
Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
FEM	110.44	-110.44	39.76	-39.76	70.68	-70.68
Dist.	-14.40	7.57	12.62	-5.38	-4.03	11.16
CO	3.79	-7.20	-2.69	6.31	5.58	-2.02
Dist.	-0.49	1.06	1.77	-2.07	-1.55	0.32
CO	0.53	-0.25	-1.03	0.88	0.16	-0.78
Dist.	-0.07	0.14	0.23	-0.18	-0.14	0.12
CO	0.07	-0.03	-0.09	0.11	0.06	-0.07
Dist.	-0.01	0.01	0.02	-0.03	-0.02	0.01
CO	0.01	0.00	-0.02	0.01	0.01	-0.01
Dist.	0.00	0.00	0.00	0.00	0.00	0.00
M^- (kip-ft)	99.85	-109.14	50.57	-40.10	70.74	-61.94
M^+ (kip-ft)	61.19		14.42		39.73	

Envelop of Maximum Moments						
						
Span	AB		BC		CD	
	M_L	M_R	M_L	M_R	M_L	M_R
Max M_u^- (Column Center Line)	100.54	-109.38	52.44	-43.07	71.20	-62.76
Max M_u^- (Column Face)	83.53	-91.92	41.57	-32.97	57.22	-49.30
Max M_u^+	61.60		17.56		40.23	

‡ Moments units are kip-ft

* Live load pattern designation in spBeam

2. Moment Redistribution

Now that the structural analysis is completed for all applicable live load patterns, and the enveloped moments are determined we can evaluate the impact of moment redistribution.

Moment redistribution is dependent on adequate ductility in plastic hinge regions. Plastic hinge regions develop at sections of maximum positive or negative moment and cause a shift in the elastic moment diagram. The usual result is a reduction in the values of maximum negative moments in the support regions and an increase in the values of positive moments between supports from those computed by elastic analysis. However, because negative moments are determined for one loading arrangement and positive moments for another, economies in reinforcement can sometimes be realized (depending on the load pattern) by reducing maximum elastic positive moments and increasing negative moments, thus narrowing the envelope of maximum negative and positive moments at any section in the span. The plastic hinges permit the utilization of the full capacity of more cross sections of a flexural member at ultimate loads. ***ACI 318-14 (R6.6.5)***

The ACI code allows the reduction of factored moments calculated by elastic theory at sections of maximum negative or maximum positive moment in any span of continuous flexural members for any assumed loading arrangement by a percentage equal to 1000 ε_t up to a maximum of 20 percent. ***ACI 318-14 (6.6.5.3)***

Redistribution of moments shall be made only when ε_t is equal to or greater than 0.0075 at the section at which moment is reduced. ***ACI 318-14 (6.6.5.1)***

Static equilibrium shall be maintained after redistribution of moments for each loading arrangement. The reduced moment shall be used for calculating redistributed moments at all other sections within the spans.

ACI 318-14 (6.6.5.4)

2.1. Reduction Percentage Calculations

Using d = 14.00 in. and cover = 1.50 in.

Calculate the coefficient of resistance using the following equation:

$$\frac{R_n}{f_c'} = \frac{M_u}{\phi \times f_c' \times b \times d^2}$$

PCA Notes on ACI 318-11 (8.4 Eq. 2)

Calculate the net tensile strain (ε_t) using the following equation:

$$\varepsilon_t = 0.003\left(\frac{\beta_1}{1-\sqrt{1-\frac{40}{17}\times\frac{R_n}{f_c'}}}-1\right)$$

PCA Notes on ACI 318-11 (8.4 Eq. 8)

For M_u use envelope value at support face. Based on ε_t calculate the adjustment. Iterate until the adjusted moments converge (start repeating) as follows: (see the following table)

For negative moment at support D:

First Iteration

$$(M_u)_1 = 49.30 \text{ kip-ft}$$

$$\left(\frac{R_n}{f'_c}\right)_1 = \frac{49.30 \times 12.00}{0.9 \times 4.00 \times 12.00 \times 14.00^2} = 0.0699$$

$$(\varepsilon_t)_1 = 0.003\left(\frac{0.85}{1-\sqrt{1-\frac{40}{17}\times 0.0699}} - 1\right) = 0.0267$$

$$Adjustment_1 = 1000 \times \varepsilon_t = 1000 \times 0.0267 = 26.7 > 20 \rightarrow Adjustment_1 = 20\%$$

Second Iteration

$$(M_u)_2 = 49.30 - 49.30 \times 0.20 = 39.44 \text{ kip-ft}$$

$$\left(\frac{R_n}{f'_c}\right)_2 = \frac{39.44 \times 12.00}{0.9 \times 4.00 \times 12.00 \times 14.00^2} = 0.0559$$

$$(\varepsilon_t)_2 = 0.003\left(\frac{0.85}{1-\sqrt{1-\frac{40}{17}\times 0.0559}} - 1\right) = 0.0345$$

$$Adjustment_2 = 1000 \times \varepsilon_t = 1000 \times 0.0345 = 34.5 > 20 \rightarrow Adjustment_2 = 20\%$$

Since Adjustment$_1$ = Adjustment$_2$ → End of Iterations

Table 2 – Moment Adjustments at Supports							
		Support					
		A	**B**		**C**		**D**
		Right	**Left**	**Right**	**Left**	**Right**	**Left**
Iteration 1	M_u **(ft-kip)**	83.53	91.92	41.57	32.97	57.22	49.30
	R_n/f_c'	0.118	0.130	0.059	0.047	0.081	0.070
	ε_t **(in./in.)**	0.014	0.012	0.032	0.042	0.022	0.027
	Adjustment (%)	13.93	12.25	20.00	20.00	20.00	20.00
Iteration 2	M_u **(ft-kip)**	71.90	80.67	33.26	26.38	45.77	39.44
	R_n/f_c'	0.102	0.114	0.047	0.037	0.065	0.056
	ε_t **(in./in.)**	0.017	0.015	0.042	0.054	0.029	0.034
	Adjustment (%)	16.91	14.59	20.00	20.00	20.00	20.00
Iteration 3	M_u **(ft-kip)**	69.41	78.52				
	R_n/f_c'	0.098	0.111				
	ε_t **(in./in.)**	0.018	0.015				
	Adjustment (%)	17.68	15.11				
Iteration 4	M_u **(ft-kip)**	68.77	78.04				
	R_n/f_c'	0.097	0.111				
	ε_t **(in./in.)**	0.018	0.015				
	Adjustment (%)	17.88	15.23				
Iteration 5	M_u **(ft-kip)**	68.60	77.92				
	R_n/f_c'	0.097	0.110				
	ε_t **(in./in.)**	0.018	0.015				
	Adjustment (%)	17.94	15.26				
Iteration 6	M_u **(ft-kip)**		77.90				
	R_n/f_c'		0.110				
	ε_t **(in./in.)**		0.015				
	Adjustment (%)		15.26				
Final Allowable Adjustment (%)		17.94	15.26	20.00	20.00	20.00	20.00

2.2. Adjustment of Moments (Redistribution)

Now the engineer can make decisions to reduce any negative moments (or positive) based on project parameters including:

- Steel detailing and placement considerations.
- New design or investigation of existing beams
- Optimize the provided reinforcement for more economical design
- Optimize the provided reinforcement for improved uniformity

It was decided to reduce the negative moments on both sides of supports B and C and accept the increase in the corresponding positive moments, and not to adjust the negative moments at the exterior supports A and D.

The following figures show the unadjusted and adjusted moment values at the columns centerlines, columns faces, and at the midspan for each load pattern and for the maximum values at each critical location (maximum moment envelops).

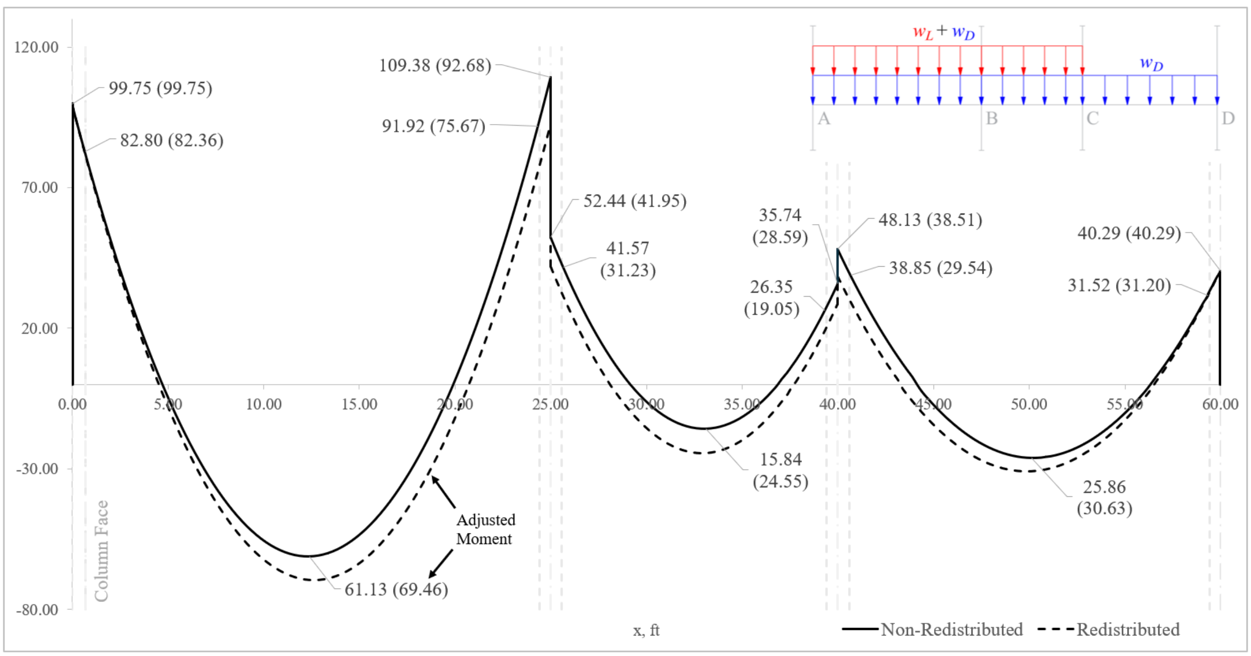


Figure 2 – Load Pattern I (moments in kip-ft)

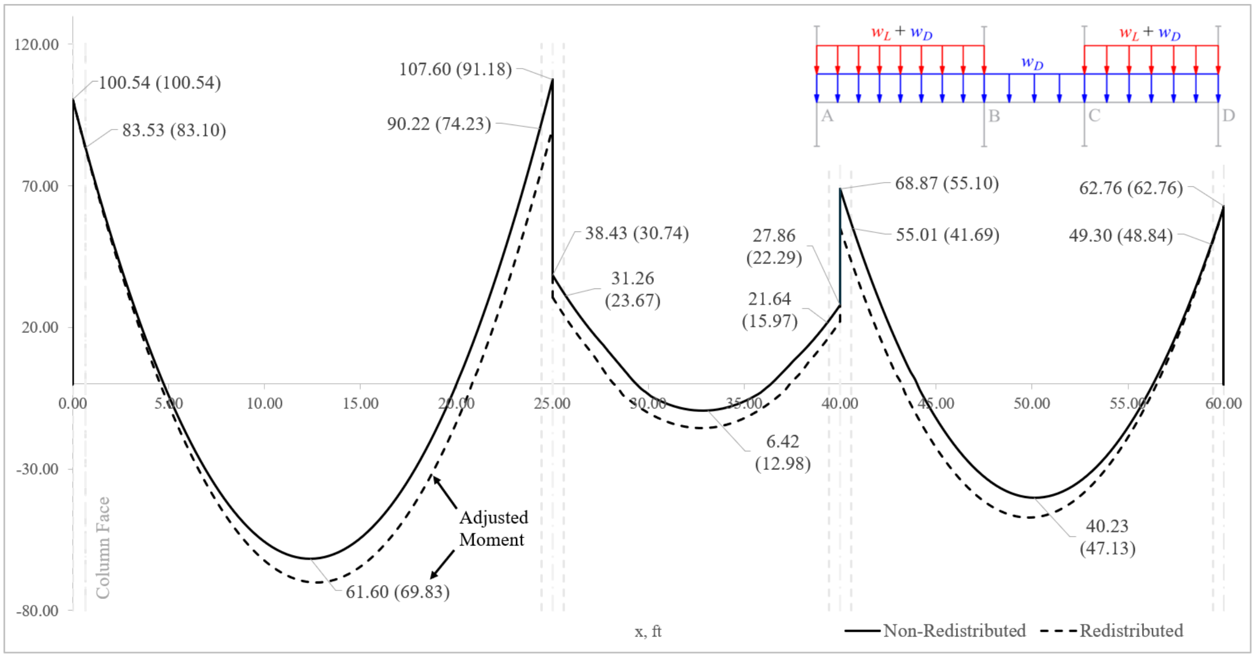


Figure 3 – Load Pattern II (moments in kip-ft)

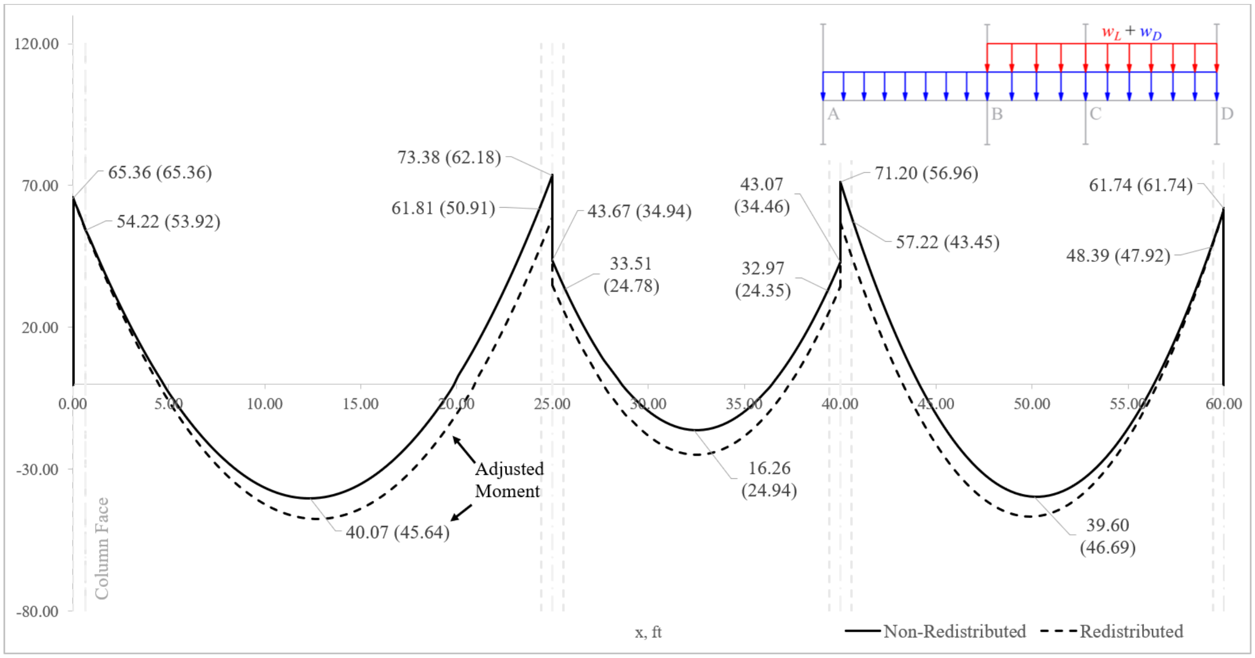


Figure 4 – Load Pattern III (moments in kip-ft)

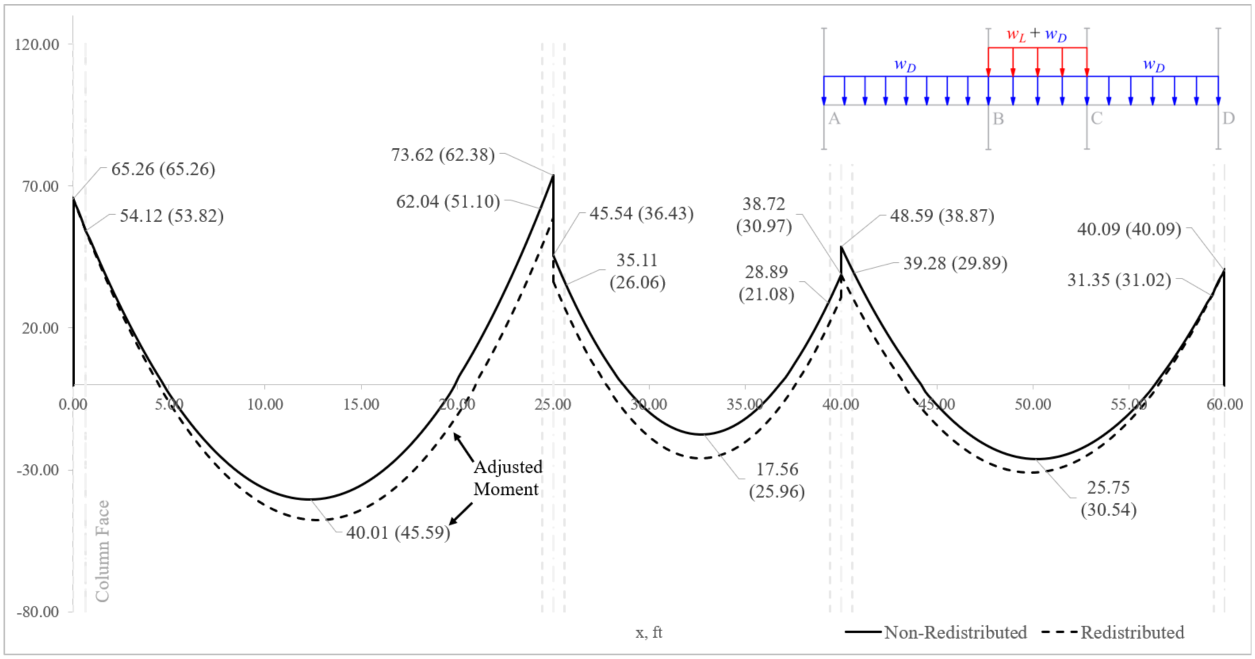


Figure 5 – Load Pattern IV (moments in kip-ft)

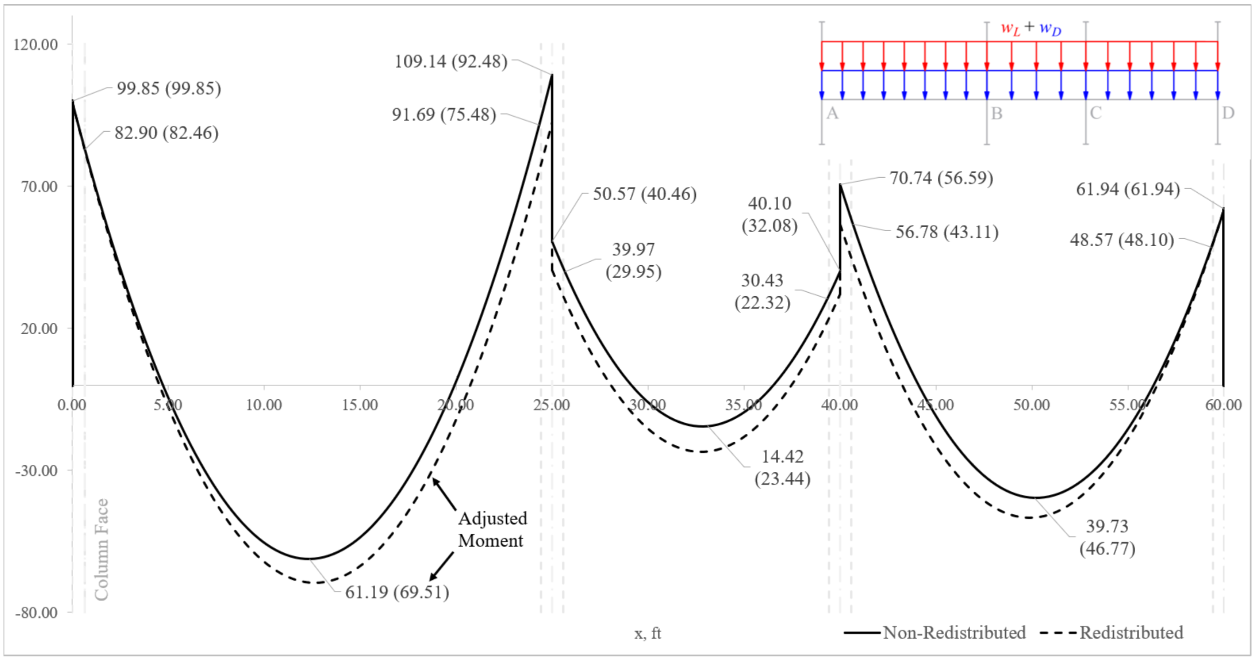


Figure 6 – Load Pattern V (moments in kip-ft)

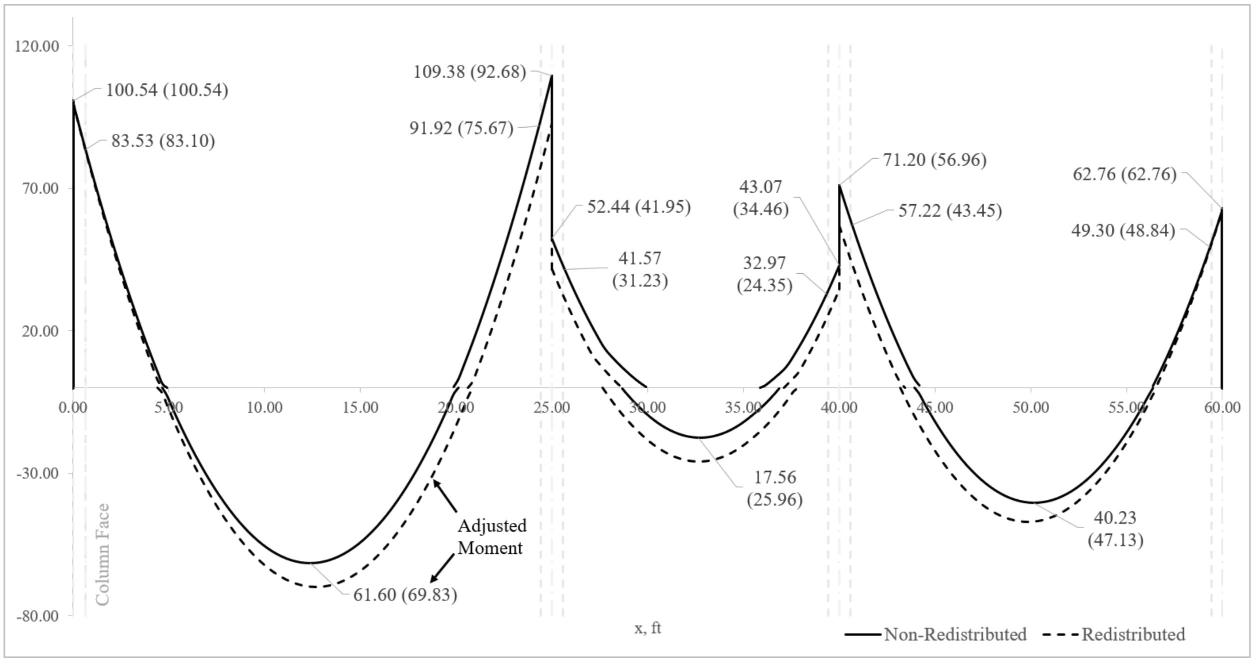


Figure 7 – Maximum Moment Envelopes for Pattern Loading (moments in kip-ft)

For load pattern I

$$M_{B,left} = 109.38 \text{ kip-ft (adjustment} = 15.26\%)$$

$$\text{Adjusted } M_{B,left} = 109.38 - 109.38 \times 0.15 = 92.68 \text{ kip-ft}$$

Increase in positive moment in span A-B

$$M_L^- = M_A = 99.75 \text{ kip-ft}$$

$$\text{Adjusted } M_R^- = M_{B,left} = 92.68 \text{ kip-ft}$$

Repeat the same procedure as Section 1.6 to calculate the maximum positive moment.

$$M_{max}^+ = \frac{w_u \times l_1^2}{8} - \frac{M_L^- + M_R^-}{2} + \frac{\left(M_L^- - M_R^-\right)^2}{2 \times w_u \times l_1^2} \text{ at distance } x_{max} = \frac{l_1}{2} + \frac{M_L^- - M_R^-}{w_u \times l_1}$$

$$M_{max}^+ = \frac{(2.12) \times 25.00^2}{8} - \frac{99.75 + 92.68}{2} + \frac{(99.75 - 92.68)^2}{2 \times (2.12) \times 25.00^2} = 69.46 \text{ kip-ft}$$

$$x_{max} = \frac{25.00}{2} + \frac{(99.75 - 92.68)}{(2.12) \times 25.00} = 12.63 \text{ ft}$$

Decrease in negative moment at the left face of support B

$$\text{Ordinate on line } M_A \text{ to } M_{B,left} = 99.75 + \frac{92.68 - 99.75}{25.00} \times 24.33 = 92.87 \text{ kip-ft}$$

$$\text{Moment due to uniform load} = \frac{1}{2} \times w_u \times x \times (l - x) = \frac{1}{2} \times 2.12 \times 24.33 \times (25.00 - 24.33) = 17.20 \text{ kip-ft}$$

$$\text{Adjusted negative moment at the left face of support B} = 92.87 - 17.20 = 75.67 \text{ kip-ft}$$

Similar calculations are made to determine the adjusted moment at other locations and for other load patterns. Results of the additional calculations are shown in the following table.

Table 3 – Moments Before and After Redistribution (Moments in kip-ft)

Location	Load Pattern I S2		Load Pattern II Odd		Load Pattern III S3		Load Pattern IV Even		Load Pattern V All	
	M_u	M_{adj}	M_u	M_{adj}	M_u	M_{adj}	M_u	M_{adj}	M_u	M_{adj}
A Center	-99.75	-99.75	-100.54	-100.54	-65.36	-65.36	-65.26	-65.26	-99.85	-99.85
A Right Face	-82.80	-82.36	-83.53	-83.10	-54.22	-53.92	-54.12	-53.82	-82.90	-82.46
Midspan A-B	61.13	69.46	61.60	69.83	40.07	45.64	40.01	45.59	61.19	69.51
B Left Face	-91.92	-75.67	-90.22	-74.23	-61.81	-50.91	-62.04	-51.10	-91.69	-75.48
B Left Center	-109.38	-92.68	-107.60	-91.18	-73.38	-62.18	-73.62	-62.38	-109.14	-92.48
B Right Center	-52.44	-41.95	-38.43	-30.74	-43.67	-34.94	-45.54	-36.43	-50.57	-40.46
B Right Face	-41.57	-31.23	-31.26	-23.67	-33.51	-24.78	-35.11	-26.06	-39.97	-29.95
Midspan B-C	15.84	24.55	6.42	12.98	16.26	24.94	17.56	25.96	14.42	23.44
C Left Face	-26.35	-19.05	-21.64	-15.97	-32.97	-24.35	-28.89	-21.08	-30.43	-22.32
C Left Center	-35.74	-28.59	-27.86	-22.29	-43.07	-34.46	-38.72	-30.97	-40.10	-32.08
C Right Center	-48.13	-38.51	-68.87	-55.10	-71.20	-56.96	-48.59	-38.87	-70.74	-56.59
C Right Face	-38.85	-29.54	-55.01	-41.69	-57.22	-43.45	-39.28	-29.89	-56.78	-43.11
Midspan C-D	25.86	30.63	40.23	47.13	39.60	46.69	25.75	30.54	39.73	46.77
D Left Face	-31.52	-31.20	-49.30	-48.84	-48.39	-47.92	-31.35	-31.02	-48.57	-48.10
D Center	-40.29	-40.29	-62.76	-62.76	-61.74	-61.74	-40.09	-40.09	-61.94	-61.94

Final design moments after redistribution for critical sections (left and right support face & midspan)

After the adjusted bending moments have been determined analytically, the adjusted bending moment diagrams for each load pattern can be determined. The adjusted moment curves were determined graphically and are indicated by the dashed lines in Figure 2 through Figure 6.

An Adjusted maximum moment envelope can now be obtained from the adjusted moment curves as shown in Figure 7 by dashed lines.

From the redistribution moment envelopes of Figure 7, the design factored moments and the required reinforcement area are obtained as shown in following table. Check example "One-Way Wide Module (Skip) Joist Concrete Floor System Design (ACI 318-14)" for detailed calculations for flexural and shear design of continuous beams.

Table 4 – Summary of Final Design (comparison of % reduction and required Reinforcement)

Location		Moment at column face (kip-ft)		Load Case	A_s (in.^{2})		
		Undistributed	Redistributed		Undistributed	Redistributed	%
Support A	Right	-83.53	-83.10	II	1.434	1.426	99.43
Midspan A-B		61.60	69.83	II	1.034	1.182	114.29
Support B	Left	-91.92	-75.67	I	1.592	1.288	80.91
	Right	-41.57	-31.23	I	0.684	<u>*0.509*</u>	74.42
Midspan B-C		17.56	25.96	IV	<u>*0.283*</u>	<u>*0.421*</u>	148.99
Support C	Left	-32.97	-24.35	III	<u>*0.539*</u>	<u>*0.395*</u>	73.28
	Right	-57.22	-43.45	III	0.956	0.717	74.95
Midspan C-D		40.23	47.13	II	0.661	0.780	117.91
Support D	Left	-49.30	-48.84	II	0.818	0.810	99.03

Italic underlined values indicate $A_{s,min}$ = 0.560 in.^{2} governs

Where $A_{s,\min} = \max\left\{\begin{matrix}\dfrac{3\times\sqrt{f'_c}}{f_y}\\ \dfrac{200}{f_y}\end{matrix}\right\}\times b_w\times d = \max\left\{\begin{matrix}0.0032\\0.0033\end{matrix}\right\}\times 12.00\times 14.00 = 0.560\text{ in.}^2$ ***<u>ACI 318-14 (9.6.1.2)</u>***

3. Continuous Beam Analysis and Design Using Moment Redistribution – spBeam Software

spBeam is widely used for analysis, design and investigation of beams, and one-way slab systems (including standard and wide module joist systems) per latest American (ACI 318-14) and Canadian (CSA A23.3-14) codes. spBeam can be used for new designs or investigation of existing structural members subjected to flexure, shear, and torsion loads. With capacity to integrate up to 20 spans and two cantilevers of wide variety of floor system types, spBeam is equipped to provide cost-effective, accurate, and fast solutions to engineering challenges.

spBeam provides top and bottom bar details including development lengths and material quantities, as well as live load patterning and immediate and long-term deflection results. Using the moment redistribution feature engineers can deliver safe designs with savings in materials and labor. Engaging this feature allows up to 20% reduction of negative moments over supports reducing reinforcement congestions in these areas.

Redistribution of negative moments applies to one-way and beam systems only. It can be engaged using the **Design & Modeling Options** from the DEFINITIONS dialog box (see the following figure). The program allows for redistribution of negative moments at supports. Only reduction in negative moments is considered. Increase of negative moments at the support is not taken into account even though it is allowed by the code. Static equilibrium is maintained meaning that bending moments and shear forces along the span are adjusted in accordance with the reduction of moments applied at the supports.

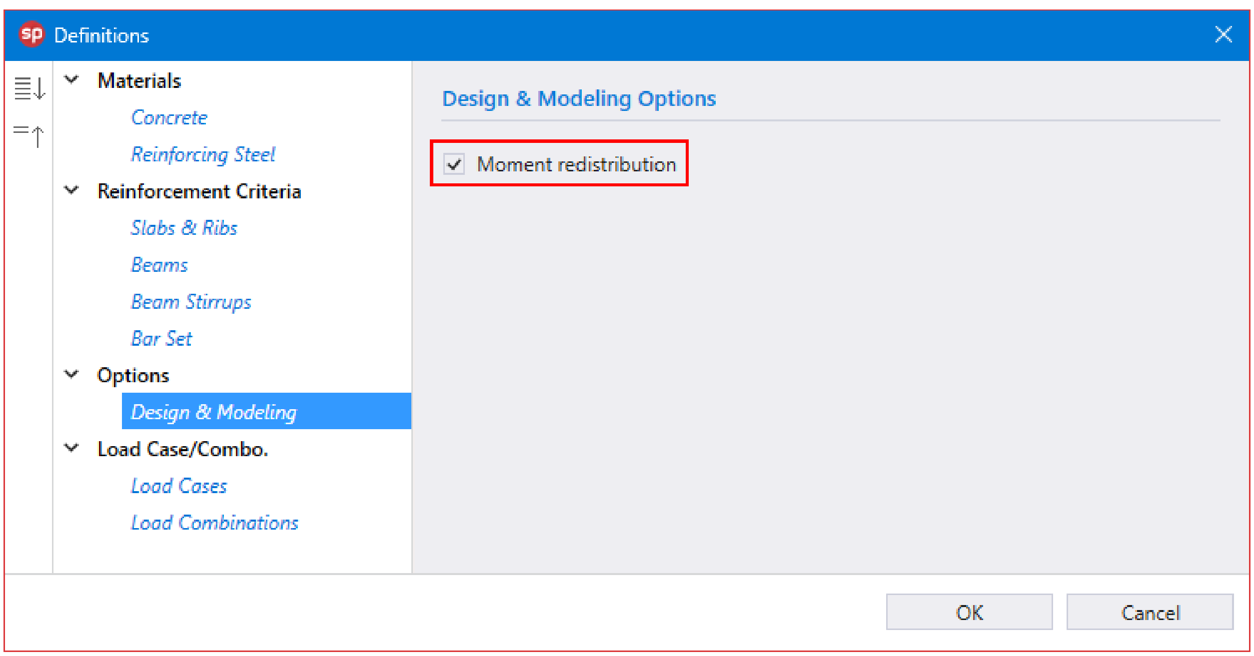


Figure 8 – Activating Moment Redistribution (spBeam)

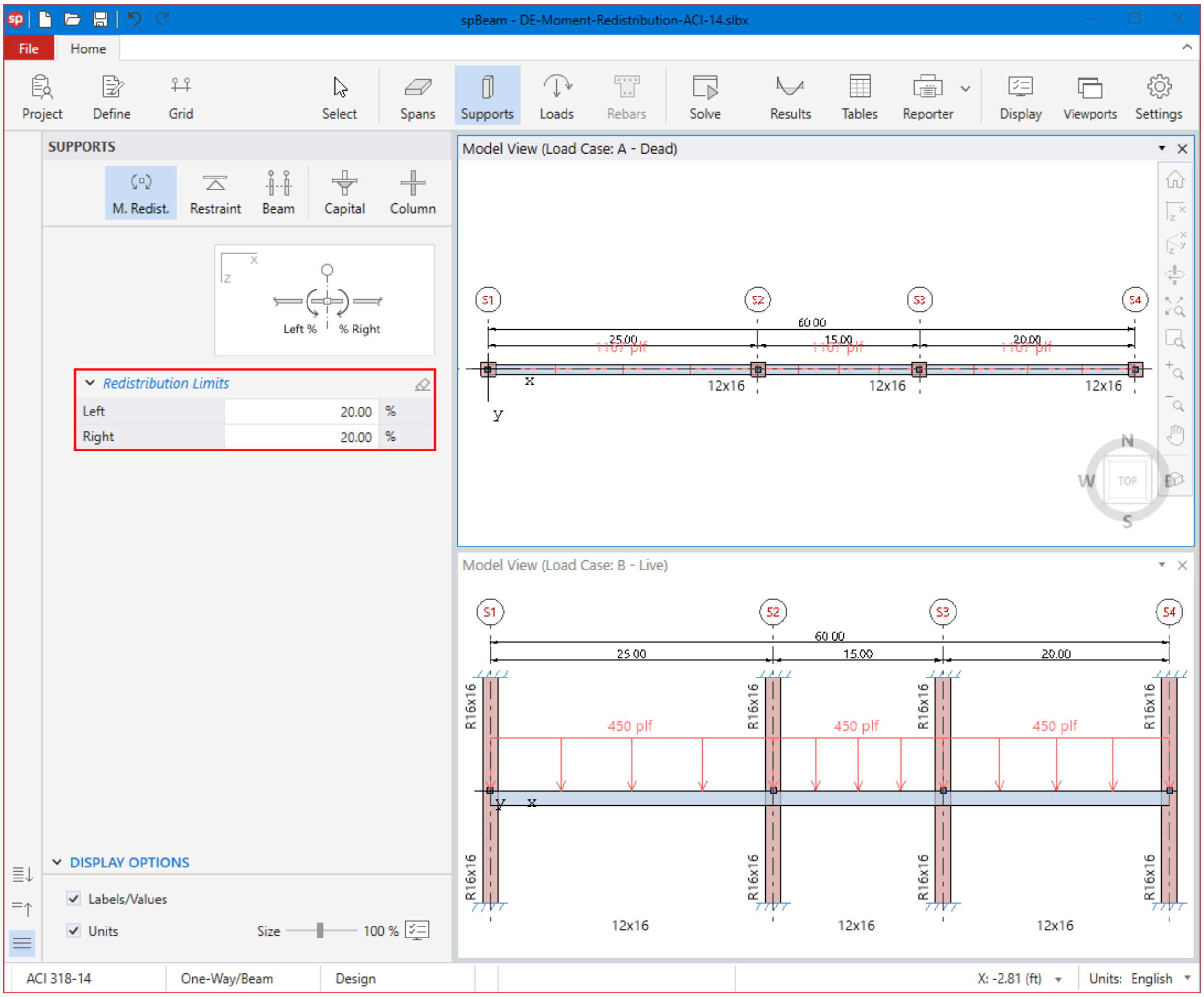


Figure 9 – Assigning Moment Redistribution Limits (spBeam)

From elastic static analysis, the largest moments from all load combinations and load patterns are determined at support faces on both ends of each span except cantilevers. These moments are used to calculate the maximum code allowable adjustment percentage of calculated moment.

In the investigation mode, program uses the area of provided reinforcement to obtain redistribution factors. In the design mode the required reinforcement area is used. The reduction percentage is limited to 20% and not to exceed the maximum values specified by the user. Negative moments at span ends are reduced by the amount of redistribution factors and new moment values are iteratively used to obtain new redistribution factors. This iterative procedure is repeated until the change in distribution factor is negligible (does not exceed 0.01%), but no more than 10 times.

For illustration and comparison purposes, the following figures provide a sample of the results obtained from an spBeam model created for the continuous beam in this example. Special emphasis can be given to Figure 12 that illustrated the maximum and adjusted moments for span 2.

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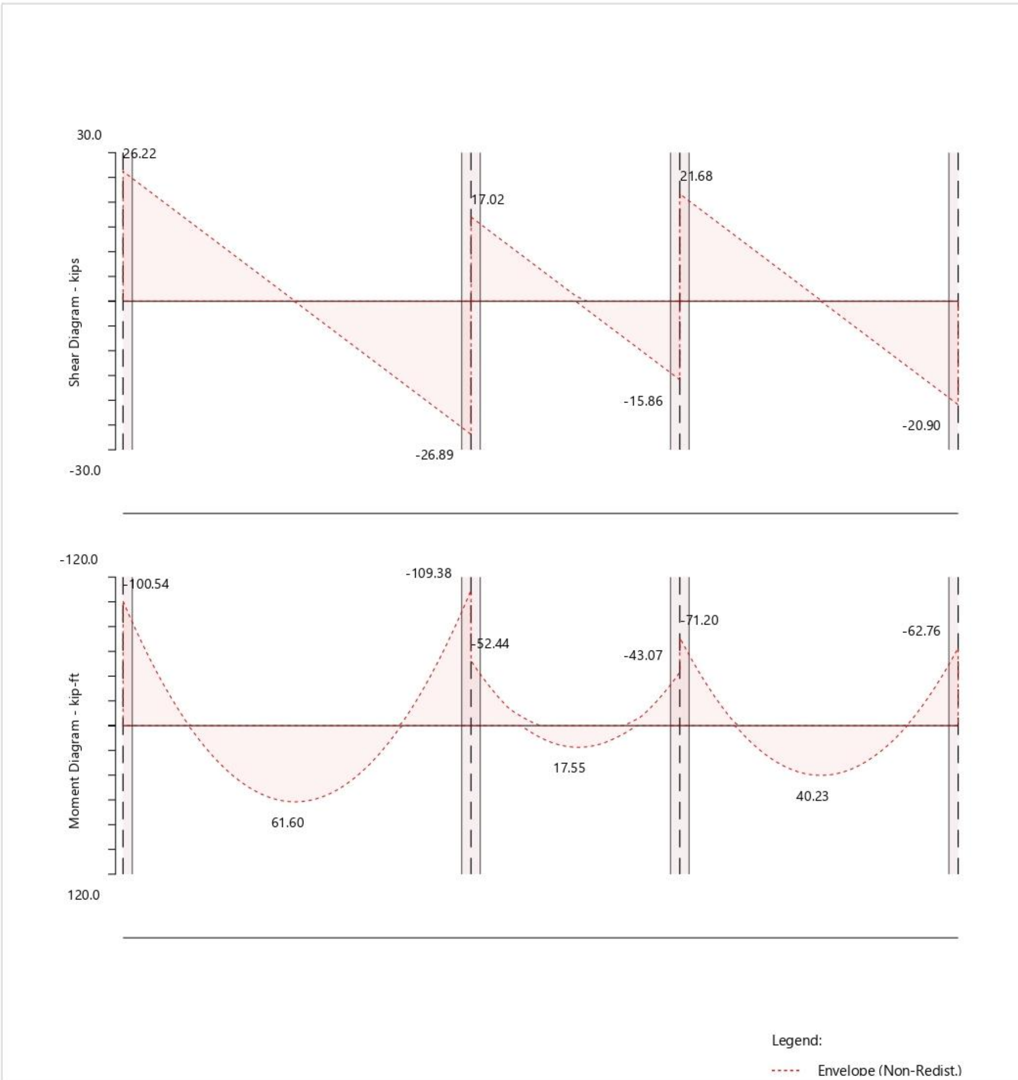


Project: Moment Redistribution
Diagram: Internal Forces

Figure 10 – Internal Forces before Moment Redistribution (spBeam)

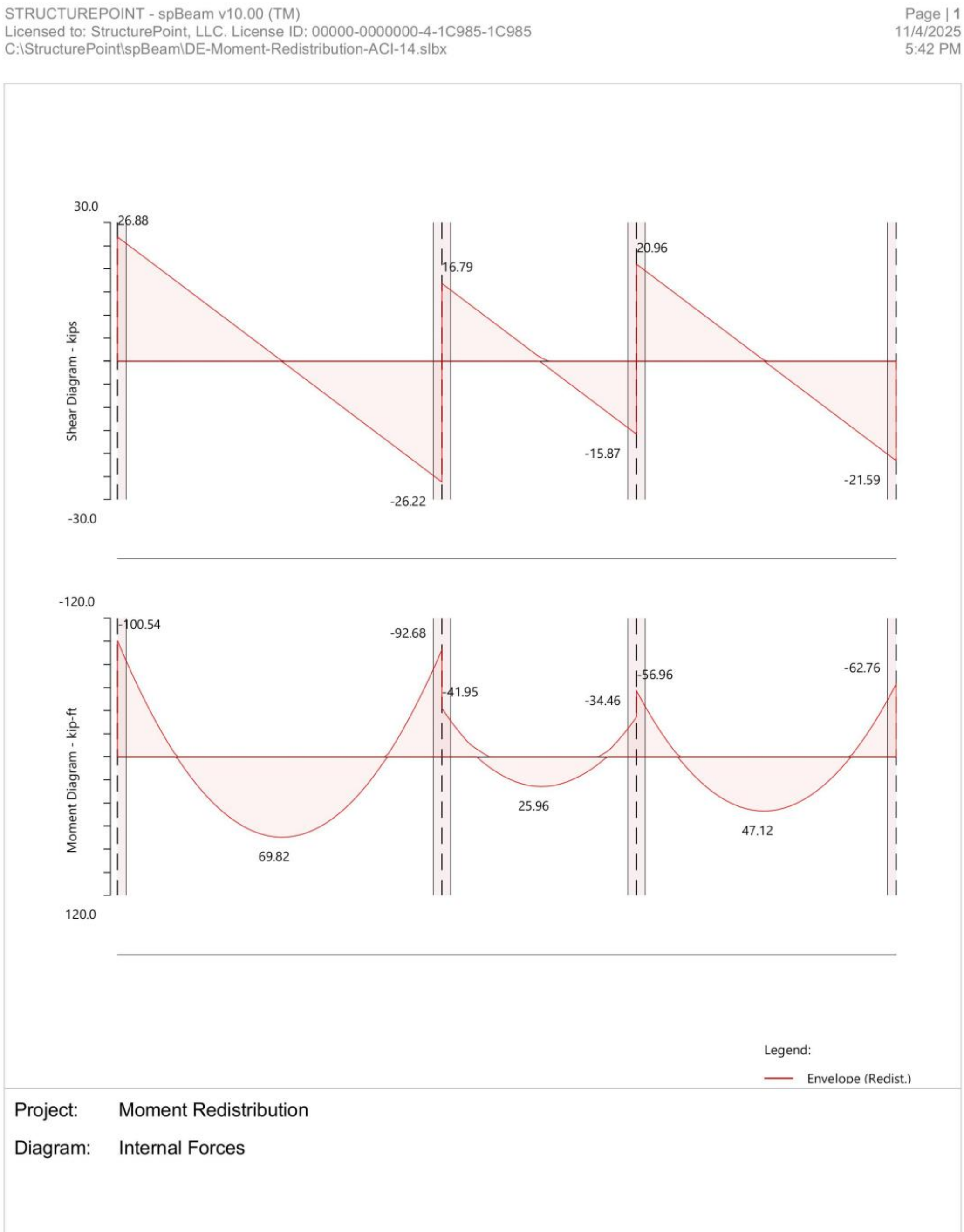


Figure 11 – Internal Forces after Moment Redistribution (spBeam)

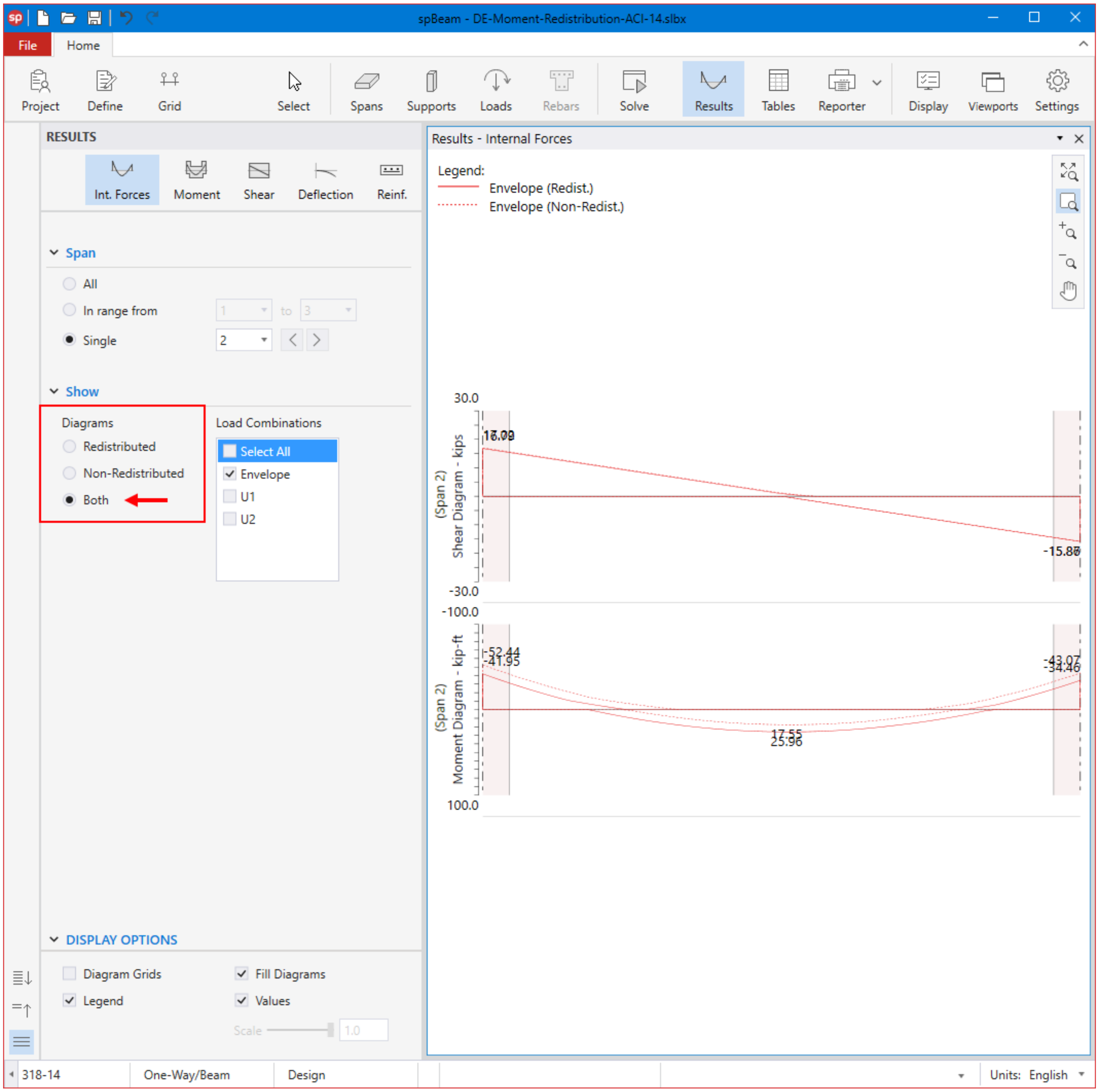
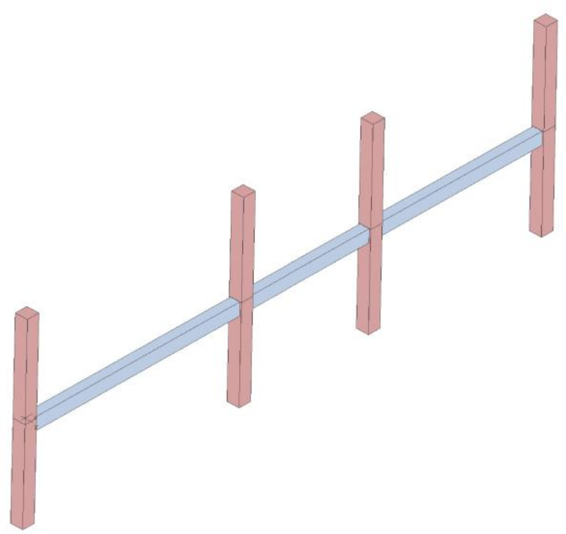


Figure 12 – Internal Forces before and after Moment Redistribution for Span 2 (spBeam)

spBeam (TM)

spBeam v10.00 (TM)
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams and One-way Slab Systems



Contents

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1. Input Echo

1.1. General Information

File Name	C:\Struct...\DE-Moment-Redistribution-ACI-14.slbx
Project	Moment Redistribution
Frame	Exterior
Engineer	SP
Code	ACI 318-14
Units	English
Reinforcement Database	ASTM A615
Mode	Design
Number of supports =	4
Floor System	One-Way/Beam

1.2. Solve Options

Live load pattern ratio = 100%
Deflections are based on cracked section properties.
In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
Moment redistribution selected.
Effective flange width calculations selected.
Rigid beam-column joint NOT selected.
Torsion analysis and design NOT selected.

1.3. Material Properties

1.3.1. Concrete: Slabs / Beams

w_c	150	pcf
f'_c	4	ksi
E_c	3834.3	ksi
f_r	0.47434	ksi

1.3.2. Concrete: Columns

w_c	150	pcf
f'_c	4	ksi
E_c	3834.3	ksi
f_r	0.47434	ksi

1.3.3. Reinforcing Steel

f_y	60	ksi
f_{yt}	60	ksi
E_s	29000	ksi
Epoxy coated bars	No	

1.4. Reinforcement Database

Size	Db	Ab	Wb	Size	Db	Ab	Wb
	in	in²	lb/ft		in	in²	lb/ft
#3	0.38	0.11	0.38	#4	0.50	0.20	0.67
#5	0.63	0.31	1.04	#6	0.75	0.44	1.50
#7	0.88	0.60	2.04	#8	1.00	0.79	2.67
#9	1.13	1.00	3.40	#10	1.27	1.27	4.30
#11	1.41	1.56	5.31	#14	1.69	2.25	7.65
#18	2.26	4.00	13.60				

1.5. Span Data

1.5.1. Slabs

Notes:

Span	Loc	L1	t	wL	wR	bEff	Hmin	
		ft	in	ft	ft	in	in	
1	ExtL	25.000	0.00	0.500	0.500	12.00	0.00	---
2	ExtL	15.000	0.00	0.500	0.500	12.00	0.00	---
3	ExtL	20.000	0.00	0.500	0.500	12.00	0.00	---

1.5.2. Ribs and Longitudinal Beams

Notes:
*b - Span depth is less than minimum. Deflection check required.

Span	Ribs			Beams		Span	
	b	h	Sp	b	h	Hmin	
	in	in	in	in	in	in	
1	0.00	0.00	0.00	12.00	16.00	16.22	*b
2	0.00	0.00	0.00	12.00	16.00	8.57	
3	0.00	0.00	0.00	12.00	16.00	12.97	

1.6. Support Data

1.6.1. Columns

Support	c1a	c2a	Ha	c1b	c2b	Hb	Red %
	in	in	ft	in	in	ft	
1	16.00	16.00	10.000	16.00	16.00	10.000	100
2	16.00	16.00	10.000	16.00	16.00	10.000	100
3	16.00	16.00	10.000	16.00	16.00	10.000	100
4	16.00	16.00	10.000	16.00	16.00	10.000	100

1.6.2. Moment Redistribution Limits

Support	Left	Right
	%	%
1	0	0
2	20	20
3	20	20
4	0	0

1.6.3. Boundary Conditions

Support	Spring		Far End	
	K_z	K_{ry}	Above	Below
	kips/in	kip-in/rad		
1	0.00	0.00	Fixed	Fixed

Support	Spring K_z kips/in	Spring K_{ry} kip-in/rad	Far End Above	Far End Below
2	0.00	0.00	Fixed	Fixed
3	0.00	0.00	Fixed	Fixed
4	0.00	0.00	Fixed	Fixed

1.7. Load Data

1.7.1. Load Cases and Combinations

Case	Dead	Live
Type	DEAD	LIVE
U1	1.400	0.000
U2	1.200	1.600

1.7.2. Line Loads

Case/Patt	Span	Wa plf	La ft	Wb plf	Lb ft
Dead	1	1167.00	0.000	1167.00	25.000
	2	1167.00	0.000	1167.00	15.000
	3	1167.00	0.000	1167.00	20.000
Live	1	450.00	0.000	450.00	25.000
	2	450.00	0.000	450.00	15.000
	3	450.00	0.000	450.00	20.000
Live/Odd	1	450.00	0.000	450.00	25.000
	3	450.00	0.000	450.00	20.000
Live/Even	2	450.00	0.000	450.00	15.000
Live/S1	1	450.00	0.000	450.00	25.000
Live/S2	1	450.00	0.000	450.00	25.000
	2	450.00	0.000	450.00	15.000
Live/S3	2	450.00	0.000	450.00	15.000
	3	450.00	0.000	450.00	20.000
Live/S4	3	450.00	0.000	450.00	20.000

1.8. Reinforcement Criteria

1.8.1. Slabs and Ribs

	Units	Top Bars Min.	Top Bars Max.	Bottom Bars Min.	Bottom Bars Max.
Bar Size		#8	#8	#8	#8
Bar spacing	in	1.00	18.00	1.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00
Clear Cover	in	1.50		1.50	

There is NOT more than 12 in of concrete below top bars.

1.8.2. Beams

	Units	Top Bars Min.	Top Bars Max.	Bottom Bars Min.	Bottom Bars Max.	Stirrups Min.	Stirrups Max.
Bar Size		#8	#8	#8	#8	#3	#3
Bar spacing	in	1.00	18.00	1.00	18.00	6.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00		
Clear Cover	in	1.50		1.50			
Layer dist.	in	1.00		1.00			
No. of legs						2	6
Side cover	in					1.50	

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	Units	Top Bars		Bottom Bars		Stirrups	
		Min.	Max.	Min.	Max.	Min.	Max.
1st Stirrup	in					3.00	

There is NOT more than 12 in of concrete below top bars.

2. Design Results

2.1. Moment Redistribution Factors

Support	Side	Org. M_u	Calculated Iter.#	ε_t	Factor	User Limit	Applied Factor
		kip-ft			%	%	%
1	Right	83.53	7	0.01796	17.96	0.00	0.00
2	Left	91.92	6	0.01526	15.26	20.00	15.26
2	Right	41.57	2	0.04168	20.00	20.00	20.00
3	Left	32.97	2	0.05368	20.00	20.00	20.00
3	Right	57.21	2	0.02909	20.00	20.00	20.00
4	Left	49.30	2	0.03446	20.00	0.00	0.00

2.2. Top Reinforcement

Notes:
*3 - Design governed by minimum reinforcement.
*8 - Reinforcement required for structural integrity.

Span	Zone	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars	
		ft	kip-ft	ft	in²	in²	in²	in		
1	Left	1.00	83.10	0.667	0.560	3.035	1.426	7.104	2-#8	
	Midspan	1.00	0.00	12.500	0.000	3.035	0.238	7.104	2-#8	*8
	Right	1.00	75.67	24.333	0.560	3.035	1.288	7.104	2-#8	
2	Left	1.00	31.23	0.667	0.560	3.035	0.509	7.104	2-#8	*3
	Midspan	1.00	0.00	7.500	0.000	3.035	0.085	7.104	2-#8	*8
	Right	1.00	24.35	14.333	0.525	3.035	0.395	7.104	2-#8	*3
3	Left	1.00	43.45	0.667	0.560	3.035	0.717	7.104	2-#8	
	Midspan	1.00	0.00	10.000	0.000	3.035	0.135	7.104	2-#8	*8
	Right	1.00	48.84	19.333	0.560	3.035	0.810	7.104	2-#8	

2.3. Top Bar Details

	Left				Continuous		Right			
Span	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
		ft		ft		ft		ft		ft
1	---		---		2-#8	25.00	---		---	
2	---		---		2-#8	15.00	---		---	
3	---		---		2-#8	20.00	---		---	

2.4. Top Bar Development Lengths

	Left				Continuous		Right			
Span	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen
		in		in		in		in		in
1	---		---		2-#8	12.00	---		---	
2	---		---		2-#8	12.00	---		---	
3	---		---		2-#8	12.00	---		---	

2.5. Bottom Reinforcement

Notes:
*3 - Design governed by minimum reinforcement.

Span	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars
	ft	kip-ft	ft	in²	in²	in²	in	
1	1.00	69.82	12.625	0.560	3.035	1.182	7.104	2-#8
2	1.00	25.96	7.624	0.560	3.035	0.421	7.104	2-#8 *3
3	1.00	47.12	9.876	0.560	3.035	0.780	7.104	2-#8

2.6. Bottom Bar Details

	Long Bars			Short Bars		
Span	Bars	Start	Length	Bars	Start	Length
		ft	ft		ft	ft
1	2-#8	0.00	25.00	---		
2	2-#8	0.00	15.00	---		
3	2-#8	0.00	20.00	---		

2.7. Bottom Bar Development Lengths

	Long Bars		Short Bars	
Span	Bars	DevLen	Bars	DevLen
		in		in
1	2-#8	26.61	---	
2	2-#8	12.00	---	
3	2-#8	17.56	---	

2.8. Flexural Capacity

	Top						Bottom				
Span	x	$A_{s,top}$	ΦM_n-	M_u-	Comb Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb Pat	Status
	ft	in²	kip-ft	kip-ft			in²	kip-ft	kip-ft		
1	0.000	1.58	-91.28	-100.54	U2 Odd	---	1.58	91.28	0.00	U1 All	---
	0.222	1.58	-91.28	-94.62	U2 Odd	---	1.58	91.28	0.00	U1 All	---
	0.667	1.58	-91.28	-83.10	U2 Odd	OK	1.58	91.28	0.00	U1 All	OK
	8.950	1.58	-91.28	0.00	U1 All	OK	1.58	91.28	55.09	U2 Odd	OK
	12.500	1.58	-91.28	0.00	U1 All	OK	1.58	91.28	69.78	U2 Odd	OK
	12.625	1.58	-91.28	0.00	U1 All	OK	1.58	91.28	69.82	U2 Odd	OK
	16.050	1.58	-91.28	0.00	U1 All	OK	1.58	91.28	57.75	U2 Odd	OK
	24.333	1.58	-91.28	-75.67	U2 S2	OK	1.58	91.28	0.00	U1 All	OK
	25.000	1.58	-91.28	-92.68	U2 S2	---	1.58	91.28	0.00	U1 All	---
2	0.000	1.58	-91.28	-41.95	U2 S2	---	1.58	91.28	0.00	U1 All	---
	0.667	1.58	-91.28	-31.23	U2 S2	OK	1.58	91.28	0.00	U1 All	OK
	5.450	1.58	-91.28	0.00	U1 All	OK	1.58	91.28	20.72	U2 Even	OK
	7.500	1.58	-91.28	0.00	U1 All	OK	1.58	91.28	25.92	U2 Even	OK
	7.624	1.58	-91.28	0.00	U1 All	OK	1.58	91.28	25.96	U2 Even	OK
	9.550	1.58	-91.28	0.00	U1 All	OK	1.58	91.28	22.21	U2 Even	OK
	14.333	1.58	-91.28	-24.35	U2 S3	OK	1.58	91.28	0.00	U1 All	OK
	15.000	1.58	-91.28	-34.46	U2 S3	---	1.58	91.28	0.00	U1 All	---

Span	Top						Bottom				
	x	$A_{s,top}$	ΦM_n-	M_u-	Comb Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb Pat	Status
	ft	in²	kip-ft	kip-ft			in²	kip-ft	kip-ft		
3	0.000	1.58	-91.28	-56.96	U2 S3	---	1.58	91.28	0.00	U1 All	---
	0.667	1.58	-91.28	-43.45	U2 S3	OK	1.58	91.28	0.00	U1 All	OK
	7.200	1.58	-91.28	0.00	U1 All	OK	1.58	91.28	39.84	U2 Odd	OK
	9.876	1.58	-91.28	0.00	U1 All	OK	1.58	91.28	47.12	U2 Odd	OK
	10.000	1.58	-91.28	0.00	U1 All	OK	1.58	91.28	47.08	U2 Odd	OK
	12.800	1.58	-91.28	0.00	U1 All	OK	1.58	91.28	37.70	U2 Odd	OK
	19.333	1.58	-91.28	-48.84	U2 Odd	OK	1.58	91.28	0.00	U1 All	OK
	20.000	1.58	-91.28	-62.76	U2 Odd	---	1.58	91.28	0.00	U1 All	---

2.9. Longitudinal Beam Transverse Reinforcement Demand and Capacity

2.9.1. Section Properties

Span	d	$(A_v/s)_{min}$	ΦV_c
	in	in²/in	kips
1	14.00	0.0100	15.94
2	14.00	0.0100	15.94
3	14.00	0.0100	15.94

2.9.2. Beam Transverse Reinforcement Demand

Notes:
*8 - Minimum transverse (stirrup) reinforcement governs.

Span	Start	End	Required				Demand	
			X_u	V_u	Comb/Patt	A_v/s	A_v/s	
	ft	ft	ft	kips		in²/in	in²/in	
1	0.917	4.881	1.833	22.99	U2/Odd	0.0112	0.0112	
	4.881	7.929	4.881	16.53	U2/Odd	0.0009	0.0100	*8
	7.929	10.976	7.929	10.07	U2/Odd	0.0000	0.0100	*8
	10.976	14.024	10.976	3.61	U2/Odd	0.0000	0.0000	
	14.024	17.071	17.071	9.41	U2/S2	0.0000	0.0100	*8
	17.071	20.119	20.119	15.87	U2/S2	0.0000	0.0100	*8
	20.119	24.083	23.167	22.34	U2/S2	0.0102	0.0102	
2	0.917	3.452	1.833	12.91	U2/S2	0.0000	0.0100	*8
	3.452	5.071	3.452	9.47	U2/S2	0.0000	0.0100	*8
	5.071	6.690	5.071	6.04	U2/S2	0.0000	0.0000	
	6.690	8.310	6.690	2.61	U2/S2	0.0000	0.0000	
	8.310	9.929	9.929	5.12	U2/S3	0.0000	0.0000	
	9.929	11.548	11.548	8.55	U2/S3	0.0000	0.0100	*8
	11.548	14.083	13.167	11.98	U2/S3	0.0000	0.0100	*8
3	0.917	4.167	1.833	17.08	U2/S3	0.0018	0.0100	*8
	4.167	6.500	4.167	12.13	U2/S3	0.0000	0.0100	*8
	6.500	8.833	6.500	7.18	U2/S3	0.0000	0.0000	
	8.833	11.167	11.167	2.86	U2/Odd	0.0000	0.0000	
	11.167	13.500	13.500	7.80	U2/Odd	0.0000	0.0000	
	13.500	15.833	15.833	12.75	U2/Odd	0.0000	0.0100	*8
	15.833	19.083	18.167	17.70	U2/Odd	0.0028	0.0100	*8

2.9.3. Beam Transverse Reinforcement Details

Span	Size	Stirrups (2 legs each unless otherwise noted)
1	#3	18 @ 6.9 + <-- 36.6 --> + 18 @ 6.9

Span	Size	Stirrups (2 legs each unless otherwise noted)
2	#3	8 @ 6.6 + <-- 58.3 --> + 8 @ 6.6
3	#3	11 @ 6.4 + <-- 84.0 --> + 11 @ 6.4

2.9.4. Beam Transverse Reinforcement Capacity

Notes:
*8 - Minimum transverse (stirrup) reinforcement governs.

			Required				Provided				
Span	Start	End	X_u	V_u	Comb/Patt	A_v/s	A_v	Sp	A_v/s	ΦV_n	
	ft	ft	ft	kips		in²/in	in²	in	in²/in	kips	
1	0.000	0.917	1.833	22.99	U2/Odd	-----	-----	-----	-----	-----	
	0.917	10.976	1.833	22.99	U2/Odd	0.0112	0.22	6.9	0.0319	36.03	
	10.976	14.024	10.976	3.61	U2/Odd	0.0000	-----	-----	-----	7.97	
	14.024	24.083	23.167	22.34	U2/S2	0.0102	0.22	6.9	0.0319	36.03	
	24.083	25.000	23.167	22.34	U2/S2	-----	-----	-----	-----	-----	
2	0.000	0.917	1.833	12.91	U2/S2	-----	-----	-----	-----	-----	
	0.917	5.071	1.833	12.91	U2/S2	0.0000	0.22	6.6	0.0331	36.79	*8
	5.071	9.929	5.071	6.04	U2/S2	0.0000	-----	-----	-----	7.97	
	9.929	14.083	13.167	11.98	U2/S3	0.0000	0.22	6.6	0.0331	36.79	*8
	14.083	15.000	13.167	11.98	U2/S3	-----	-----	-----	-----	-----	
3	0.000	0.917	1.833	17.08	U2/S3	-----	-----	-----	-----	-----	
	0.917	6.500	1.833	17.08	U2/S3	0.0018	0.22	6.4	0.0345	37.66	*8
	6.500	13.500	13.500	7.80	U2/Odd	0.0000	-----	-----	-----	7.97	
	13.500	19.083	18.167	17.70	U2/Odd	0.0028	0.22	6.4	0.0345	37.66	*8
	19.083	20.000	18.167	17.70	U2/Odd	-----	-----	-----	-----	-----	

2.10. Slab Shear Capacity

Span	b	d	V_{ratio}	ΦV_c	V_u	X_u
	in	in		kips	kips	ft
1 --- Not checked ---						
2 --- Not checked ---						
3 --- Not checked ---						

2.11. Material TakeOff

2.11.1. Reinforcement in the Direction of Analysis

Top Bars	320.4	lb	<=>	5.34	lb/ft	<=>	5.340	lb/ft²
Bottom Bars	320.4	lb	<=>	5.34	lb/ft	<=>	5.340	lb/ft²
Stirrups	102.0	lb	<=>	1.70	lb/ft	<=>	1.700	lb/ft²
Total Steel	742.8	lb	<=>	12.38	lb/ft	<=>	12.380	lb/ft²
Concrete	80.0	ft³	<=>	1.33	ft³/ft	<=>	1.333	ft³/ft²

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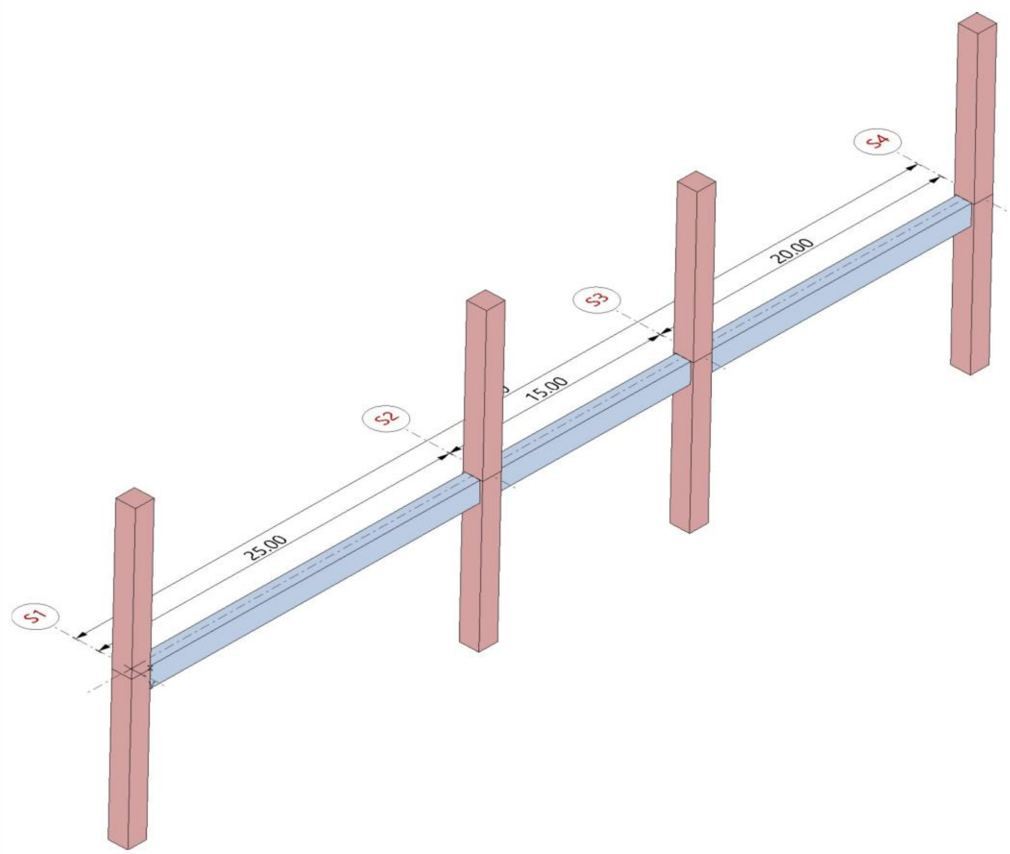
3. Screenshots

3.1. Extrude 3D view

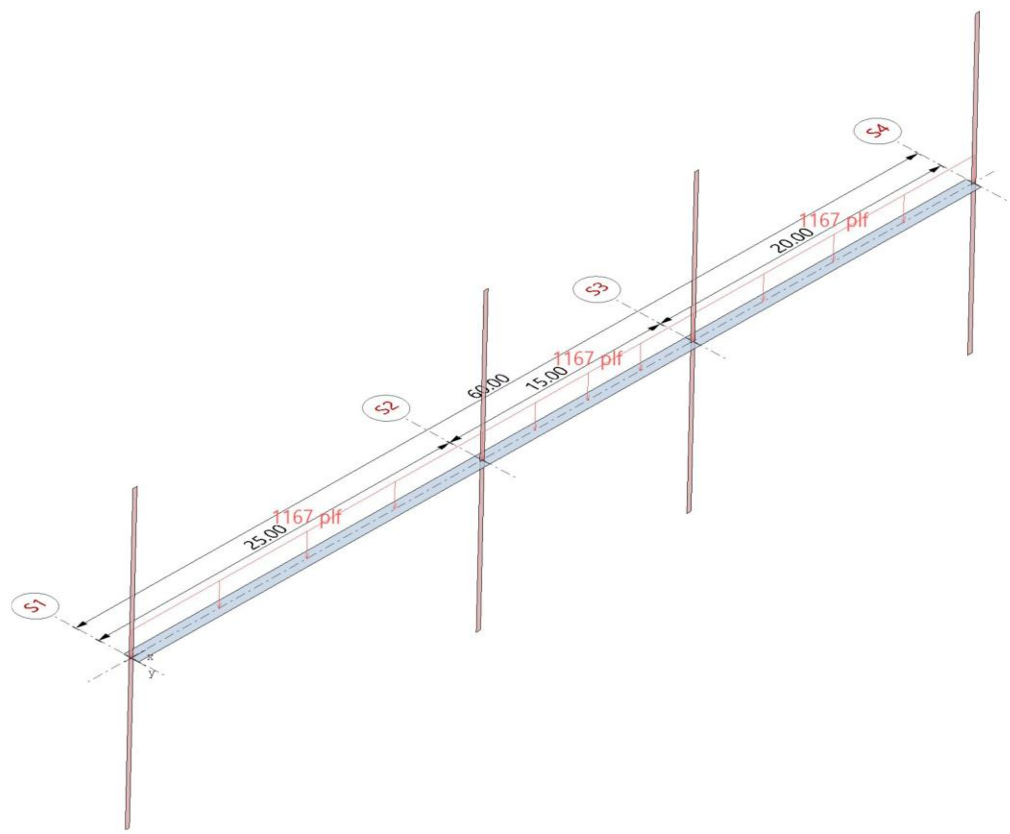


Project: Moment Redistribution

Diagram: Model View (Extrude)

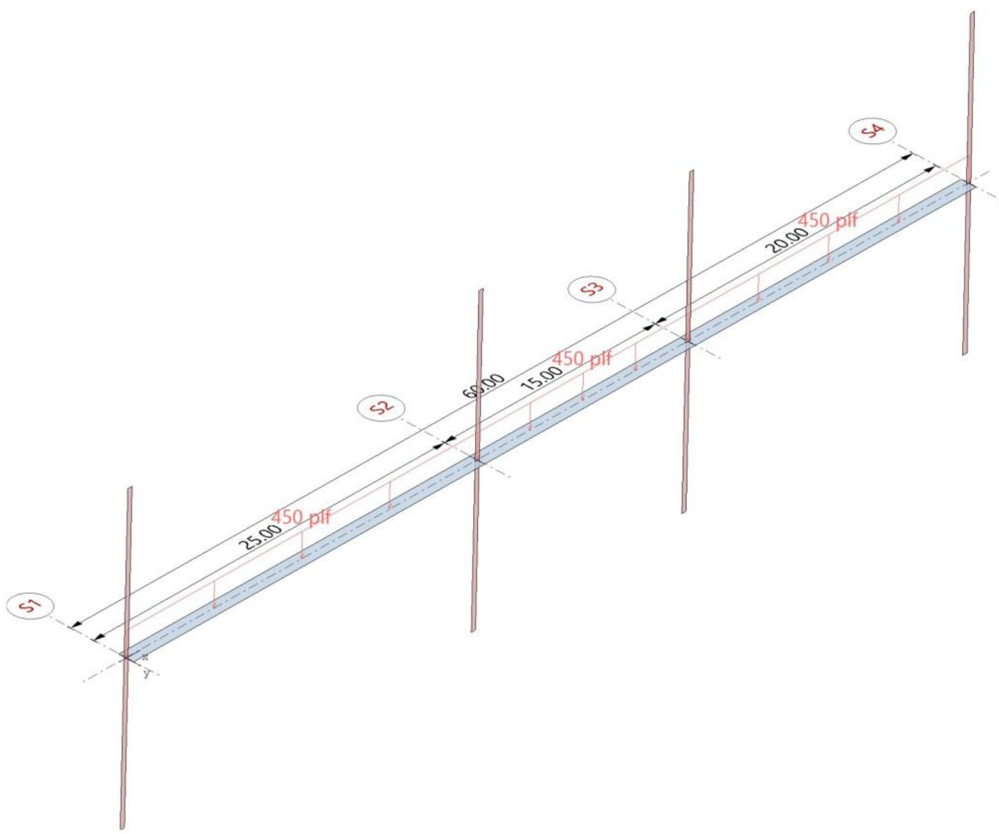
Longitudinal beams

3.2. Loads - Case A - Dead



Project:	Moment Redistribution
Diagram:	Model View (Load Case: A - Dead)
	Columns; Longitudinal beams

3.3. Loads - Case B - Live



Project:	Moment Redistribution
Diagram:	Model View (Load Case: B - Live)
	Columns; Longitudinal beams

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3.4. Internal Forces

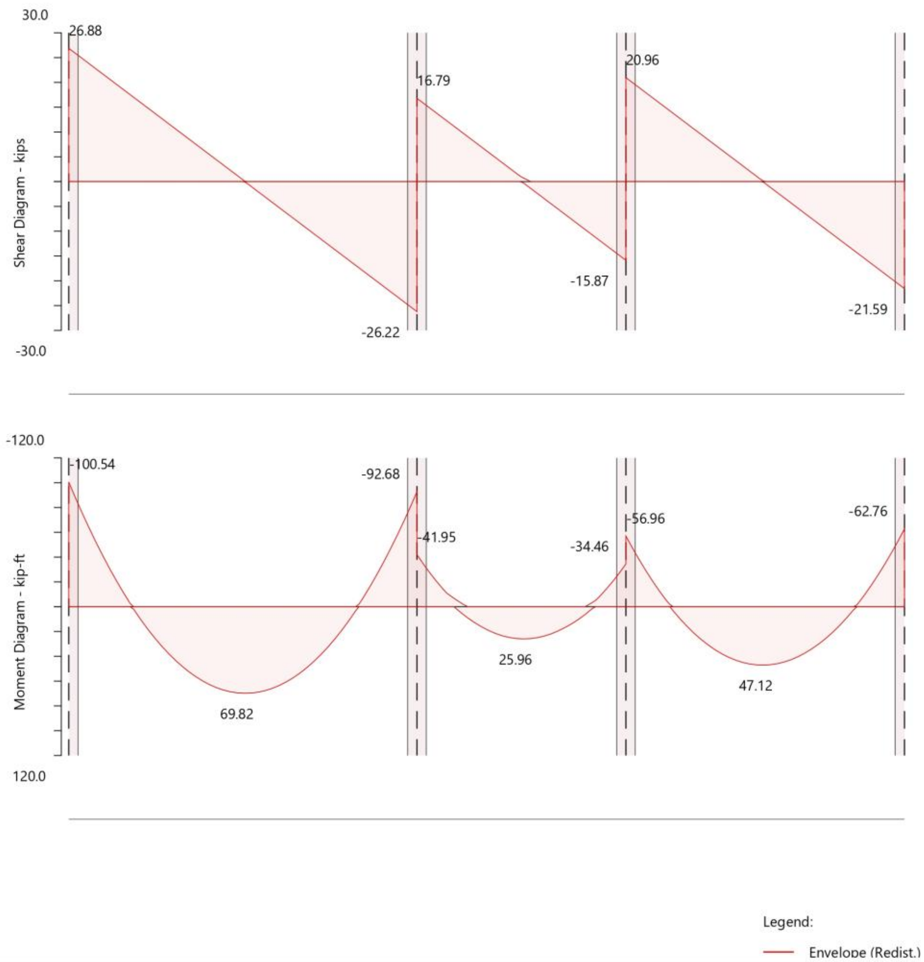

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Diagram: Internal Forces

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3.5. Moment Capacity

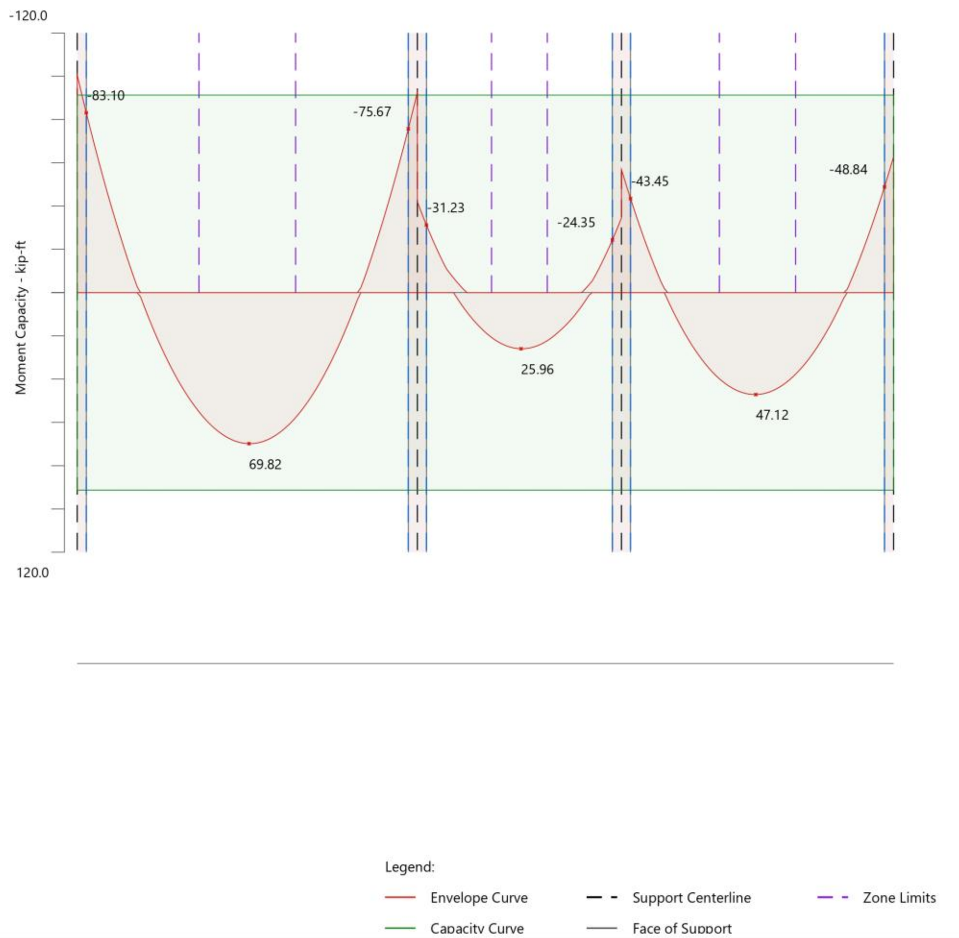

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Diagram: Moment Capacity

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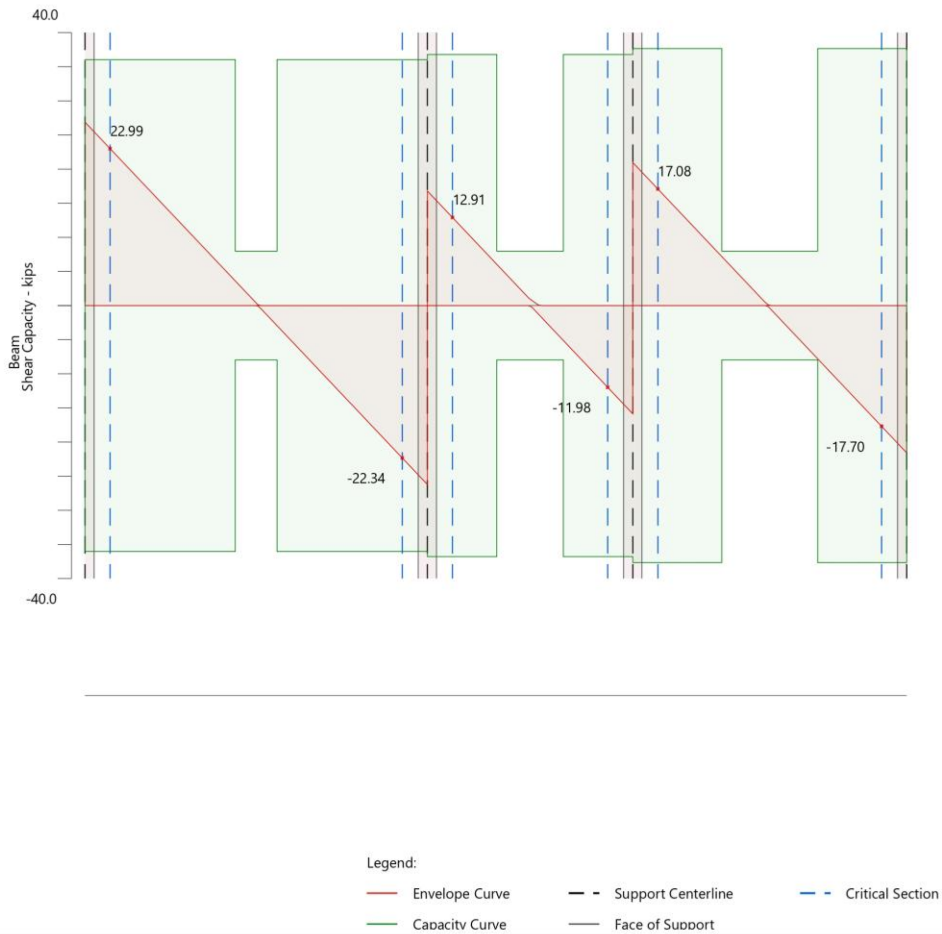
3.6. Shear Capacity



Project: Moment Redistribution

Diagram: Shear Capacity

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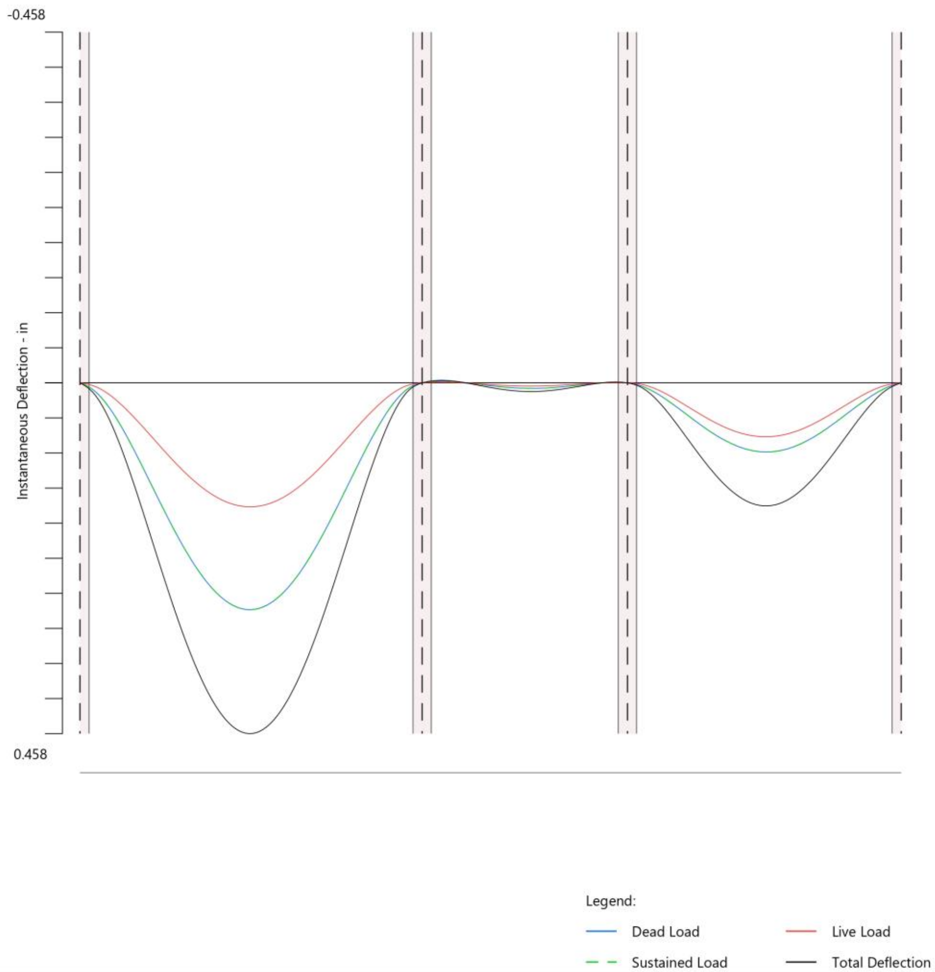
3.7. Deflection



Project: Moment Redistribution

Diagram: Deflection

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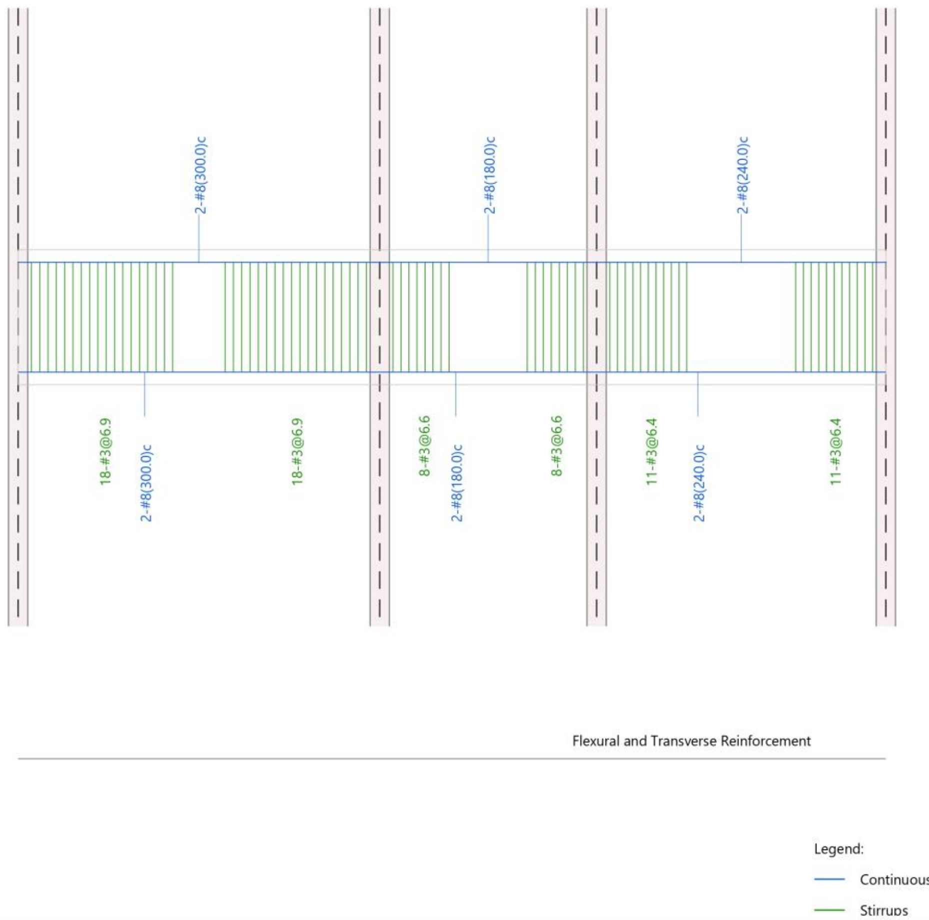
3.8. Reinforcement



Project: Moment Redistribution

Diagram: Reinforcement

4. Design Results Comparison and Conclusions

The following table shows the comparison between hand results and spBeam model results.

Table 5 – Comparison of the Continuous Beam Analysis and Design Results							
Location		M_u (kip-ft) Before Redistribution		M_u (kip-ft) After Redistribution		$A_{s,req}$ (in.^{2})	
		Hand	spBeam	Hand	spBeam	Hand	spBeam
Support A	Right Face	-83.53	-83.53	-83.10	-83.10	1.426	1.426
Midspan A-B		61.60	61.60	69.83	69.82	1.182	1.182
Support B	Left Face	-91.92	-91.92	-75.67	-75.67	1.288	1.288
	Right Face	-41.57	-41.57	-31.23	-31.23	0.509*	0.509*
Midspan B-C		17.56	17.55	25.96	25.96	0.421*	0.421*
Support C	Left Face	-32.97	-32.97	-24.35	-24.35	0.395*	0.395*
	Right Face	-57.22	-57.21	-43.45	-43.45	0.717	0.717
Midspan C-D		40.23	40.23	47.13	47.12	0.780	0.780
Support D	Left Face	-49.30	-49.30	-48.84	-48.84	0.810	0.810

* $A_{s,min}$ governs

The results of all the hand calculations used illustrated above are in precise agreement with the automated exact results obtained from the spBeam program.

The moment redistribution is often utilized for the investigation of existing structures for conditions such as change of use, additional loading, or verifying adequacy for the latest design code. In these conditions, any reserve capacity from existing reinforcement layout at mid-span (or support) of a span may be utilized to compensate for the inadequacy of the support (or mid-span) of the same span.

The moment redistribution can also be utilized in the design of a new structure. One such example of its application may help reduce the negative moment at an interior support and corresponding top reinforcement while increasing the positive moment at mid-span. The advantage of this may be the alleviation of the congestion of rebar at support top regions.

The calculation of moment redistribution is a tedious process especially while considering live load patterning as presented in this example. The procedure gets far more complicated if point loads or partial line loads are present. The spBeam software program performs the moment redistribution calculations with speed and accuracy.