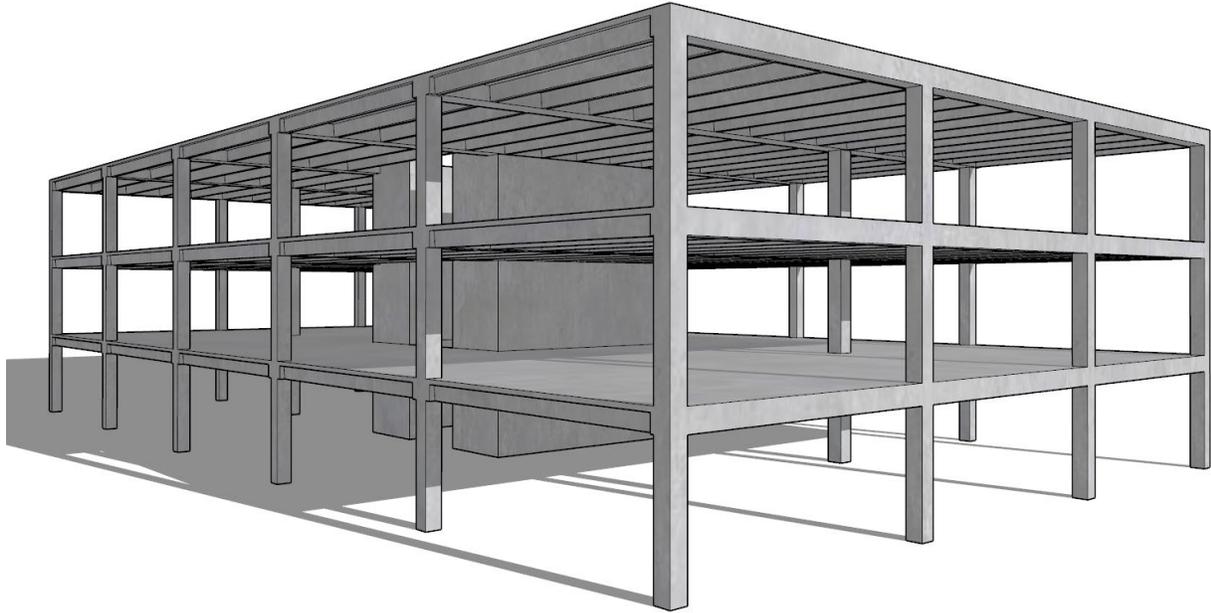


One-Way Wide Module (Skip) Joist Concrete Floor System Design (ACI 318-14)



One-Way Wide Module (Skip) Joist Concrete Floor System Design (ACI 318-14)

A typical floor plan of a 5-story office building is shown below. Wide-module joists, or “skip” joists, are similar to standard one-way joists, except the pans are 53 in. or 66 in. wide. For the 53 in. pans, the pan depth varies from 16 in. to 24 in., and for 66 in. pans, the range is 14 in. to 24 in. Wide-module systems are economical for long spans with heavy loads and improved vibration resistance (see references). The gravity loads treatment is shown in this example and the lateral load effects are resisted by reinforced concrete shear walls. The design procedures shown in ACI 318-14 are illustrated in detail in this example. The hand solution is also used for a detailed comparison with the analysis and design results of the engineering software program [spBeam](#).

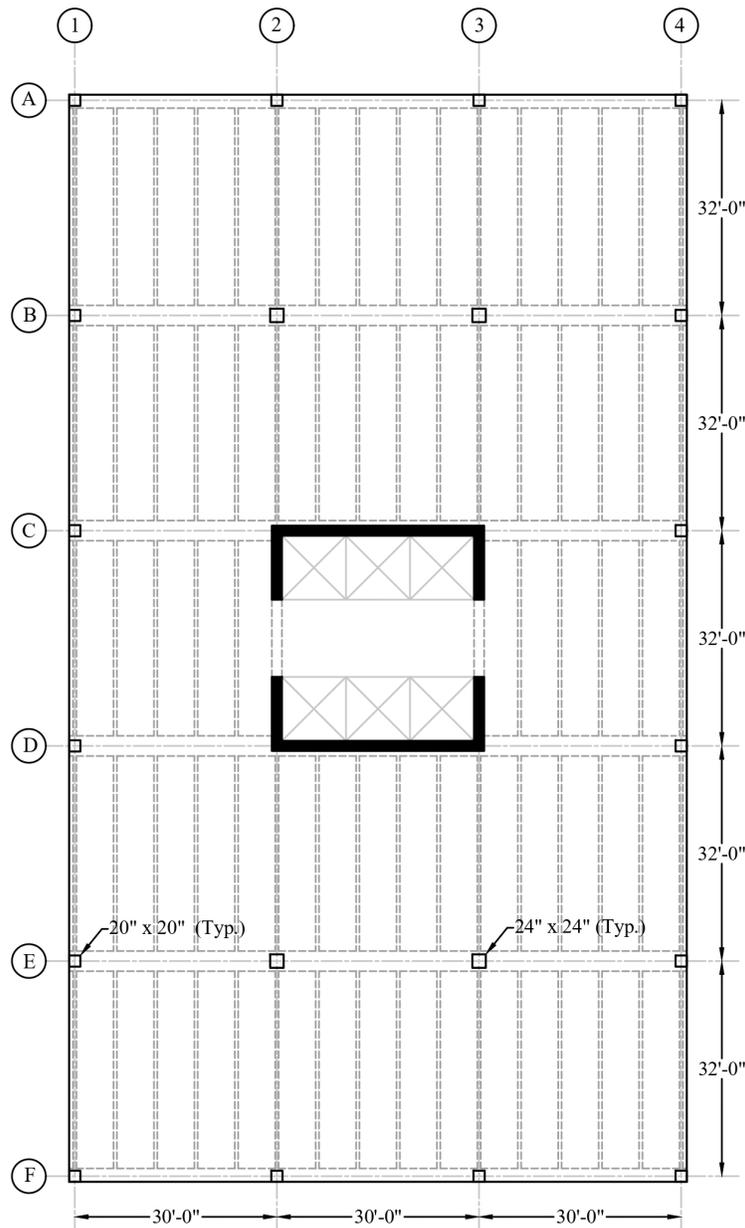


Figure 1 – One-Way Wide Module Joist Concrete Floor Framing System

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Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Minimum Design Loads for Buildings and Other Structures (ASCE/SEI 7-10)

International Code Council, 2012 International Building Code, Washington, D.C., 2012

References

- Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association.
- Concrete Floor Systems (Guide to Estimating and Economizing), Second Edition, 2002 David A. Fanella
- Simplified Design of Reinforced Concrete Buildings, Fourth Edition, 2011 Mahmoud E. Kamara and Lawrence C. Novak
- [spBeam Engineering Software Program Manual v5.50](#), [STRUCTUREPOINT](#), 2018

Design Data

Floor-to-Floor Height = 12 ft (provided by architectural drawings)

$w_c = 150$ pcf

$f_c' = 5,000$ psi

$f_y = 60,000$ psi (For flexural reinforcement)

$f_{yt} = 60,000$ psi (For shear and torsional reinforcement)

Superimposed dead load, $SDL = 20$ psf framed partitions, wood studs plaster 2 sides

ASCE/SEI 7-10 (Table C3-1)

Typical Floor Level, Live load, $L_o = 80$ psf (Office building)

ASCE/SEI 7-10 (Table 4-1)

Roof Live Load, $L_o = 20$ psf (Ordinary flat roofs)

ASCE/SEI 7-10 (Table 4-1)

Required fire resistance rating = 2 hours

1. Preliminary Member Sizing

1.1. Preliminary slab thickness and joist dimensions

In this example deflection will be calculated and checked to satisfy project deflection limits. Minimum member thickness and depths from ACI 318-14 will be used for preliminary sizing.

a) One-way Slab

Using minimum thickness for solid one-way slabs in Table 7.3.1.1 for the solid slab spanning between the ribs.

$$\text{End Spans: } h_s = \frac{l}{24} = \frac{72}{24} = 3.00 \text{ in.} \quad \text{ACI 318-14 (Table 7.3.1.1)}$$

$$\text{Interior Spans: } h_s = \frac{l}{28} = \frac{72}{28} = 2.57 \text{ in.} \quad \text{ACI 318-14 (Table 7.3.1.1)}$$

The minimum slab thickness for wide-module joists for 2-hour fire rating is 4.60 in.

IBC 2012 (Table 720.1(3))

Therefore, select a slab thickness of 5 in. for all spans.

b) One-way Joist

The wide-module joist systems do not meet the limitations of ACI 318-14, 9.8.1.1 through 9.8.1.4. Therefore, the structural members of this type of joist construction shall be designed with standard provisions for slabs and beams.

ACI 318-14 (9.8.1.8)

Using minimum thickness for non-prestressed beams in Table 9.3.1.1. For the ribs (part of the joists) supporting the solid slab.

$$\text{End Span: } h = \frac{l}{18.5} = \frac{384}{18.5} = 20.76 \text{ in. (governs)} \quad \text{ACI 318-14 (Table 9.3.1.1)}$$

$$\text{Interior Span: } h = \frac{l}{21} = \frac{384}{21} = 18.29 \text{ in.} \quad \text{ACI 318-14 (Table 9.3.1.1)}$$

Therefore, select rib depth of 16 in. for a total joist depth of 21 in.

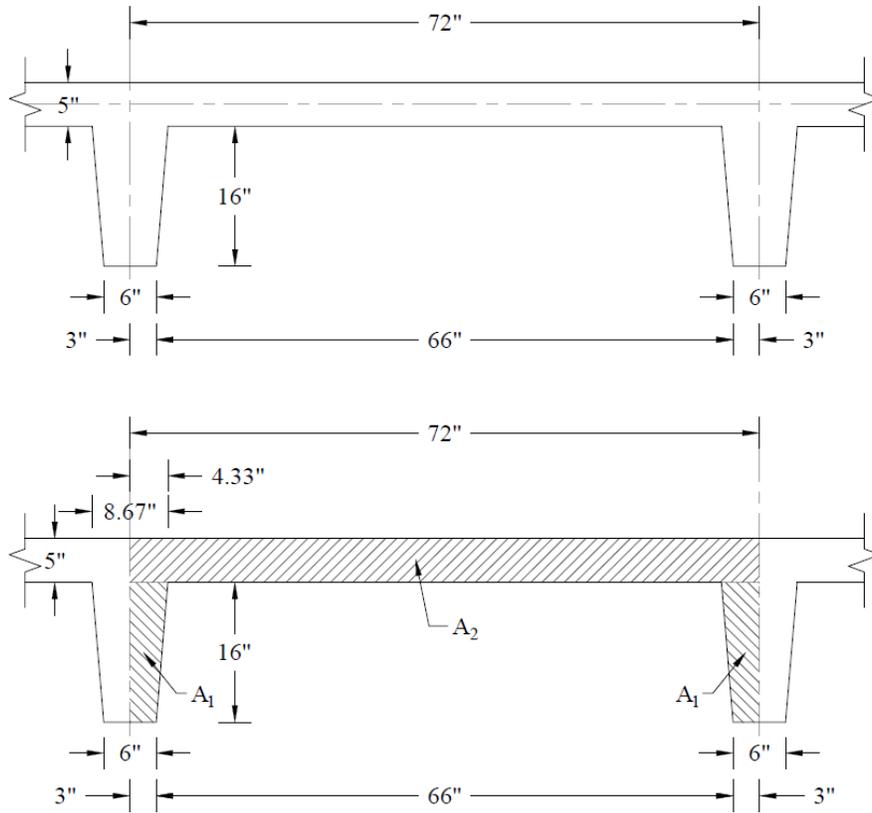


Figure 2 – Slab and Joist Dimensions

1.2. Preliminary column sizes

a) Interior Columns

Select a preliminary size based on the axial load demand. Determine interior column loads as follows:

The governing load combination: $U = 1.20 \times D + 1.60 \times L + 0.50 \times L_r$ ACI 318-14 (Eq. 5.3.1b)

Where:

D = Dead Load, L = Live Load, L_r = Roof Live Load

Typical Floor Level Loads

of Floors = 4

Dead Loads, D

Self-weight of wide-module joist system (see Figure 2):

$$\text{Joist average thickness} = \frac{2 \times A_1 + A_2}{\text{Total Width}} = \frac{2 \times \left(\frac{3 + 4.33}{2} \times 16 \right) + 5 \times 72}{72} = 6.63 \text{ in.} = 0.55 \text{ ft}$$

Weight of the joist = $0.55 \times 150 \text{ pcf} = 82.88 \text{ psf}$.

Superimposed dead load = 20 psf

Live Load, L : Calculate the live load reduction per ASCE/SEI 7-10

$$L = L_o \times \left(0.25 + \frac{15}{\sqrt{K_{LL} \times A_T}} \right) \quad \text{ASCE/SEI 7-10 (Eq. 4-1)}$$

Where:

L = Reduced design live load per ft² of area supported by the member

L_o = Unreduced design live load per ft² of area supported by the member = 80 psf

K_{LL} = Live load element factor = 4 ASCE/SEI 7-10 (Table 4-2)

A_T = Tributary area = $(30 \times 32) = 960 \text{ ft}^2$

$$L = 80 \times \left(0.25 + \frac{15}{\sqrt{4 \times 960}} \right) = 39.36 \text{ psf}$$

Which satisfies $0.40 \times L_o$ requirement for members supporting two or more floors. ASCE/SEI 7-10 (4.7.2)

Roof Level Loads

Dead Loads, D

Self-weight of wide-module joist system (see Figure 2):

$$\text{Joist average thickness} = \frac{2 \times A_1 + A_2}{\text{Total Width}} = \frac{2 \times \left(\frac{3 + 4.33}{2} \times 16 \right) + 5 \times 72}{72} = 6.63 \text{ in.} = 0.55 \text{ ft}$$

Weight of the joist = $0.55 \times 150 \text{ pcf} = 82.88 \text{ psf}$.

No superimposed dead load at the roof

Roof Live Load, L_r : Calculate the roof live load reduction

$$L_r = L_o \times R_1 \times R_2; \quad 12 \leq L_r \leq 20 \quad \text{ASCE/SEI 7-10 (Eq 4-2)}$$

Where:

$$L_o = 20 \text{ psf}$$

$$R_1 = 0.60 \text{ since } A_T = 960 \text{ ft}^2 \geq 600 \text{ ft}^2$$

$$R_2 = 1.00 \text{ for flat roof}$$

$$L_r = 20 \times 0.60 \times 1.00 = 12.00 \text{ psf}$$

Total Factored Load on 1st story interior column (@ 1st interior support)

$$\text{Total Floor Load} = 4 \times [1.20 \times (82.88 + 20) + 1.60 \times 39.36] \times 960 = 715,906.05 \text{ lb} = 715.91 \text{ kips}$$

$$\text{Total Roof Load} = [1.20 \times 82.88 + 1.60 \times 12.00] \times 960 = 113,904.00 \text{ lb} = 113.90 \text{ kips}$$

Assume 24 in square column with 4 – #11 vertical bars with design axial strength, $\phi P_{n,max}$ of

$$\phi P_{n,max} = 0.80 \times \phi \times \left[0.85 \times f'_c \times (A_g - A_{st}) + f_y \times A_{st} \right] \quad \text{ACI 318-14 (22.4.2)}$$

$$\phi P_{n,max} = 0.80 \times 0.65 \times \left[0.85 \times 5,000 \times ((24 \times 24 - 4 \times 1.56) + 60,000 \times 4 \times 1.56) \right] = 1,453,857.60 \text{ lb}$$

$$\phi P_{n,max} = 1,453.86 \text{ kips}$$

$$\text{Column Self-weight} = \left[1.20 \times \left(\frac{24 \times 24}{144} \right) \times 0.15 \right] \times (5 \times 12) = 43.20 \text{ kips}$$

$$\text{Total Reaction @ 1st interior support} = 1.15 \times (715.91 + 113.90) + 43.20 = 997.48 \text{ kips} < 1,453.86 \text{ kips}$$

Therefore, the preliminary interior column size of 24 in. \times 24 in. is adequate.

b) Edge (Exterior) Columns

Select a preliminary size based on the axial load demand. Therefore, the load take-down for an edge column is done as follows:

The governing load combination: $U = 1.20 \times D + 1.60 \times L + 0.50 \times L_r$ ACI 318-14 (Eq. 5.3.1b)

Typical Floor Level Loads

of Floors = 4

Dead Loads, D

Self-weight of wide-module joist system (see Figure 2):

Weight of the joist = 82.88 psf

Superimposed dead load = 20 psf

Live Load, L : Calculate the live load reduction per ASCE/SEI 7-10

$$L = L_o \times \left(0.25 + \frac{15}{\sqrt{K_{LL} \times A_T}} \right) \quad \text{ASCE/SEI 7-10 (Eq. 4-1)}$$

Where:

L = Reduced design live load per ft² of area supported by the member

L_o = Unreduced design live load per ft² of area supported by the member = 80 psf

K_{LL} = Live load element factor = 4 ASCE/SEI 7-10 (Table 4-2)

$$A_T = \text{Tributary area} = \left(\frac{30 \times 32}{2} \right) = 480 \text{ ft}^2$$

$$L = 80 \times \left(0.25 + \frac{15}{\sqrt{4 \times 480}} \right) = 47.39 \text{ psf}$$

Which satisfies $0.40 \times L_o$ requirement for members supporting two or more floors. ASCE/SEI 7-10 (4.7.2)

Roof Level Loads

Dead Loads, D

Weight of the joist = 82.88 psf

No superimposed dead load at the roof

Roof Live Load, L_r : Calculate the roof live load reduction

$$L_r = L_o \times R_1 \times R_2; \quad 12 \leq L_r \leq 20 \quad \text{ASCE/SEI 7-10 (Eq 4-2)}$$

Where:

$$L_o = 20 \text{ psf}$$

$$R_1 = 1.20 - 0.001 \times A_T = 1.20 - 0.001 \times 480 = 0.72, \text{ since } 200 \text{ ft}^2 < A_T = 480 \text{ ft}^2 < 600 \text{ ft}^2$$

$$R_2 = 1.00 \text{ for flat roof}$$

$$L_r = 20 \times 0.72 \times 1.00 = 14.40 \text{ psf}$$

Total Factored Load on 1st story edge column (@ 1st interior support)

$$\text{Total Floor Load} = 4 \times [1.20 \times (82.88 + 20) + 1.60 \times 47.39] \times 480 = 382,594.18 \text{ lb} = 382.59 \text{ kips}$$

$$\text{Total Roof Load} = [1.20 \times 82.88 + 1.60 \times 14.40] \times 480 = 58,795.20 \text{ lb} = 58.80 \text{ kips}$$

Assume 20 in square column with 4 - #11 vertical bars with design axial strength, $\phi P_{n,max}$ of

$$\phi P_{n,max} = 0.80 \times \phi \times \left[0.85 \times f'_c \times (A_g - A_{st}) + f_y \times A_{st} \right] \quad \text{ACI 318-14 (22.4.2)}$$

$$\phi P_{n,max} = 0.80 \times 0.65 \times \left[0.85 \times 5,000 \times ((20 \times 20 - 4 \times 1.56) + 60,000 \times 4 \times 1.56) \right] = 1,064,897.60 \text{ lb}$$

$$\phi P_{n,max} = 1,064.90 \text{ kips}$$

$$\text{Column Self-weight} = \left[1.20 \times \left(\frac{20 \times 20}{144} \right) \times 0.15 \right] \times (5 \times 12) = 30.00 \text{ kips}$$

$$\text{Total Reaction @ 1}^{\text{st}} \text{ interior support} = 1.15 \times (382.59 + 58.80) + 30.00 = 537.60 \text{ kips} < 1,064.90 \text{ kips}$$

Therefore, the preliminary edge column size of 20 in. \times 20 in. is adequate.

e) Corner Columns

Select a preliminary size based on the axial load demand. Therefore, the load take-down for a corner column is done as follows:

The governing load combination: $U = 1.20 \times D + 1.60 \times L + 0.50 \times L_r$ ACI 318-14 (Eq. 5.3.1b)

Typical Floor Level Loads

of Floors = 4

Dead Loads, D

Self-weight of wide-module joist system (see Figure 2):

Weight of the joist = 82.88 psf

Superimposed dead load = 20 psf

Live Load, L : Calculate the live load reduction per ASCE/SEI 7-10

$$L = L_o \times \left(0.25 + \frac{15}{\sqrt{K_{LL} \times A_T}} \right) \quad \text{ASCE/SEI 7-10 (Eq. 4-1)}$$

Where:

L = Reduced design live load per ft² of area supported by the member

L_o = Unreduced design live load per ft² of area supported by the member = 80 psf

K_{LL} = Live load element factor = 4 ASCE/SEI 7-10 (Table 4-2)

$$A_T = \text{Tributary area} = \left(\frac{30 \times 32}{4} \right) = 240 \text{ ft}^2$$

$$L = 80 \times \left(0.25 + \frac{15}{\sqrt{4 \times 240}} \right) = 58.73 \text{ psf}$$

Which satisfies $0.40 \times L_o$ requirement for members supporting two or more floors.

ASCE/SEI 7-10 (4.7.2)

Roof Level Loads

Dead Loads, D

Weight of the joist = 82.88 psf

No superimposed dead load at the roof

Roof Live Load, L_r : Calculate the roof live load reduction

$$L_r = L_o \times R_1 \times R_2; \quad 12 \leq L_r \leq 20 \quad \text{ASCE/SEI 7-10 (Eq 4-2)}$$

Where:

$$L_o = 20 \text{ psf}$$

$$R_1 = 1.20 - 0.001 \times A_T = 1.20 - 0.001 \times 240 = 0.96 \text{ since } 200 \text{ ft}^2 < A_T = 240 \text{ ft}^2 < 600 \text{ ft}^2$$

$$R_2 = 1.00 \text{ for flat roof}$$

$$L_r = 20 \times 0.96 \times 1.00 = 19.20 \text{ psf}$$

Total Factored Load on 1st story corner column (@ exterior support)

$$\text{Total Floor Load} = 4 \times [1.20 \times (82.88 + 20) + 1.60 \times 59.73] \times 240 = 208,721.02 \text{ lb} = 208.72 \text{ kips}$$

$$\text{Total Roof Load} = [1.20 \times 82.88 + 1.60 \times 19.20] \times 240 = 31,240.80 \text{ lb} = 31.24 \text{ kips}$$

Assume 20 in square column with 4 - #11 vertical bars with design axial strength, $\phi P_{n,max}$ of

$$\phi P_{n,max} = 0.80 \times \phi \times [0.85 \times f'_c \times (A_g - A_{st}) + f_y \times A_{st}] \quad \text{ACI 318-14 (22.4.2)}$$

$$\phi P_{n,max} = 0.80 \times 0.65 \times [0.85 \times 5,000 \times ((20 \times 20 - 4 \times 1.56) + 60,000 \times 4 \times 1.56)] = 1,064,897.60 \text{ lb}$$

$$\phi P_{n,max} = 1,064.90 \text{ kips}$$

$$\text{Column Self-weight} = \left[1.20 \times \left(\frac{20 \times 20}{144} \right) \times 0.15 \right] \times (5 \times 12) = 30.00 \text{ kips}$$

$$\text{Total Reaction @ 1st interior support} = 208.72 + 31.24 + 30.00 = 269.96 \text{ kips} < 1,064.90 \text{ kips}$$

Therefore, the preliminary edge column size of 20 in. × 20 in. is adequate.

2. Design of Structural Members

The design of the following structural members is performed and compared with results of the engineering software program [spBeam](#):

- 2.1. One-Way Slab
- 2.2. One-Way Joist
- 2.3. Interior Beam
- 2.4. Exterior Beam
- 2.5. Interior Column

2.1. One-Way Slab Design

A unit strip of 1 ft is considered for the design of slab spanning between ribs. Note that ACI 318-14 does not allow live load reduction for one-way slabs.

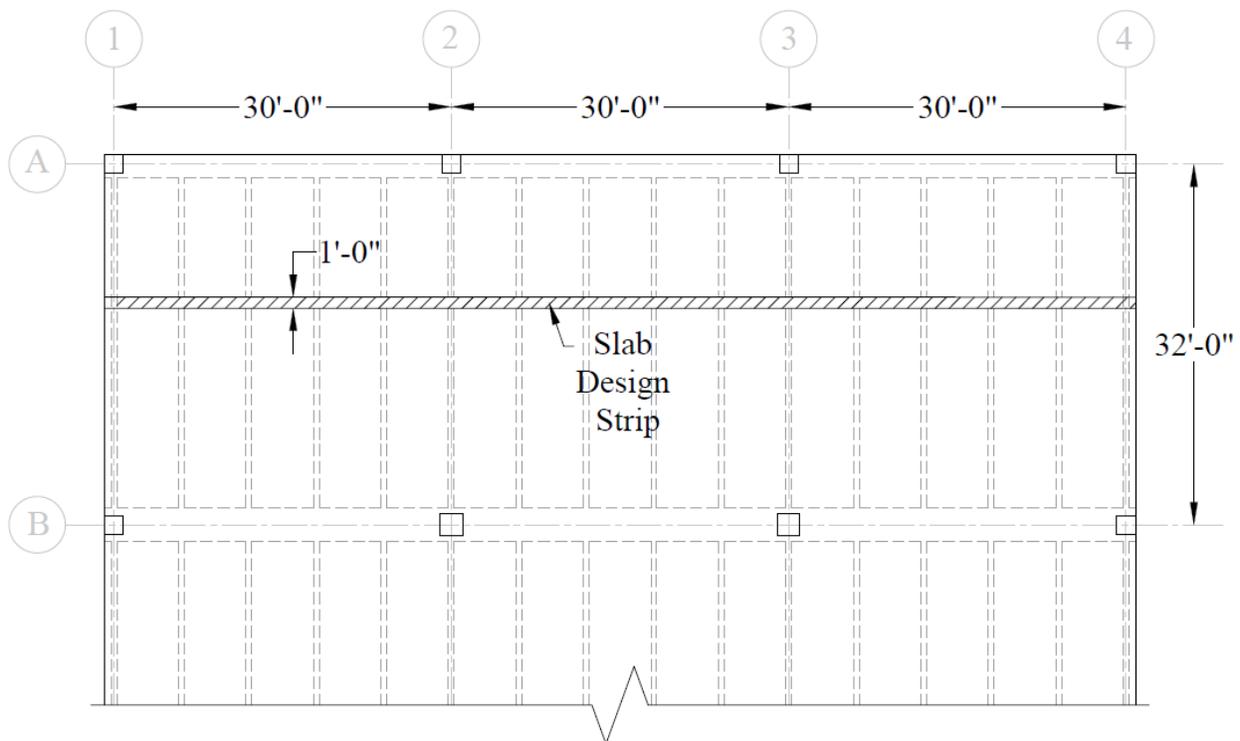


Figure 3 – Partial Plan View Illustrating Slab Design Strip

Slab design involves the following steps:

- 2.1.1. Determination of span loads
- 2.1.2. Determination of design moments and shears

- 2.1.3. Flexural Design
- 2.1.4. Shear Design
- 2.1.5. Deflections
- 2.1.6. Computer Program Solution
- 2.1.7. Summary and comparison of design results
- 2.1.8. Conclusions and observations

2.1.1. Determination of span loads

The following gravity load combinations are considered:

$$U = 1.40 \times D \quad \text{ACI 318-14 (Eq. 5.3.1a)}$$

$$w_u = 1.40 \times \left(\left(\frac{5}{12} \times 0.15 \right) + 0.02 \right) = 0.12 \text{ kips/ft per ft}$$

$$U = 1.20 \times D + 1.60 \times L \quad \text{ACI 318-14 (Eq. 5.3.1b)}$$

$$w_u = 1.20 \times \left(\left(\frac{5}{12} \times 0.15 \right) + 0.02 \right) + 1.60 \times 0.08 = 0.23 \text{ kips/ft per ft}$$

Span loads are governed by the second load combination.

2.1.2. Determination of design moment and shear

The factored moment and shear can be determined using the simplified method if the requirements are satisfied: ACI 318-14 (6.5.1)

- ✓ Members are prismatic.
- ✓ Loads are uniformly distributed.
- ✓ $L \leq 3D$ (0.08 kips/ft per ft $\leq 3 \times 0.0825$ kips/ft per ft)
- ✓ There are at least two spans.
- ✓ The longer of two adjacent spans does not exceed the shorter by more than 20 percent.

Thus, the approximate coefficients can be used. The factored moments and shears are determined and summarized in the following tables. ACI 318-14 (Table 6.5.2 and Table 6.5.3)

Table 1 – One-Way Slab Design Moment Values		
Location		Design Moment Value
End Spans	Exterior Support Negative	$\frac{w_u \times l_n^2}{24} = \frac{0.23 \times 5.50^2}{24} = 0.29 \frac{\text{ft-kips}}{\text{ft}}$
	Mid-span	$\frac{w_u \times l_n^2}{14} = \frac{0.23 \times 5.50^2}{14} = 0.49 \frac{\text{ft-kips}}{\text{ft}}$
	Interior Support Negative	$\frac{w_u \times l_n^2}{10} = \frac{0.23 \times 5.50^2}{10} = 0.69 \frac{\text{ft-kips}}{\text{ft}}$
Interior Spans	Mid-span Positive	$\frac{w_u \times l_n^2}{16} = \frac{0.23 \times 5.50^2}{16} = 0.43 \frac{\text{ft-kips}}{\text{ft}}$
	Support Negative	$\frac{w_u \times l_n^2}{11} = \frac{0.23 \times 5.50^2}{11} = 0.62 \frac{\text{ft-kips}}{\text{ft}}$

Table 2 – One-Way Slab Design Shear Values	
Location	Design Shear Value
End Span at Face of First Interior Support	$1.15 \times \frac{w_u \times l_n}{2} = 1.15 \times \frac{0.23 \times 5.50}{2} = 0.72 \frac{\text{kips}}{\text{ft}}$
At Face of all other Supports	$\frac{w_u \times l_n}{2} = \frac{0.23 \times 5.50}{2} = 0.62 \frac{\text{kips}}{\text{ft}}$

2.1.3. Flexural Design

For the one-way slab of a wide-module joist system, a single layer of longitudinal reinforcement is provided. The first interior support negative moment governs the design as tabulated in [Table 1](#). Therefore, it is favorable to place the single layer reinforcement closer to the top fiber of the concrete slab. The required reinforcement shall be calculated for the first interior support negative moment first. The required reinforcement for the end span positive moment shall also be calculated as the low effective depth due to the reinforcement location may govern the required reinforcement amount. Finally, the required reinforcement for design shall be checked against the minimum shrinkage and temperature reinforcement requirement per [ACI 318-14 \(24.4.3.2\)](#).

Calculate the required reinforcement to resist the first interior support negative moment:

$$M_u = 0.69 \text{ ft-kips/ft}$$

Use welded wire fabric reinforcement, 6×6 -W5.5 \times W5.5 with 1.5 in. concrete cover. The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, d , is calculated below:

$$d = 5.00 - 1.50 - \frac{0.26}{2} = 3.37 \text{ in.}$$

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section (jd). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.9, and jd will be taken equal to $0.992 \times d$. The assumptions will be verified once the area of steel is finalized.

$$\text{Assume } jd = 0.992 \times d = 0.992 \times 3.37 = 3.34 \text{ in.}$$

Unit strip width, $b = 12$ in.

$$A_s = \frac{M_u}{\phi \times f_y \times jd} = \frac{0.69 \times 12,000}{0.90 \times 60,000 \times 3.34} = 0.046 \frac{\text{in}^2}{\text{ft}}$$

Recalculate 'a' for the actual $A_s = 0.046 \text{ in.}^2$ per ft:

$$a = \frac{A_s \times f_y}{0.85 \times f'_c \times b} = \frac{0.046 \times 60,000}{0.85 \times 5,000 \times 12} = 0.054 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.054}{0.85} = 0.063 \text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c} \right) \times d_t - 0.003 = \left(\frac{0.003}{0.063} \right) \times 3.37 - 0.003 = 0.157 > 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi \times f_y \times \left(d - \frac{a}{2}\right)} = \frac{0.69 \times 12,000}{0.90 \times 60,000 \times \left(3.37 - \frac{0.054}{2}\right)} = 0.046 \frac{\text{in}^2}{\text{ft}}$$

Calculate the required reinforcement to resist the positive moment:

$$M_u = 0.49 \text{ ft-kips/ft}$$

The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement:

$$d = 5.00 - 3.37 = 1.63 \text{ in.}$$

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section (jd). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.9, and jd will be taken equal to $0.975 \times d$. The assumptions will be verified once the area of steel is finalized.

$$\text{Assume } jd = 0.975 \times d = 0.975 \times 1.63 = 1.59 \text{ in.}$$

Unit strip width, $b = 12 \text{ in.}$

$$A_s = \frac{M_u}{\phi \times f_y \times jd} = \frac{0.49 \times 12,000}{0.90 \times 60,000 \times 1.59} = 0.068 \frac{\text{in}^2}{\text{ft}}$$

Recalculate 'a' for the actual $A_s = 0.068 \text{ in.}^2$:

$$a = \frac{A_s \times f_y}{0.85 \times f'_c \times b} = \frac{0.070 \times 60,000}{0.85 \times 5,000 \times 12} = 0.081 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.081}{0.85} = 0.095 \text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right) \times d_t - 0.003 = \left(\frac{0.003}{0.095}\right) \times 1.63 - 0.003 = 0.049 > 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi \times f_y \times \left(d - \frac{a}{2}\right)} = \frac{0.49 \times 12,000}{0.90 \times 60,000 \times \left(1.63 - \frac{0.081}{2}\right)} = 0.068 \frac{\text{in}^2}{\text{ft}}$$

Check the shrinkage and temperature reinforcement requirement:

$$A_s = 0.0018 \times b \times h = 0.0018 \times 12 \times 5 = 0.108 \frac{\text{in}^2}{\text{ft}} \geq 0.0014 \frac{\text{in}^2}{\text{ft}} \quad \text{ACI 318-14 (Table 24.4.3.2)}$$

Check reinforcement spacing for crack control:

The maximum spacing of the flexural reinforcement closest to the tension face of the slab shall be:

$$s = 15 \times \left(\frac{40,000}{f_s} \right) - 2.5 \times c_c, \text{ but not greater than } 12 \times \left(\frac{40,000}{f_s} \right) \quad \text{ACI 318-14 (Table 24.3.2)}$$

Where:

s = Maximum reinforcement spacing for crack control, in.

f_s = Calculated stress in reinforcement closest to the tension face at service load, ksi

c_c = The least distance from surface of reinforcement to the tension face, in.

$$\text{Use } f_s = \frac{2}{3} \times f_y = \frac{2}{3} \times 60,000 = 40,000 \text{ psi} \quad \text{ACI 318-14 (24.3.2.1)}$$

c_c = 1.50 in. for reinforcement resisting negative moment at supports (i.e. tension at the top)

c_c = 3.24 in. for reinforcement resisting positive moment at mid-span (i.e. tension at the bottom)

Thus,

At supports

$$s = 15 \times \left(\frac{40,000}{f_s} \right) - 2.50 \times c_c = 15 \times \left(\frac{40,000}{40,000} \right) - 2.50 \times 1.50 = 11.25 \text{ in. (governs @ support)}$$

$$\text{But not greater than } s = 12 \times \left(\frac{40,000}{f_s} \right) = 12 \times \left(\frac{40,000}{40,000} \right) = 12 \text{ in.}$$

At mid-span

$$s = 15 \times \left(\frac{40,000}{f_s} \right) - 2.50 \times c_c = 15 \times \left(\frac{40,000}{40,000} \right) - 2.50 \times 3.24 = 6.90 \text{ in. (governs @ mid-span)}$$

$$\text{But not greater than } s = 12 \times \left(\frac{40,000}{f_s} \right) = 12 \times \left(\frac{40,000}{40,000} \right) = 12 \text{ in.}$$

Therefore, for this one-way slab, the shrinkage and temperature reinforcement requirement per ACI 318-14 (Table 24.4.3.2) governs the required reinforcement area ($A_s = 0.108 \text{ in}^2/\text{ft}$) and crack control requirement per ACI 318-14 (Table 24.3.2) governs the reinforcement spacing ($s = 6.90 \text{ in.}$).

The most feasible reinforcement solution that meets both requirements mentioned above is to provide welded wire fabric reinforcement, 6×6 -W5.5 \times W5.5. Note that the welded wire reinforcement selected provides minimum shrinkage and temperature reinforcement in the slab direction parallel to the joists as well. Alternately, deformed bars can be utilized in lieu of welded wire fabric. It should be noted that two conditions specific to this design contribute to having such a stringent spacing requirement.

These are listed below:

- The 5 in. slab has a single layer reinforcement that is placed near the top surface (i.e. clear cover from the top surface to the reinforcement is 1.50 in. This result in a high c_c value for the calculation of reinforcement spacing for crack control due to positive moment.
- The stress in reinforcement closest to the tension face at service load, f_s , is taken as $2/3 f_y$ as permitted by ACI 318-14 without calculation. It is very likely that under the loading considered, the stress in the steel will be lower than $2/3 f_y$. The f_s value is expected to be in the range of $1/3 f_y$ to $1/2 f_y$. Even if it is assumed to be $1/2 f_y$, s value will be 12 in.

2.1.4. Shear Design

From [Table 2](#) above, the shear value in end span at face of first interior support governs.

$$V_u = 1.15 \times \frac{w_u \times l_n}{2} = 1.15 \times \frac{0.23 \times 5.50}{2} = 0.72 \frac{\text{kips}}{\text{ft}}$$

The design shear at a distance, d , away from the face of support,

$$V_u = 0.72 - 0.23 \times \frac{1.63}{12} = 0.69 \frac{\text{kips}}{\text{ft}}$$

Shear strength provided by concrete

$$\phi V_c = \phi \times (2 \times \sqrt{f'_c} \times b_w \times d)$$

ACI 318-14 (Eq. 22.5.5.1)

$$\phi V_c = 0.75 \times (2.00 \times 1.00 \times \sqrt{5,000} \times 12 \times 1.63) = 2077.20 \frac{\text{lb}}{\text{ft}} = 2.08 \frac{\text{kips}}{\text{ft}}$$

$$V_u = 0.69 \frac{\text{kips}}{\text{ft}} < \phi V_c = 2.08 \frac{\text{kips}}{\text{ft}}$$

Therefore, the slab shear capacity is adequate.

2.1.5. Deflections

Since the preliminary slab thickness met minimum thickness requirement, the deflection calculations are not required. Unless governed by fire rating requirements; as in this example; lesser thicknesses and consequently cost savings can be achieved through deflection computations. Deflection values are calculated and provided for every model created by [spBeam](#) Program and can be used by the engineer to make additional optimization decisions.

2.1.6. Computer Program Solution

[spBeam](#) Program can be utilized to analysis and design beams and one-way slab systems. The one-way slab is modeled as 1-ft unit strip supported on ribs. The ribs provide some rotational stiffness at the supports. In [spBeam](#) solution, the rotational stiffness is assumed as 32,000* kip-in/rad for modeling the joist supports. Also, for one-way slab run, the rib widths assumed as 6 in. and modeled through dummy columns of 6 in. × 12 in. with zero height (i.e. column stiffness is zero, but the 6 in. dimension of the column is utilized to push the design moments 3 in. from the support centerline). In this example, user-defined bar size #2 is defined in [spBeam](#) to represent welded wire fabric, W5.5, with the cross-sectional area of 0.055 in² (see [Figure 4](#)).

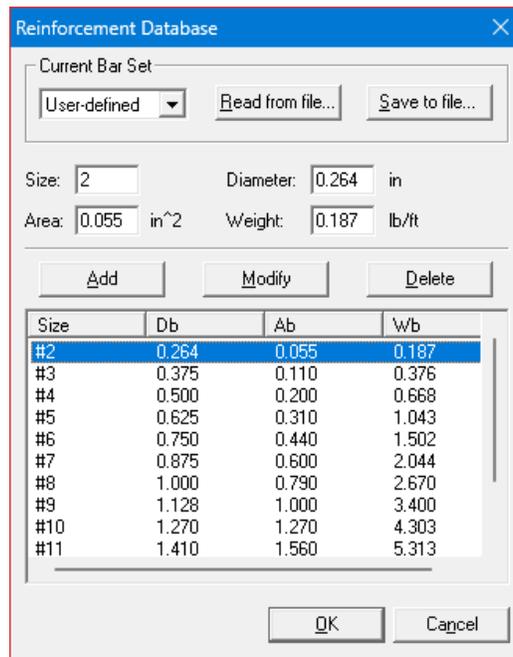


Figure 4 – [spBeam](#) Reinforcement Database – User-defined Bar Set

The program calculates the internal forces (shear force and bending moment), moment and shear capacities, immediate and long-term deflections, and required reinforcements. The graphical and text results are provided below for input and output of the [spBeam](#) program. The graphical and text results are provided here for both input and output of the [spBeam](#) model.

* Refer to [spBeam](#) manual (Chapter 2 – Special Considerations for One and Two-Way Joist Systems)

* Refer to [spBeam](#) manual (Chapter 4 – Defining Boundary Conditions, Rotational Stiffness)

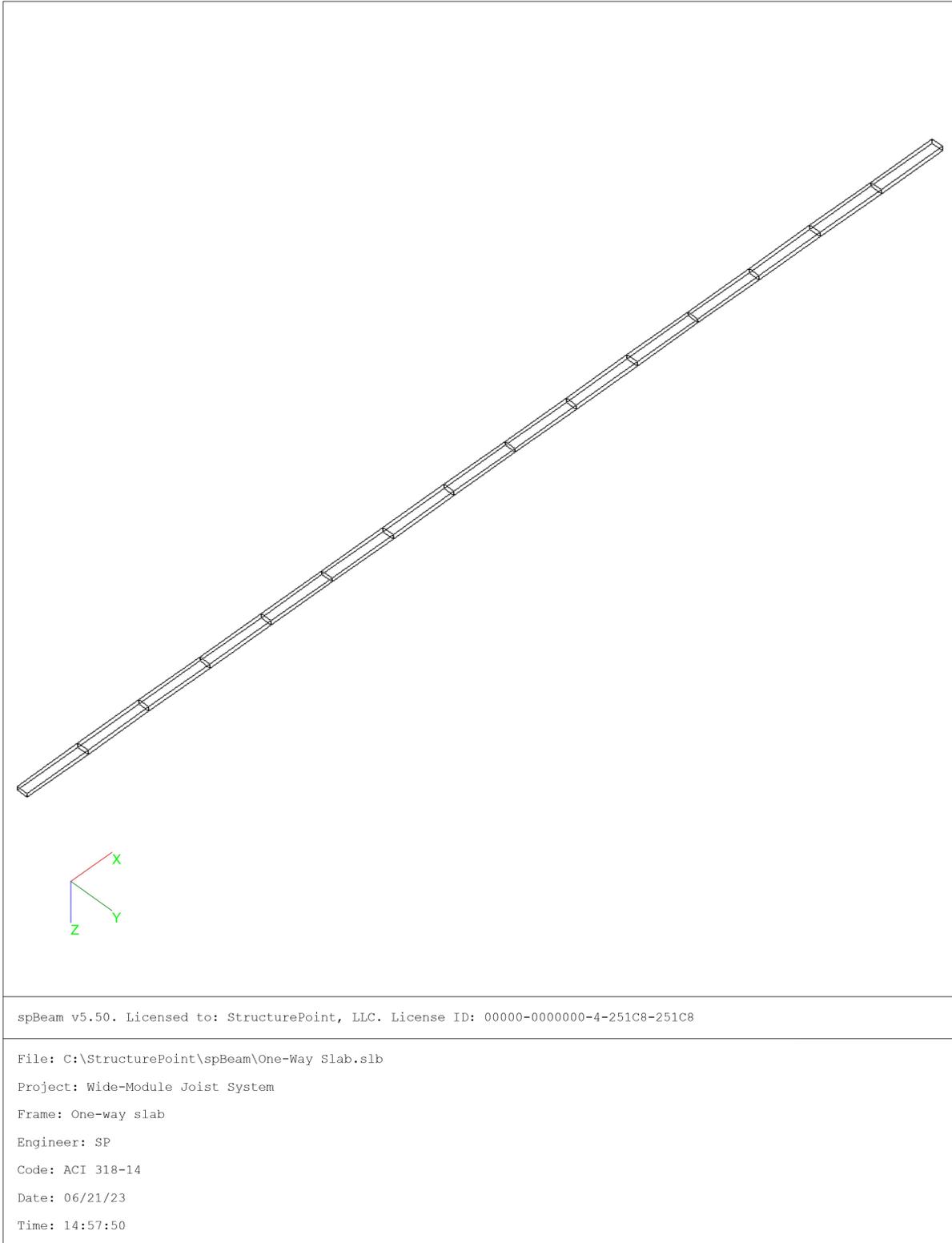


Figure 5 – spBeam Model – Isometric View of 15 Span – 1ft Wide Unit Strip of One-Way Slab

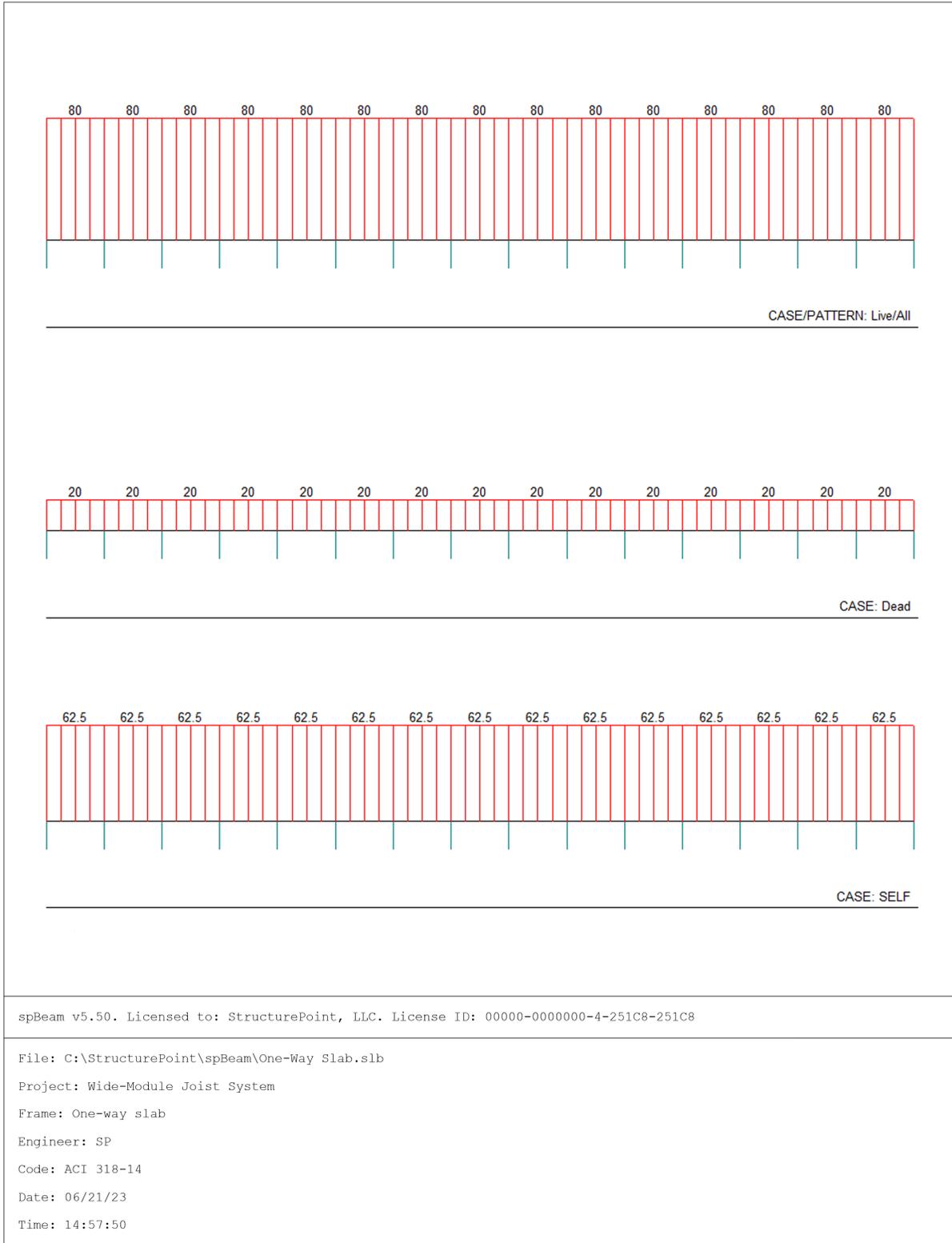


Figure 6 – [spBeam](#) Model – Loads (Including Live Load Patterning) units in lb/ft²

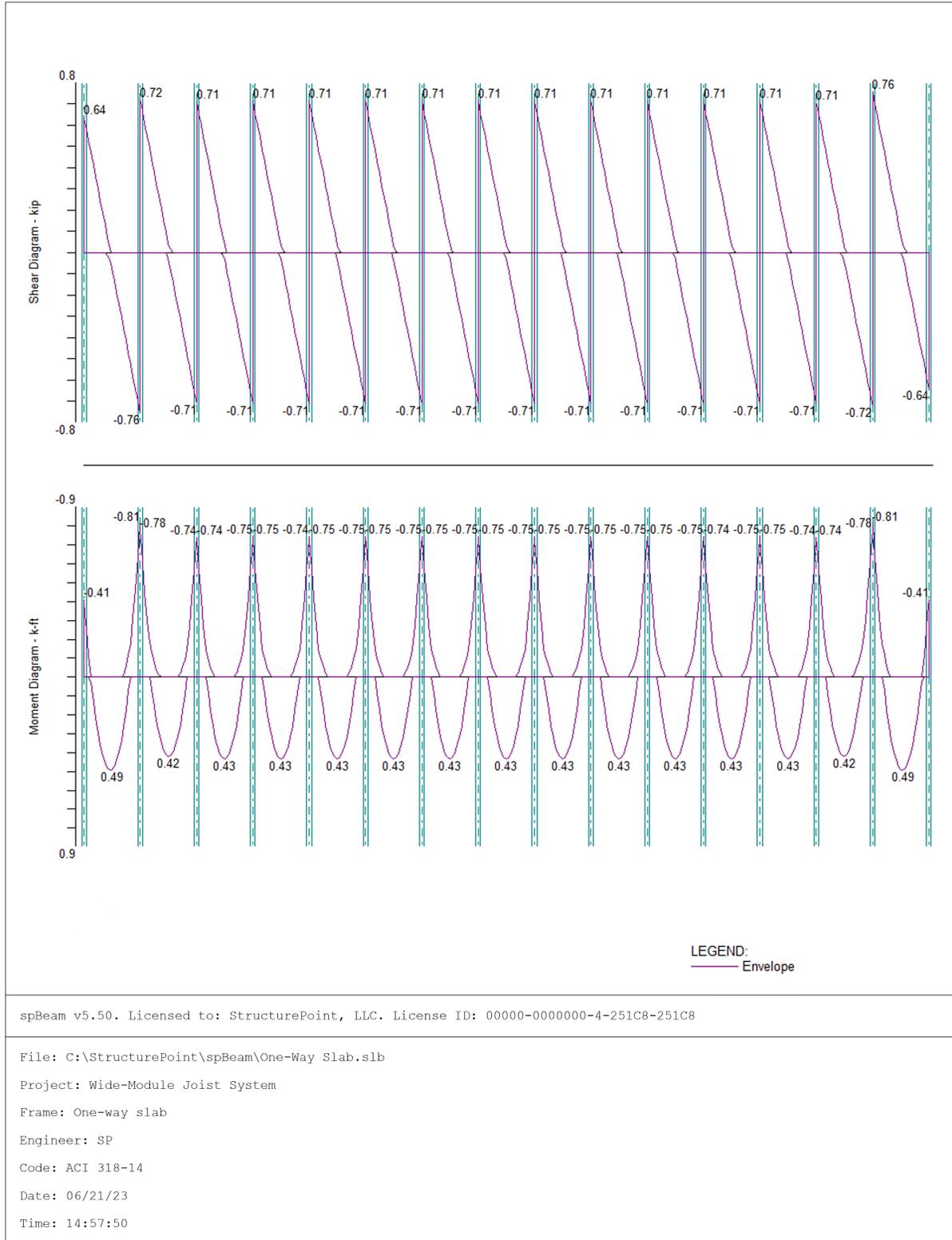


Figure 7 – spBeam Model – Internal Forces (Shear Force Diagram and Bending Moment Diagram)

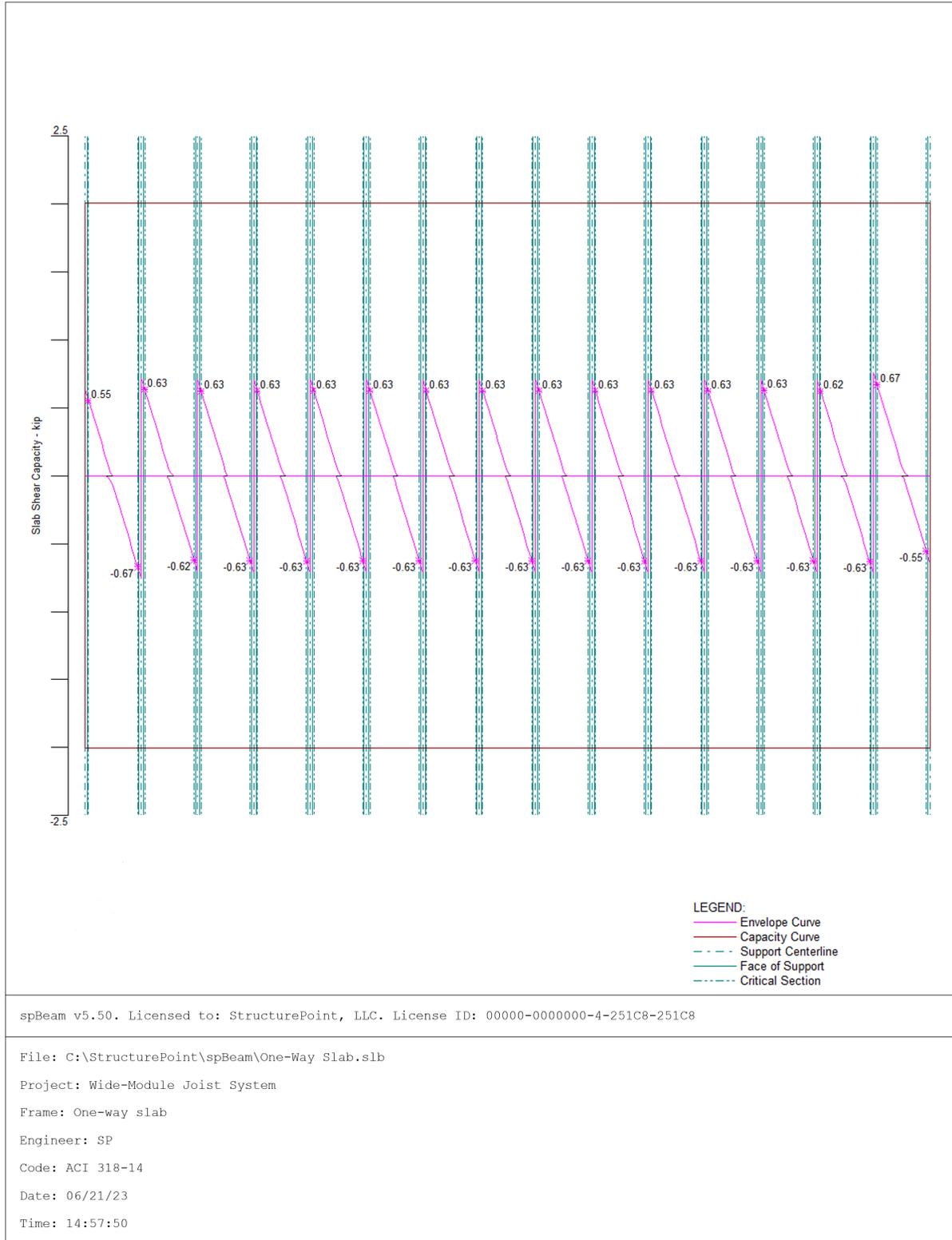


Figure 9 – spBeam Model – Shear Capacity Diagram

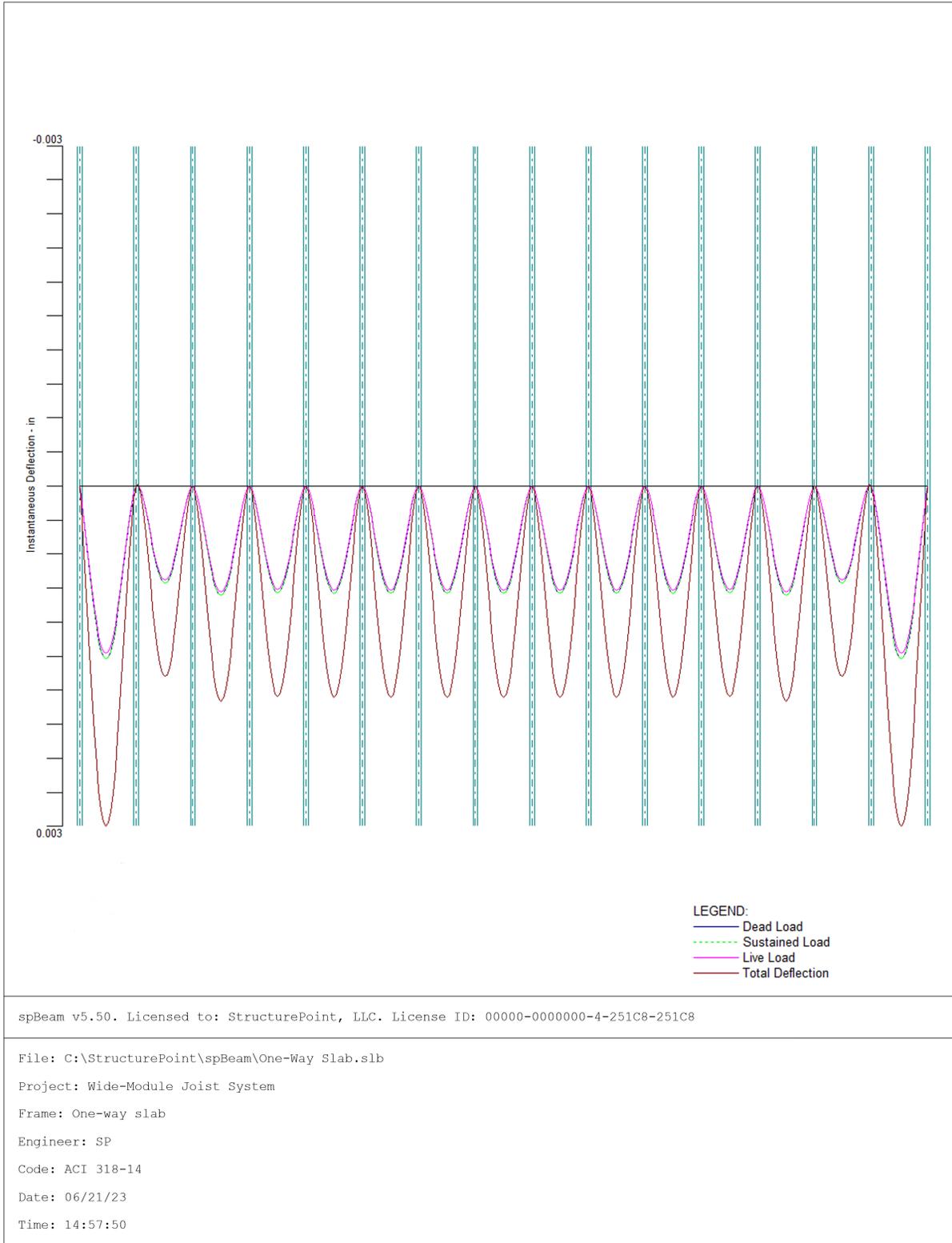


Figure 10 – spBeam Model – Immediate Deflection Diagram

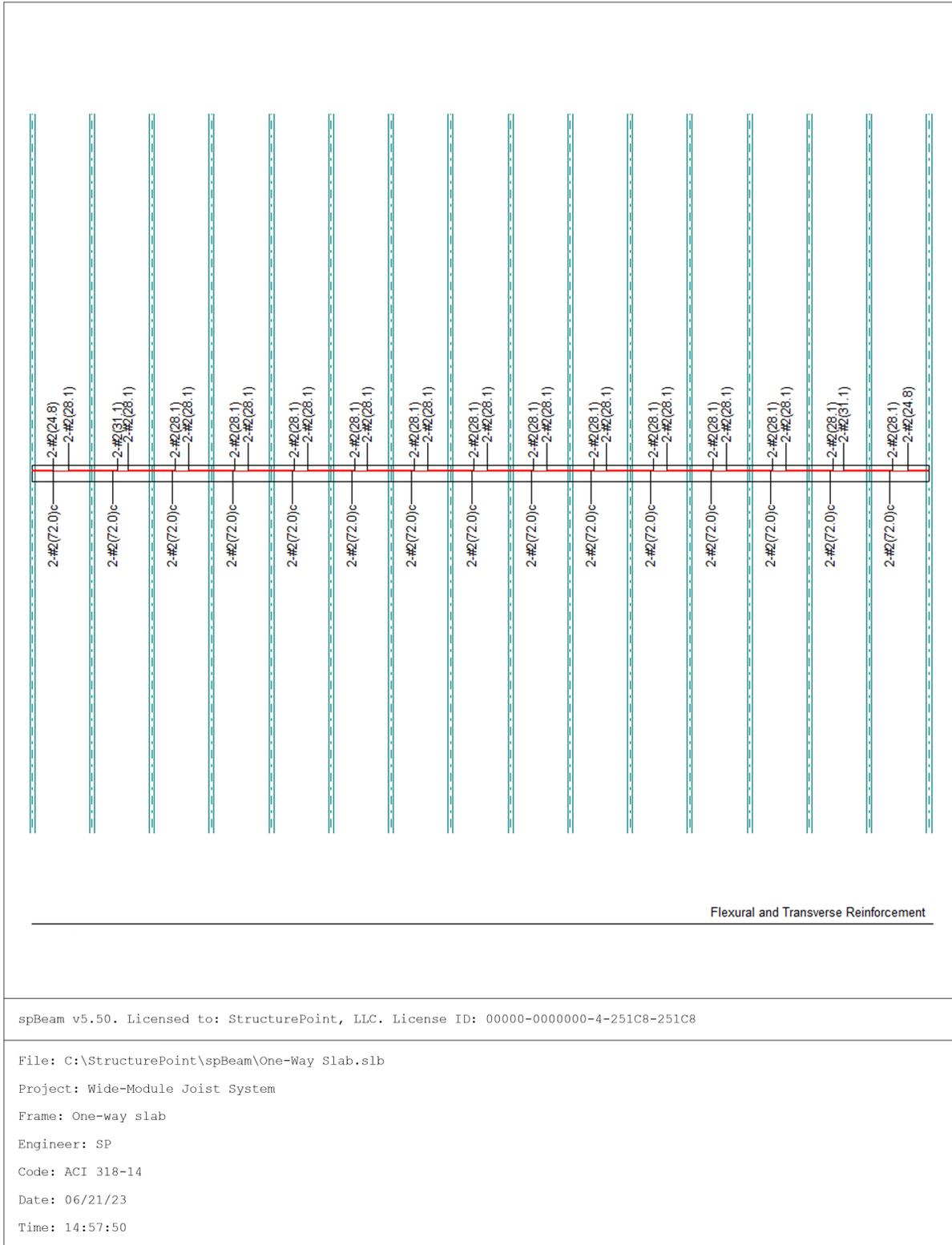


Figure 11 – spBeam Model – Reinforcement Diagram

2.1.7. Summary and Comparison of Results

1. Design Results

1.1. Top Reinforcement

Notes:

*3 - Design governed by minimum reinforcement.

Span Zone	Width ft	M _{max} k-ft	X _{max} ft	A _{s,min} in ²	A _{s,max} in ²	A _{s,req} in ²	Sp _{Prov} in	Bars
1 Left	1.00	0.26	0.250	0.108	0.859	0.017	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.63	5.750	0.108	0.859	0.042	6.000	2-#2 *3
2 Left	1.00	0.60	0.250	0.108	0.859	0.040	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.57	5.750	0.108	0.859	0.038	6.000	2-#2 *3
3 Left	1.00	0.57	0.250	0.108	0.859	0.038	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.58	5.750	0.108	0.859	0.038	6.000	2-#2 *3
4 Left	1.00	0.57	0.250	0.108	0.859	0.038	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.57	5.750	0.108	0.859	0.038	6.000	2-#2 *3
5 Left	1.00	0.57	0.250	0.108	0.859	0.038	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.57	5.750	0.108	0.859	0.038	6.000	2-#2 *3
6 Left	1.00	0.57	0.250	0.108	0.859	0.038	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.57	5.750	0.108	0.859	0.038	6.000	2-#2 *3
7 Left	1.00	0.57	0.250	0.108	0.859	0.038	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.57	5.750	0.108	0.859	0.038	6.000	2-#2 *3
8 Left	1.00	0.57	0.250	0.108	0.859	0.038	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.57	5.750	0.108	0.859	0.038	6.000	2-#2 *3
9 Left	1.00	0.57	0.250	0.108	0.859	0.038	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.57	5.750	0.108	0.859	0.038	6.000	2-#2 *3
10 Left	1.00	0.57	0.250	0.108	0.859	0.038	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.57	5.750	0.108	0.859	0.038	6.000	2-#2 *3
11 Left	1.00	0.57	0.250	0.108	0.859	0.038	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.57	5.750	0.108	0.859	0.038	6.000	2-#2 *3
12 Left	1.00	0.57	0.250	0.108	0.859	0.038	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.57	5.750	0.108	0.859	0.038	6.000	2-#2 *3
13 Left	1.00	0.58	0.250	0.108	0.859	0.038	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.57	5.750	0.108	0.859	0.038	6.000	2-#2 *3
14 Left	1.00	0.57	0.250	0.108	0.859	0.038	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.60	5.750	0.108	0.859	0.040	6.000	2-#2 *3
15 Left	1.00	0.63	0.250	0.108	0.859	0.042	6.000	2-#2 *3
Midspan	1.00	0.00	3.000	0.000	0.859	0.000	0.000	---
Right	1.00	0.26	5.750	0.108	0.859	0.017	6.000	2-#2 *3

1.2. Bottom Reinforcement

Notes:

*3 - Design governed by minimum reinforcement.

Span	Width ft	M _{max} k-ft	X _{max} ft	A _{s,min} in ²	A _{s,max} in ²	A _{s,req} in ²	Sp _{Prov} in	Bars
1	1.00	0.49	2.750	0.108	0.416	0.069	6.000	2-#2 *3
2	1.00	0.42	3.000	0.108	0.416	0.058	6.000	2-#2 *3
3	1.00	0.43	3.000	0.108	0.416	0.061	6.000	2-#2 *3
4	1.00	0.43	3.000	0.108	0.416	0.060	6.000	2-#2 *3
5	1.00	0.43	3.000	0.108	0.416	0.060	6.000	2-#2 *3
6	1.00	0.43	3.000	0.108	0.416	0.060	6.000	2-#2 *3
7	1.00	0.43	3.000	0.108	0.416	0.060	6.000	2-#2 *3
8	1.00	0.43	3.000	0.108	0.416	0.060	6.000	2-#2 *3
9	1.00	0.43	3.000	0.108	0.416	0.060	6.000	2-#2 *3
10	1.00	0.43	3.000	0.108	0.416	0.060	6.000	2-#2 *3
11	1.00	0.43	3.000	0.108	0.416	0.060	6.000	2-#2 *3
12	1.00	0.43	3.000	0.108	0.416	0.060	6.000	2-#2 *3
13	1.00	0.43	3.000	0.108	0.416	0.061	6.000	2-#2 *3
14	1.00	0.42	3.000	0.108	0.416	0.058	6.000	2-#2 *3
15	1.00	0.49	3.250	0.108	0.416	0.069	6.000	2-#2 *3

1.3. Slab Shear Capacity

Span	b in	d in	V _{ratio}	ΦV _c kip	V _u kip	X _u ft
1	12.00	1.63	1.000	2.08	0.67	5.61
2	12.00	1.63	1.000	2.08	0.63	0.39
3	12.00	1.63	1.000	2.08	0.63	5.61
4	12.00	1.63	1.000	2.08	0.63	0.39
5	12.00	1.63	1.000	2.08	0.63	5.61
6	12.00	1.63	1.000	2.08	0.63	0.39
7	12.00	1.63	1.000	2.08	0.63	5.61
8	12.00	1.63	1.000	2.08	0.63	0.39
9	12.00	1.63	1.000	2.08	0.63	0.39
10	12.00	1.63	1.000	2.08	0.63	5.61
11	12.00	1.63	1.000	2.08	0.63	0.39
12	12.00	1.63	1.000	2.08	0.63	5.61
13	12.00	1.63	1.000	2.08	0.63	0.39
14	12.00	1.63	1.000	2.08	0.63	5.61
15	12.00	1.63	1.000	2.08	0.67	0.39

Table 3 - Comparison of Hand Solution with spBeam Solution for One-Way Slab						
Flexural Design						
Span Location	Design Moment (ft-kips/ft)		Reinforcement Required for Flexure (in ² /ft)		Minimum Reinforcement (in ² /ft) (Shrinkage & Temperature Reinforcement)	
	Hand	spBeam	Hand	spBeam	Hand	spBeam
Interior Negative	0.69	0.63	0.046	0.042	0.108 (2-#2)	0.108 (2-#2)
Positive	0.49	0.49	0.068	0.069	0.108 (2-#2)	0.108 (2-#2)
Shear Design						
Span Location	V_u (kips/ft)			ϕV_n (kips/ft)		
	Hand	spBeam		Hand	spBeam	
Interior Negative	0.69		0.67	2.08		2.08

2.1.8. Conclusions and Observations

Minimum reinforcement requirement governed flexural design in this example. [spBeam](#) program enables the user to enter the rotational support springs as boundary conditions for joist supports and evaluate various analysis and design options beyond the limitations of the simplified method. The coefficients traditionally used to determine moments do not address various types of support and geometry.

Typically, in wide-module joist construction, one-way slab is reinforced with single layer reinforcement placed near the top in the primary direction. As seen in this example, this may cause crack control criteria to govern the reinforcement spacing and consequently, it may warrant the use of welded wire fabric reinforcement instead of deformed bar.

The maximum calculated total immediate (instantaneous) deflection (DL + LL) = 0.003 in., this value can be compared with maximum permissible calculated deflection limitation per project criteria in accordance to [ACI 318-14](#). [ACI 318-14 \(Table 24.2.2\)](#)

In addition to deflection results, parametric studies can be performed in [spBeam](#) to optimize design and detailing results. Note in the reinforcement diagram ([Figure 11](#)) 2-#2 for top reinforcement in the span left and right zones as well as span bottom reinforcement. One layer is suitable by inspection ([Figure 8](#)) to meet the required area of steel for top and bottom reinforcement.

2.2. One-Way Joist Design

The wide-module joists in this floor are considered as beams per ACI 318-14 (9.8.1.8). Therefore, the design of the joist shall conform to the requirements of T-beams per ACI 318-14 (9.2.4).

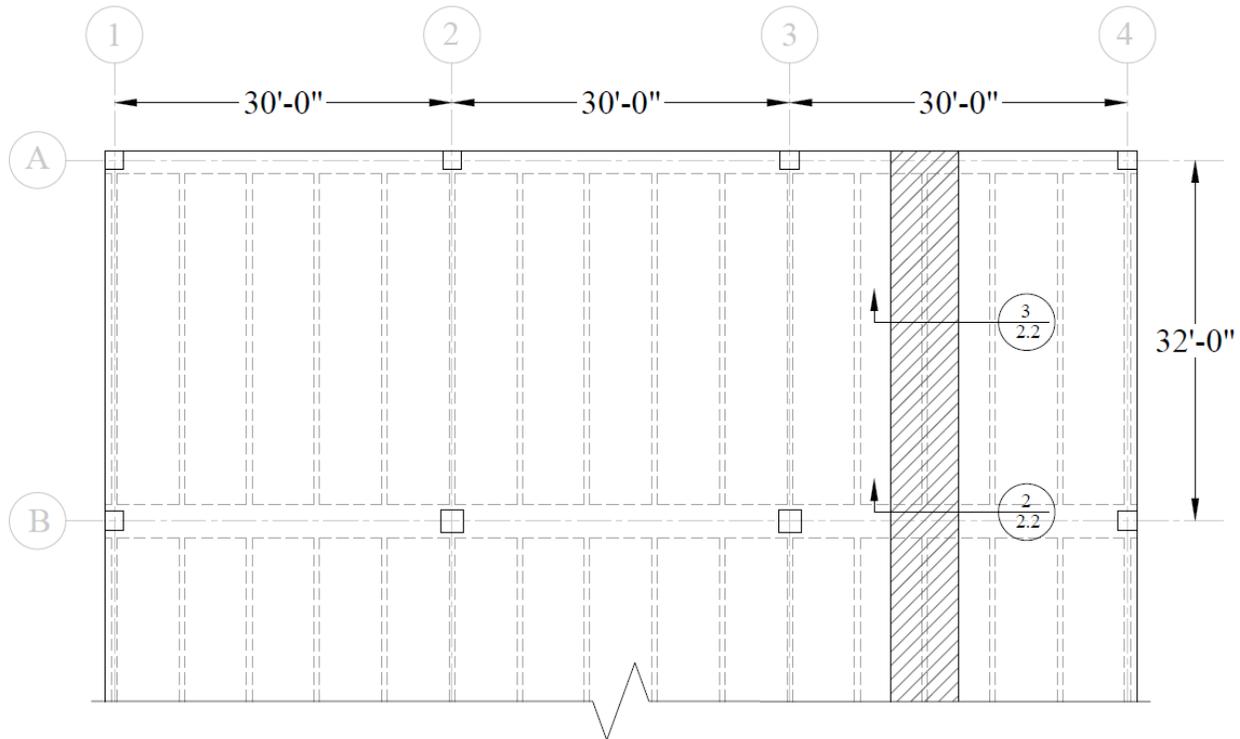


Figure 12 – Partial Plan View Illustrating One-Way Joist to be Design

Joist design involves the following steps:

- 2.2.1. Determination of span loads
- 2.2.2. Determination of design moments and shears
- 2.2.3. Flexural Design
- 2.2.4. Shear Design
- 2.2.5. Deflections
- 2.2.6. Computer Program Solution
- 2.2.7. Summary and comparison of design results
- 2.2.8. Conclusions and observations

2.2.1. Determination of span loads

The following gravity load combinations are considered:

$$U = 1.40 \times D \quad \text{ACI 318-14 (Eq. 5.3.1a)}$$

$$w_u = 1.40 \times \left[\left(\left(\frac{5}{12} \times 0.15 \right) + 0.02 \right) \times 6 + \left(\frac{(6+8.67)/2}{12} \times \frac{16}{12} \right) \times 0.15 \right] = 1.40 \times 0.62 = 0.86 \frac{\text{kips}}{\text{ft}}$$

$$U = 1.20 \times D + 1.60 \times L \quad \text{ACI 318-14 (Eq. 5.3.1b)}$$

$$w_u = 1.20 \times \left[\left(\left(\frac{5}{12} \times 0.15 \right) + 0.02 \right) \times 6 + \left(\frac{(6+8.67)/2}{12} \times \frac{16}{12} \right) \times 0.15 \right] + 1.60 \times 0.08 \times 6$$

$$w_u = 1.20 \times 0.62 + 1.60 \times 0.48 = 1.51 \frac{\text{kips}}{\text{ft}}$$

Span loads are governed by the second load combination.

Note that for Floor Live Load Reduction per ASCE/SEI 7-10:

$$L = L_o \times \left(0.25 + \frac{15}{\sqrt{K_{LL} \times A_T}} \right) \quad \text{ASCE/SEI 7-10 (Eq. 4-1)}$$

Where:

Live Load Element Factor, $K_{LL} = 2$ for interior beams ASCE/SEI 7-10 (Table 4-2)

Tributary Area $A_T = (6 \times 32) = 192 \text{ ft}^2$

Since $K_{LL} \times A_T = 2 \times 192 = 384 \text{ ft}^2 < 400 \text{ ft}^2$ live load reduction is not applicable.

2.2.2. Determination of design moment and shear

The factored moment and shear can be determined using the simplified method if the requirements are satisfied: ACI 318-14 (6.5.1)

- ✓ Members are prismatic.
- ✓ Loads are uniformly distributed.
- ✓ $L \leq 3D$ ($0.48 \text{ kips/ft} \leq 3 \times 0.62 \text{ kips/ft}$)
- ✓ There are at least two spans.
- ✓ The longer of two adjacent spans does not exceed the shorter by more than 20 percent.

Thus, the approximate coefficients can be used. The factored moments and shears are determined and summarized in the following tables. **ACI 318-14 (Table 6.5.2 and Table 6.5.3)**

Table 4 - One-Way Joist Design Moment Values		
Location		Design Moment Value
End Spans	Exterior Support Negative	$\frac{w_u \times l_n^2}{24} = \frac{1.51 \times 30.17^2}{24} = 57.21 \text{ ft-kips}$
	Mid-span	$\frac{w_u \times l_n^2}{14} = \frac{1.51 \times 30.17^2}{14} = 98.07 \text{ ft-kips}$
	Interior Support Negative	$\frac{w_u \times l_n^2}{10} = \frac{1.51 \times 30.08^2}{10} = 136.54 \text{ ft-kips}$
Interior Spans	Mid-span Positive	$\frac{w_u \times l_n^2}{16} = \frac{1.51 \times 30^2}{16} = 84.86 \text{ ft-kips}$
	Support Negative	$\frac{w_u \times l_n^2}{11} = \frac{1.51 \times 30^2}{11} = 123.44 \text{ ft-kips}$

Table 5 - One-Way Joist Design Shear Values	
Location	Design Shear Value
End Span at Face of First Interior Support	$1.15 \times \frac{w_u \times l_n}{2} = 1.15 \times \frac{1.51 \times 30.17}{2} = 26.17 \text{ kips}$
At Face of all other Supports	$\frac{w_u \times l_n}{2} = \frac{1.51 \times 30.17}{2} = 22.76 \text{ kips}$

* When support beam is wider than the column, the clear span, l_n , of the joists is measured from the face of the column. For calculating negative moments, l_n , is taken as the average of the adjacent clear spans.

ACI 318-14 (6.5.2)

2.2.3. Flexural Design

For the one-way joist of a wide-module joist system, the end span moment values govern the design as tabulated in [Table 4](#).

Calculate the required reinforcement to resist the first interior support negative moment:

$$M_u = 136.54 \text{ ft-kips}$$

Use #5 reinforcement with 1.5 in. concrete cover. The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, d , is calculated below:

$$d = 21 - \left(1.50 + \frac{3}{8} + 0.50 \times \frac{5}{8} \right) = 18.81 \text{ in.}$$

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section (jd). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.9, and jd will be taken equal to $0.909 \times d$ since we are designing for the negative moment in a T-beam (narrow compression zone). The assumptions will be verified once the area of steel is finalized.

$$\text{Assume } jd = 0.909 \times d = 0.909 \times 18.81 = 17.11 \text{ in.}$$

$$\text{Joist average width, } b = \frac{6 + 8.67}{2} = 7.33 \text{ in.}$$

The required reinforcement at initial trial is calculated as follows:

$$A_s = \frac{M_u}{\phi \times f_y \times jd} = \frac{136.54 \times 12,000}{0.90 \times 60,000 \times 17.11} = 1.774 \text{ in.}^2$$

Recalculate 'a' for the actual $A_s = 1.774 \text{ in.}^2$:

$$a = \frac{A_s \times f_y}{0.85 \times f'_c \times b} = \frac{1.774 \times 60,000}{0.85 \times 5,000 \times 7.33} = 3.414 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{3.414}{0.85} = 4.016 \text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c} \right) \times d_t - 0.003 = \left(\frac{0.003}{4.016} \right) \times 18.81 - 0.003 = 0.011 > 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi \times f_y \times \left(d - \frac{a}{2}\right)} = \frac{136.54 \times 12,000}{0.90 \times 60,000 \times \left(18.81 - \frac{3.414}{2}\right)} = 1.774 \text{ in.}^2$$

The minimum reinforcement shall not be less than

$$A_{s,\min} = \frac{3 \times \sqrt{f'_c}}{f_y} \times b_w \times d = \frac{3 \times \sqrt{5,000}}{60,000} \times 7.34 \times 18.81 = 0.49 \text{ in.}^2 \quad \text{\underline{ACI 318-14 (9.6.1.2(a))}}$$

And not less than

$$A_{s,\min} = \frac{200}{f_y} \times b_w \times d = \frac{200}{60,000} \times 7.34 \times 18.81 = 0.46 \text{ in.}^2 \quad \text{\underline{ACI 318-14 (9.6.1.2(b))}}$$

Part of the negative-moment steel shall be distributed over a width equal to the smaller of the effective flange width (72 in.) and $\frac{l}{10} = \frac{384}{10} = 38.40 \text{ in.}$ ACI 318-14 (24.3.4)

Where the effective width of the overhanging flange on each side of the beam web is the smallest of the following: ACI 318-14 (6.3.2.1)

$$b_w + 2 \times (8 \times h) = 8.67 + 2 \times (8 \times 5) = 88.67 \text{ in.}, \text{ where } h \text{ is the slab thickness.}$$

$$b_w + 2 \times \left(\frac{s_w}{2}\right) = 6 + 2 \times \left(\frac{5.50 \times 12}{2}\right) = 72.00 \text{ in.}, \text{ where } s_w \text{ is the clear distance to the adjacent web.}$$

$$b_w + 2 \times \left(\frac{l_n}{8}\right) = 8.67 + 2 \times \left(\frac{30.17 \times 12}{8}\right) = 99.17 \text{ in.}$$

Therefore, the effective flange width is 72 in.

Provide 6-#5 bars within 38.4 in width. $A_{s,\text{prov}} = (6 \times 0.31) = 1.86 \text{ in.}^2 > 1.774 \text{ in.}^2$ **o.k.**

Calculate the required reinforcement to resist the positive moment:

$$M_u = 98.07 \text{ ft-kips}$$

In the positive moment regions, the beam acts as a T-shaped beam. The effective flange width as was calculated earlier is 72 in.

By assuming #3 bars for joist stirrups and the maximum bar size for joist bottom reinforcement as #7 and following the 1.50 in. concrete cover to reinforcement requirement of beam stirrups per ACI 318-14 (20.6.1), the distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, d , is calculated below:

$$d = 21 - \left(1.50 + \frac{3}{8} + 0.50 \times \left(\frac{7}{8} \right) \right) = 18.69 \text{ in.}$$

Since we are designing for the positive moment in a T-Beam (wide compression zone), select a moment arm, jd approximately equal to $0.994 \times d$.

$$\text{Assume that } jd = 0.994 \times d = 0.994 \times 18.69 = 18.57 \text{ in.}$$

$$A_s = \frac{M_u}{\phi \times f_y \times jd} = \frac{98.07 \times 12,000}{0.90 \times 60,000 \times 18.57} = 1.173 \text{ in.}^2$$

Recalculate 'a' for the actual $A_s = 1.173 \text{ in.}^2$:

$$a = \frac{A_s \times f_y}{0.85 \times f'_c \times b} = \frac{1.173 \times 60,000}{0.85 \times 5,000 \times 72} = 0.230 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.230}{0.85} = 0.271 \text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c} \right) \times d_t - 0.003 = \left(\frac{0.003}{0.27} \right) \times 18.69 - 0.003 = 0.204 > 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi \times f_y \times \left(d - \frac{a}{2} \right)} = \frac{98.07 \times 12,000}{0.90 \times 60,000 \times \left(18.69 - \frac{0.230}{2} \right)} = 1.173 \text{ in.}^2$$

Use 2-#7 bundled bars with $A_{s,prov} = (2 \times 0.60) = 1.20 \text{ in.}^2 > 1.173 \text{ in.}^2$

o.k.

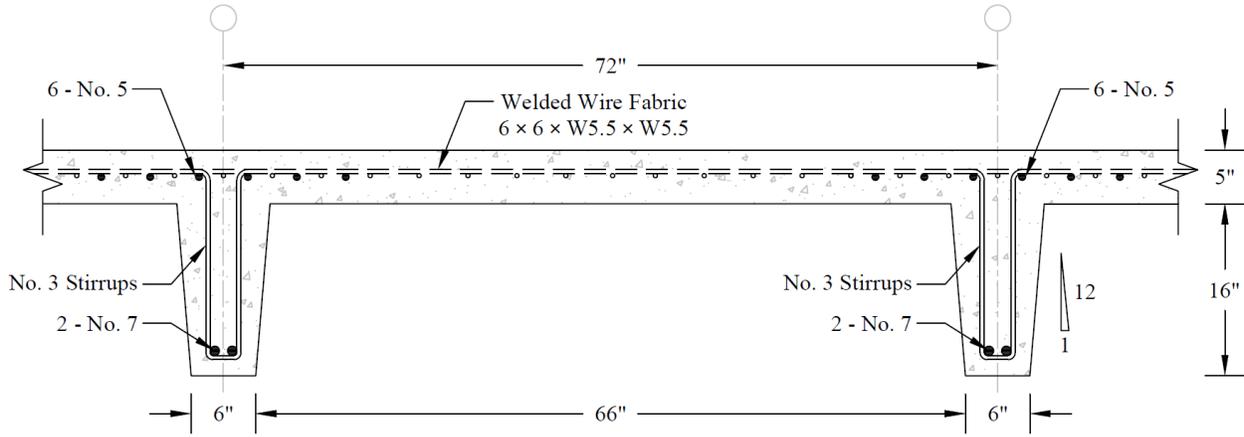


Figure 13 – Cross-Sectional View at Joist Mid-Span (Section 2/2.2 in Figure 12)

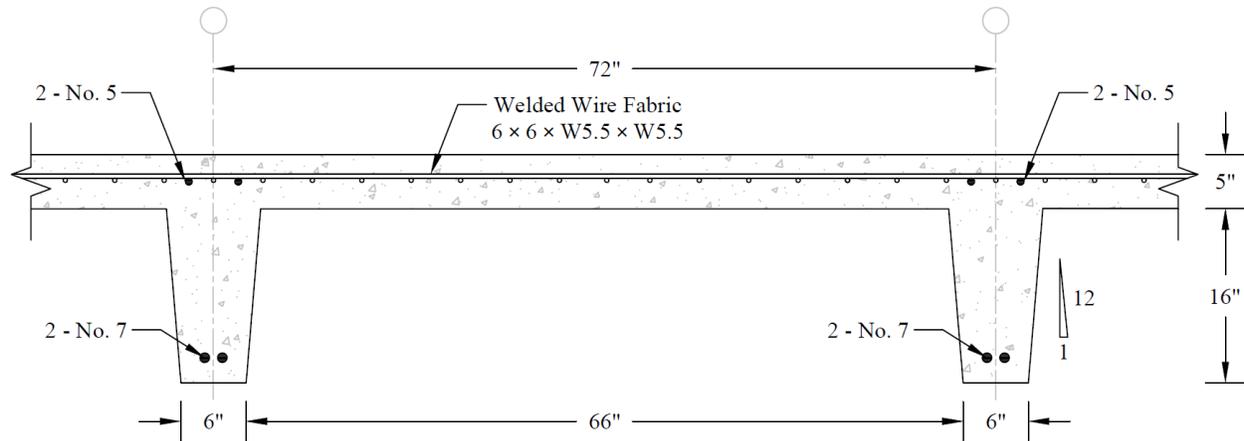


Figure 14 – Cross-Sectional View at Joist Near Support Face (Section 3/2.2 in Figure 12)

2.2.4. Shear Design

From Table 5 above, the shear value in end span at face of first interior support governs.

$$V_u = 1.15 \times \frac{w_u \times l_n}{2} = 1.15 \times \frac{1.51 \times 30.17}{2} = 26.17 \text{ kips}$$

The design shear at a distance, d , away from the face of support,

$$V_u = 26.17 - 1.51 \times \frac{18.69}{12} = 23.82 \text{ kips}$$

Shear strength provided by concrete

$$\phi V_c = \phi \times (2 \times \sqrt{f'_c} \times b_w \times d) \quad \text{ACI 318-14 (Eq. 22.5.5.1)}$$

$$\phi V_c = 0.75 \times (2.00 \times 1.00 \times \sqrt{5,000} \times 7.34 \times 18.69) = 14,538.77 \text{ lb} = 14.54 \text{ kips}$$

Since $V_u > \frac{\phi V_c}{2}$, shear reinforcement is required.

Try #3, Grade 60 double-leg stirrups with a 90° hook.

The nominal shear strength required to be provided by shear reinforcement is

$$V_s = V_n - V_c = \frac{V_u}{\phi} - V_c = \left(\frac{23.82}{0.75} \right) - 19.39 = 12.38 \text{ kips}$$

Check whether V_s is less than $8 \times \sqrt{f'_c} \times b_w \times d$

If V_s is greater than $8 \times \sqrt{f'_c} \times b_w \times d$, then the cross-section has to be revised as ACI 318-14 limits the shear capacity to be provided by stirrups to $8 \times \sqrt{f'_c} \times b_w \times d$ ACI 318-14 (22.5.1.2)

$$8 \times \sqrt{f'_c} \times b_w \times d = 8 \times \sqrt{5,000} \times 7.34 \times 18.69 = 77,540.09 \text{ lb} = 77.54 \text{ kips}$$

Since V_s does not exceed $8 \times \sqrt{f'_c} \times b_w \times d$, the cross section is adequate.

Assume #3 stirrups with two legs ($A_v = 0.22 \text{ in.}^2$)

Calculate the required stirrup spacing as

$$S_{req'd} = \frac{\phi \times A_v \times f_{yt} \times d}{V_u - \phi V_c} = \frac{0.75 \times 0.22 \times 60 \times 18.69}{23.82 - 14.54} = 19.93 \text{ in.} \quad \text{ACI 318-14 (22.5.10.5.3)}$$

Check whether the required spacing based on the shear demand meets the spacing limits for shear reinforcement per ACI 318-14 (9.7.6.2.2).

Check whether V_s is less than $4 \times \sqrt{f'_c} \times b_w \times d$

$$4 \times \sqrt{f'_c} \times b_w \times d = 4 \times \sqrt{5,000} \times 7.34 \times 18.69 = 38,770.05 \text{ lb} = 38.77 \text{ kips} > V_s = 12.38 \text{ kips}$$

Therefore, maximum stirrup spacing shall be the smallest of $d/2$ and 24 in. ACI 318-14 (Table 9.7.6.2.2)

$$s_{\max} = \text{lesser of } \left[\frac{d}{2} \right] = \text{lesser of } \left[\frac{18.69}{2} \right] = \text{lesser of } \left[\frac{9.34 \text{ in.}}{24 \text{ in.}} \right] = 9.34 \text{ in.}$$

This value governs over the required stirrup spacing of 19.93 in. which was based on the demand.

Joist minimum shear reinforcement requirements must be checked since wide-module joists do not satisfy ACI 318-14 (9.8).

Check the maximum stirrup spacing based on minimum shear reinforcement

$$s_{\max} \leq \frac{A_v \times f_{yt}}{0.75 \times \sqrt{f'_c} \times b_w} = \frac{0.22 \times 60,000}{0.75 \times \sqrt{5,000} \times 7.34} = 33.93 \text{ in. (does not govern)} \quad \text{ACI 318-14 (10.6.2.2(a))}$$

$$s_{\max} \leq \frac{A_v \times f_{yt}}{50 \times b_w} = \frac{0.22 \times 60,000}{50 \times 7.34} = 35.99 \text{ in. (does not govern)} \quad \text{ACI 318-14 (10.6.2.2(b))}$$

Therefore, s_{\max} value is governed by the spacing limit per ACI 318-14 (9.7.6.2.2), and is equal to 9.34 in.

Use #3 @ 9 in. stirrups

$$V_n = \frac{A_v \times f_{yt} \times d}{s} + V_c \quad \text{ACI 318-14 (22.5.1.1 and 22.5.10.5.3)}$$

$$V_n = \frac{0.22 \times 60,000 \times 18.69}{9} + 19.39 = 26.40 + 19.39 = 45.79 \text{ kips}$$

$$\phi V_n = 0.75 \times 45.79 = 34.34 \text{ kips} > V_u = 23.82 \text{ kips} \quad \text{o.k.}$$

Compute where $\frac{V_u}{\phi}$ is equal to $\frac{V_c}{2}$, and the stirrups can be stopped

$$x = \frac{\left(\frac{V_u}{\phi}\right) - \left(\frac{V_c}{2}\right)}{\frac{V_u}{\phi}} \times \frac{l_n}{2} = \frac{\left(\frac{23.82}{0.75}\right) - \left(\frac{19.39}{2}\right)}{\left(\frac{23.82}{0.75}\right)} \times \frac{30.17 \times 12}{2} = 125.76 \text{ in.}$$

At interior end of the exterior span, use 16-#3 @ 9 in o.c., Place 1st stirrup 2 in. from the face of supporting girder.

2.2.5. Deflections

Since the preliminary joist depth met minimum depth requirement, the deflection calculations are not required. A lesser depth maybe possible and consequently cost savings can be achieved through deflection computations. Deflection values are calculated and provided for every model created by [spBeam](#) Program and can be used by the engineer to make additional optimization decisions.

2.2.6. Computer Program Solution

[spBeam](#) Program can be utilized to analysis and design the one-way wide-module joist. A single wide-module joist is modeled as a five span continuous-beam.

The program calculates the internal forces (shear force and bending moment), moment and shear capacities, immediate and long-term deflection results, and required flexural reinforcement. The graphical and text results are provided here for both input and output of the [spBeam](#) model.

The ribs are modeled as a rectangular longitudinal beam with an equivalent width of 7.33 in. and 21 in. depth to reflect the sloped sides of the forming pans.

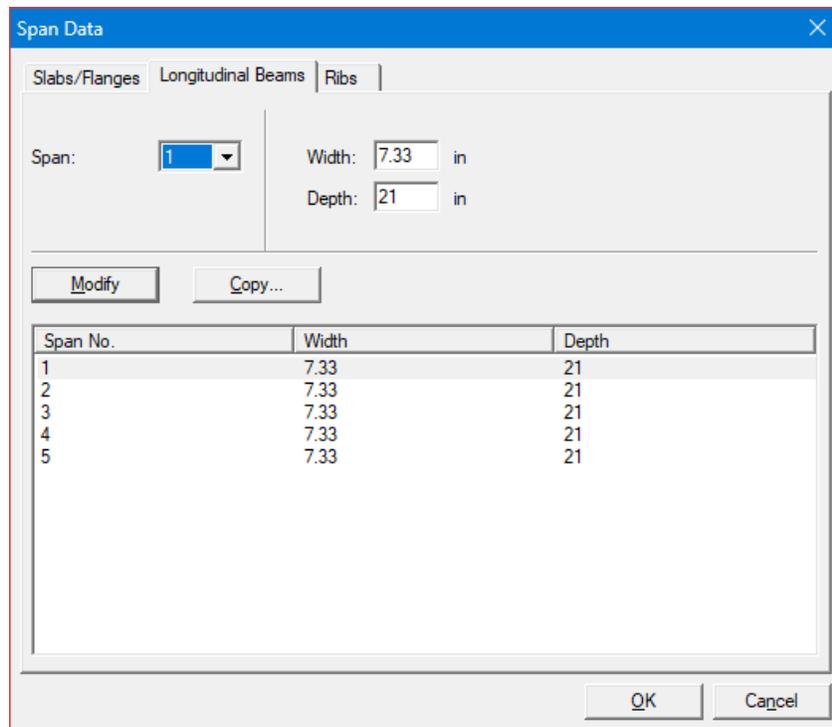
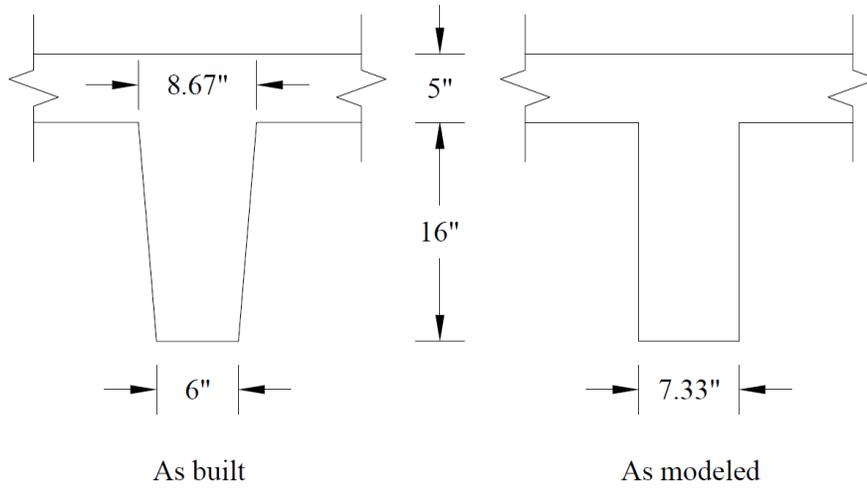


Figure 15 – spBeam Model – One-Way Joist Section

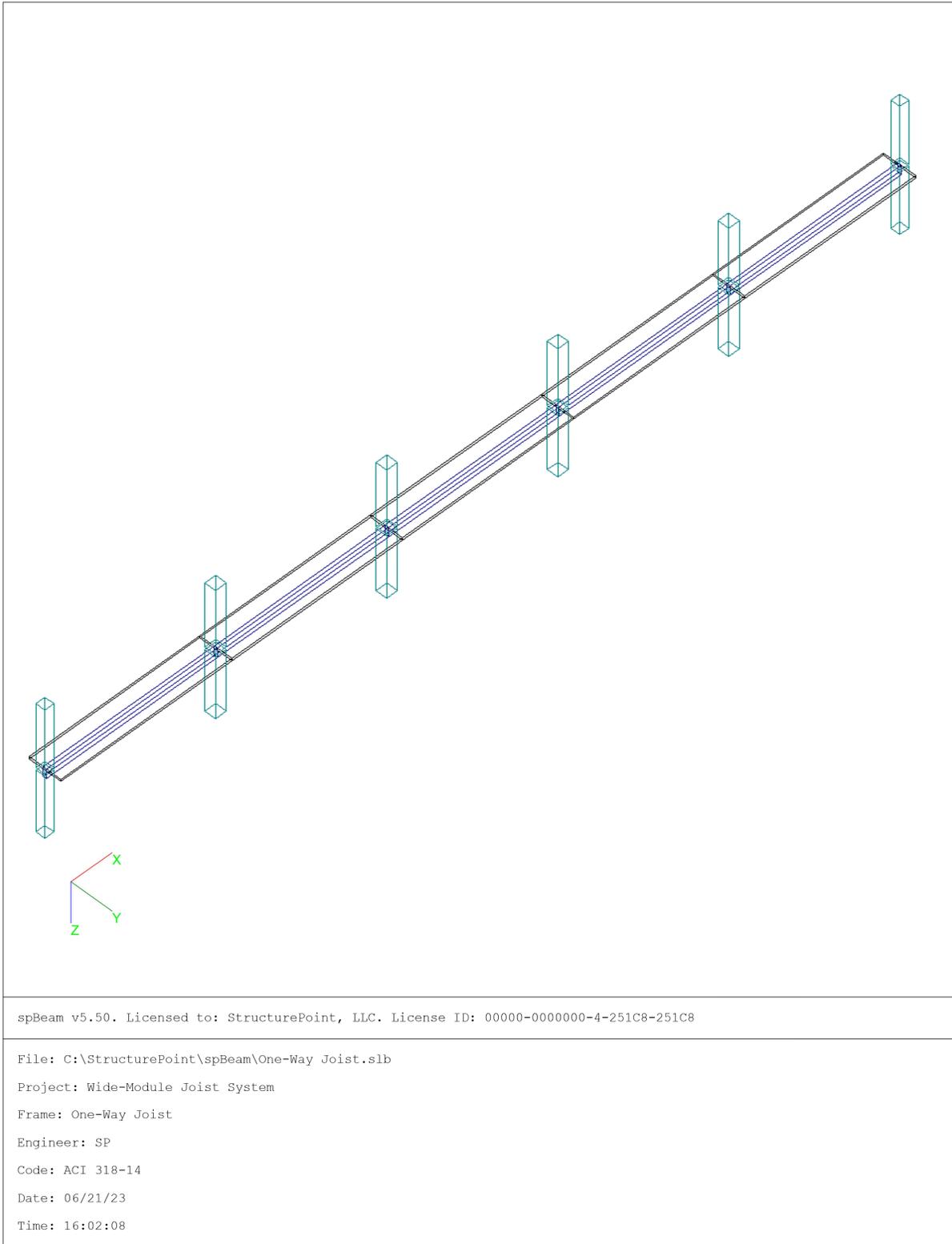


Figure 16 – [spBeam](#) Model – Isometric View – One-Way Joist

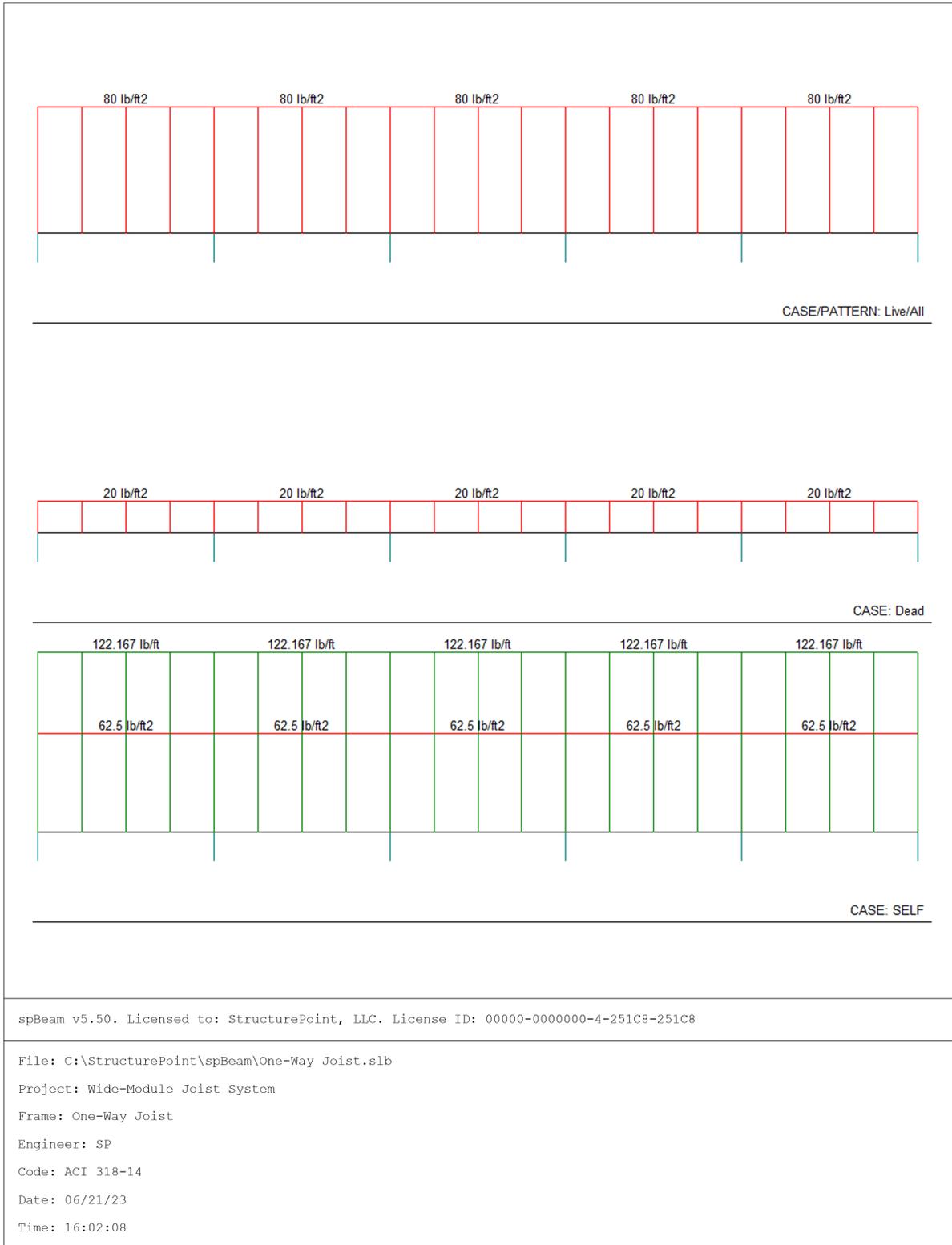


Figure 17 – spBeam Model – Loads (Including Live Load Patterning)

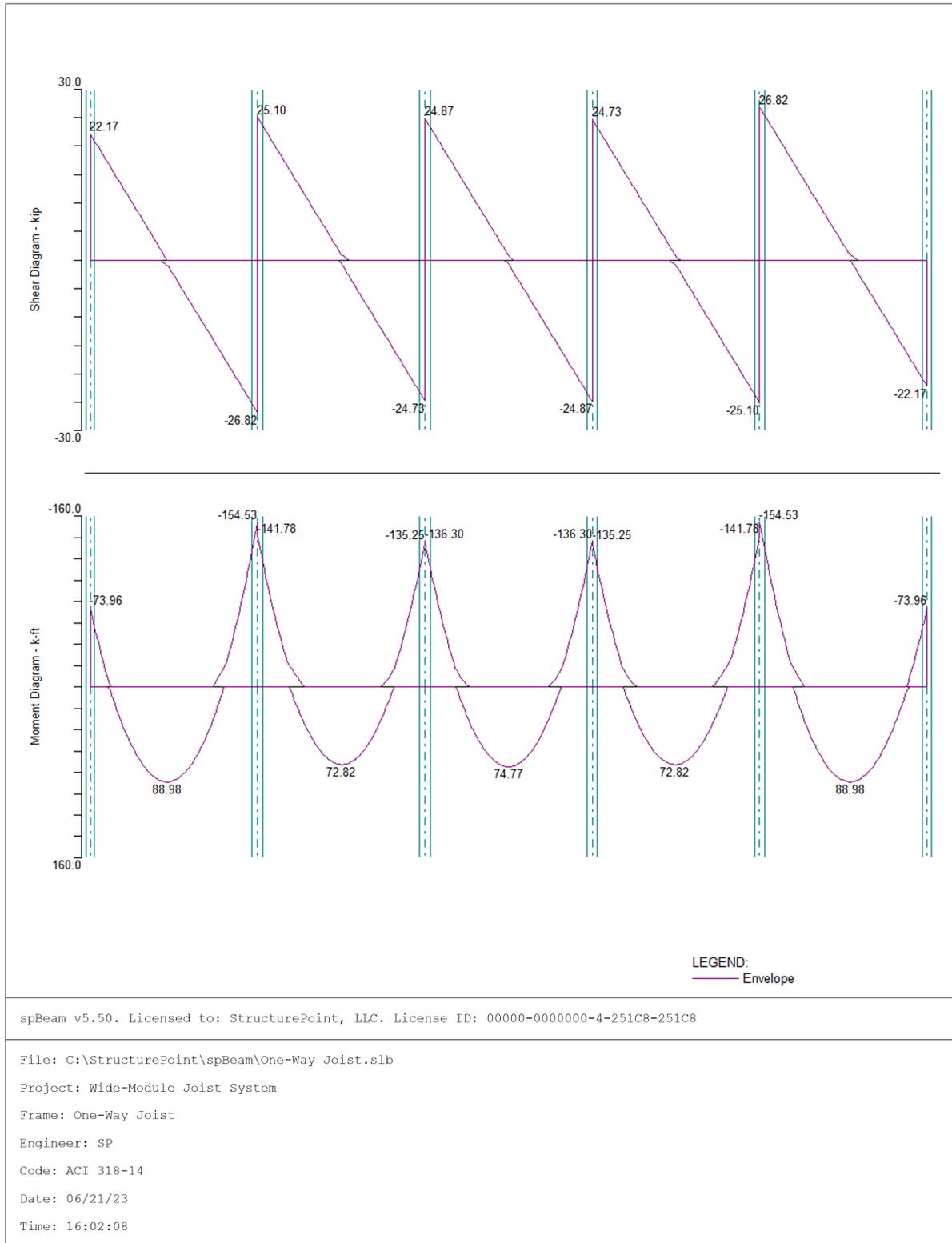


Figure 18 – spBeam Model – Internal Forces (Shear Force Diagram and Bending Moment Diagram)

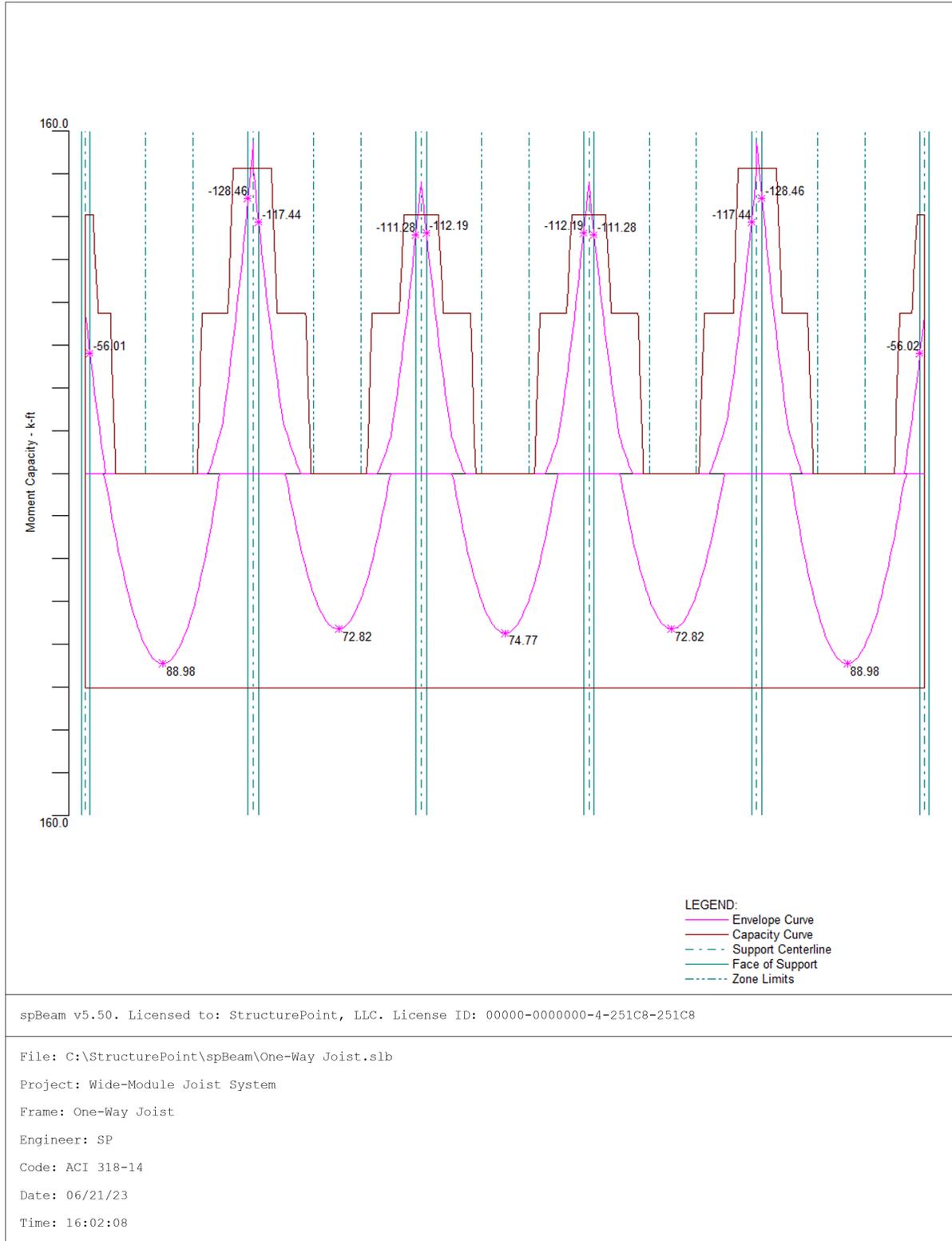


Figure 19 – spBeam Model – Moment Capacity Diagram

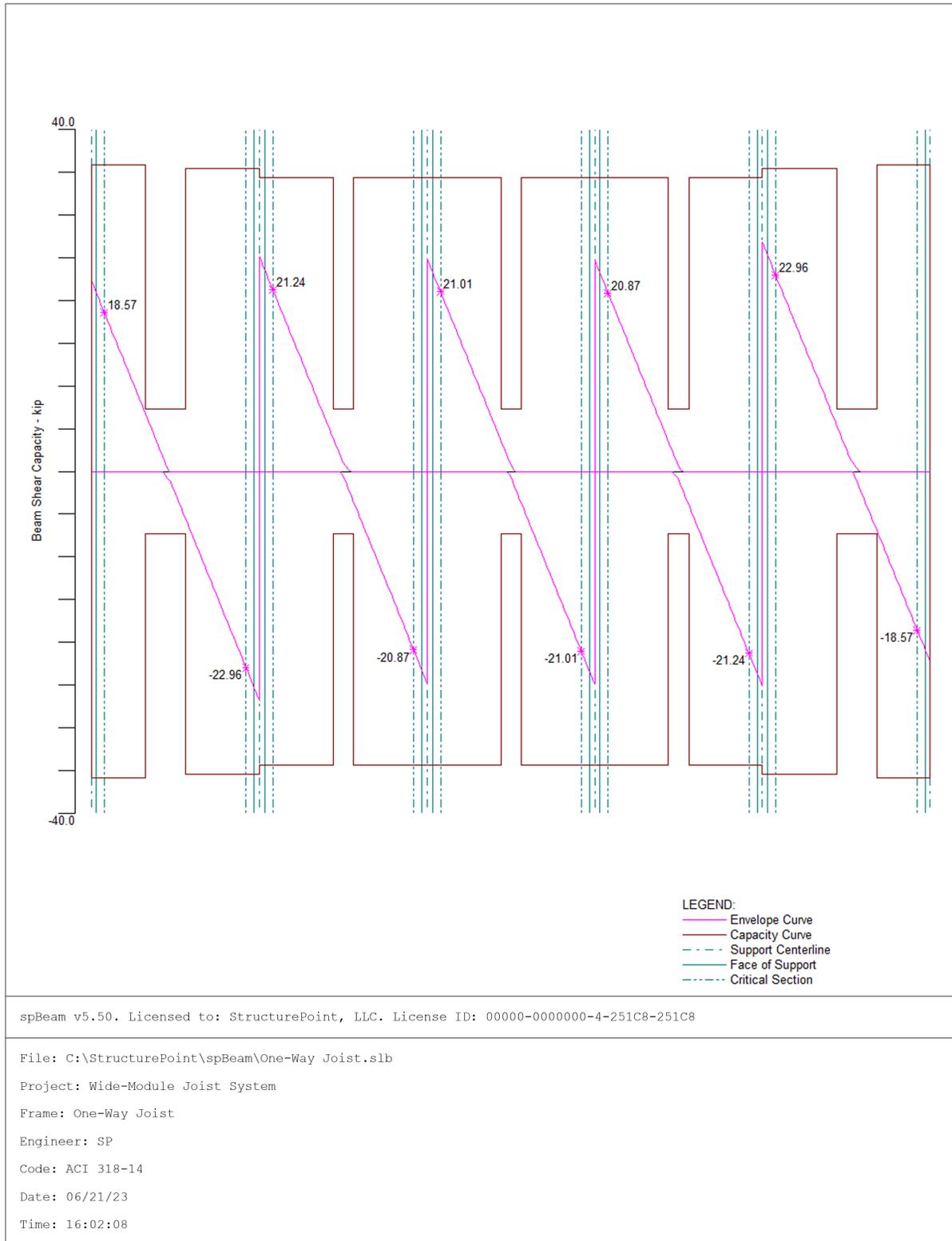


Figure 20 – spBeam Model – Shear Capacity Diagram

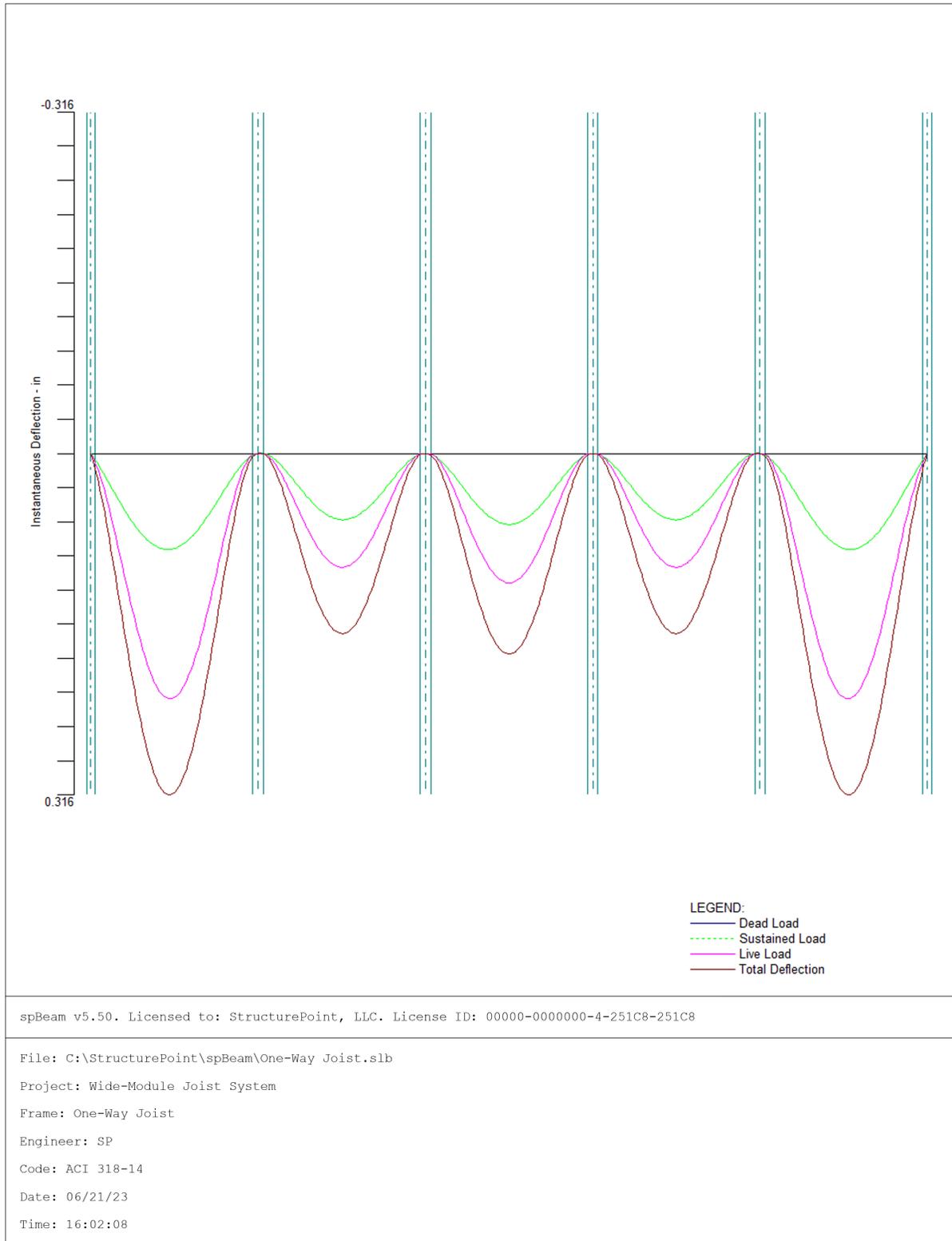


Figure 21 – spBeam Model – Immediate Deflection Diagram

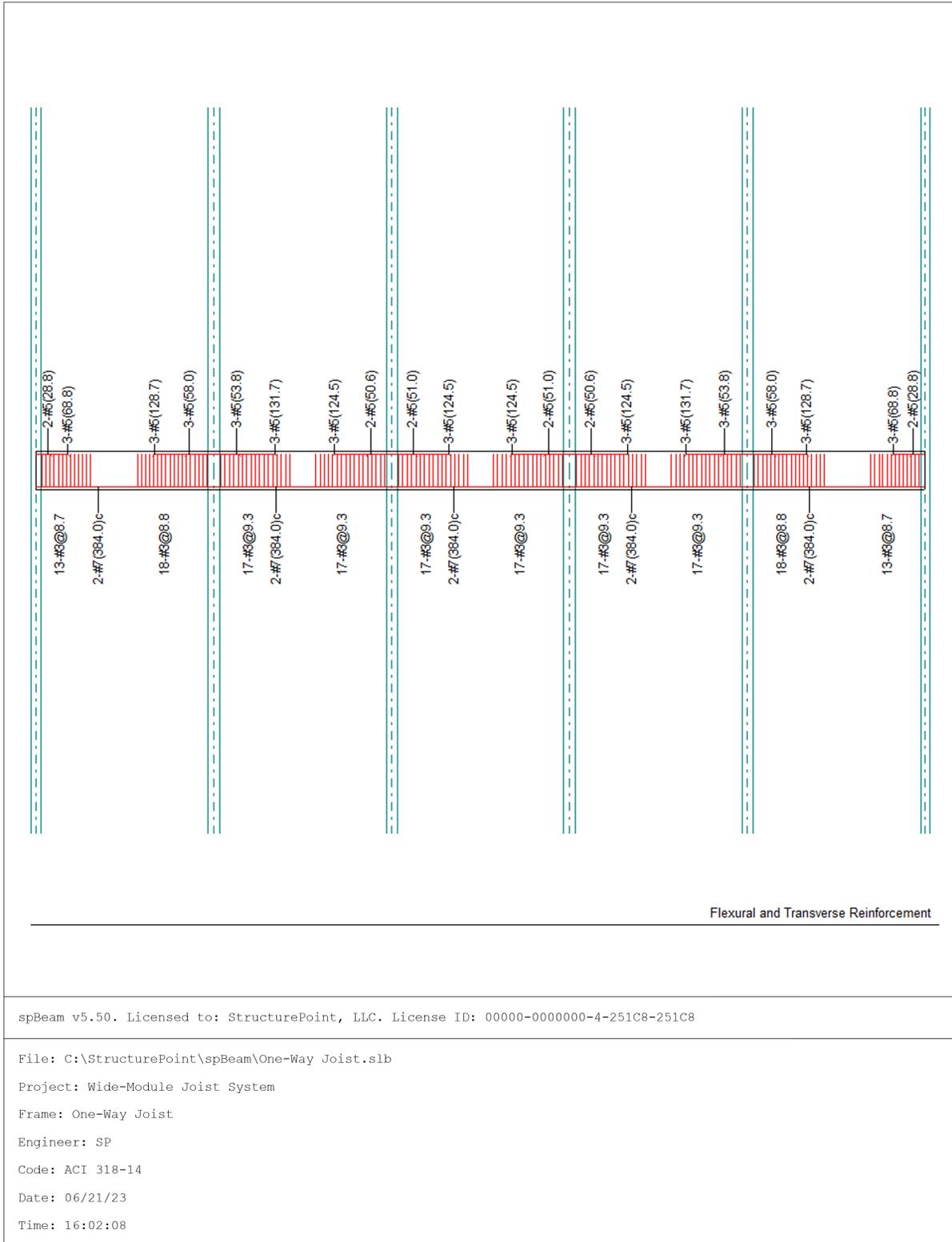


Figure 22 – spBeam Model – Reinforcement Diagram

2.2.7. Summary and Comparison of Results

1. Design Results

1.1. Top Reinforcement

Notes:

*5 - Number of bars governed by maximum allowable spacing.

Span Zone	Width ft	M _{max} k-ft	X _{max} ft	A _{s,min} in ²	A _{s,max} in ²	A _{s,req} in ²	Sp _{Prov} in	Bars
1 Left	3.20	56.01	0.833	0.488	2.930	0.686	8.817	5-#5 *5
Midspan	3.20	0.00	15.917	0.000	2.930	0.000	0.000	---
Right	3.20	128.46	31.000	0.488	2.930	1.658	7.054	6-#5
2 Left	3.20	117.44	1.000	0.488	2.930	1.503	7.054	6-#5
Midspan	3.20	0.00	16.000	0.000	2.930	0.000	0.000	---
Right	3.20	111.28	31.000	0.488	2.930	1.417	8.817	5-#5
3 Left	3.20	112.19	1.000	0.488	2.930	1.430	8.817	5-#5
Midspan	3.20	0.00	16.000	0.000	2.930	0.000	0.000	---
Right	3.20	112.19	31.000	0.488	2.930	1.430	8.817	5-#5
4 Left	3.20	111.28	1.000	0.488	2.930	1.417	8.817	5-#5
Midspan	3.20	0.00	16.000	0.000	2.930	0.000	0.000	---
Right	3.20	117.44	31.000	0.488	2.930	1.503	7.054	6-#5
5 Left	3.20	128.46	1.000	0.488	2.930	1.658	7.054	6-#5
Midspan	3.20	0.00	16.083	0.000	2.930	0.000	0.000	---
Right	3.20	56.02	31.167	0.488	2.930	0.686	8.817	5-#5 *5

1.2. Bottom Reinforcement

Span	Width ft	M _{max} k-ft	X _{max} ft	A _{s,min} in ²	A _{s,max} in ²	A _{s,req} in ²	Sp _{Prov} in	Bars
1	0.61	88.98	14.710	0.484	25.815	1.064	4.022	2-#7
2	0.61	72.82	16.300	0.484	25.815	0.870	4.022	2-#7
3	0.61	74.77	16.000	0.484	25.815	0.893	4.022	2-#7
4	0.61	72.82	15.700	0.484	25.815	0.870	4.022	2-#7
5	0.61	88.98	17.290	0.484	25.815	1.064	4.022	2-#7

1.3. Longitudinal Beam Transverse Reinforcement Demand and Capacity

1.3.1. Beam Transverse Reinforcement Capacity

Notes:

*8 - Minimum transverse (stirrup) reinforcement governs.

Span	Start ft	End ft	Required				Provided				
			X _u ft	V _u kip	Comb/Patt	A _v /s in ² /in	A _v in ²	Sp in	A _v /s in ² /in	ϕV _n kip	
1	0.000	1.083	2.391	18.57	U1/Odd	----	----	----	----	----	
	1.083	10.120	2.391	18.57	U1/Odd	0.0048	0.22	8.7	0.0254	35.86 *8	
	10.120	17.849	10.120	6.91	U1/Odd	0.0000	----	----	----	7.26	
	17.849	30.750	29.443	22.96	U1/S2	0.0100	0.22	8.8	0.0249	35.44	
	30.750	32.000	29.443	22.96	U1/S2	----	----	----	----	----	
2	0.000	1.250	2.557	21.24	U1/S2	----	----	----	----	----	
	1.250	14.080	2.557	21.24	U1/S2	0.0080	0.22	9.3	0.0236	34.36	
	14.080	17.920	14.080	3.86	U1/S2	0.0000	----	----	----	7.26	
	17.920	30.750	29.443	20.87	U1/S3	0.0075	0.22	9.3	0.0236	34.36	
	30.750	32.000	29.443	20.87	U1/S3	----	----	----	----	----	
3	0.000	1.250	2.557	21.01	U1/S3	----	----	----	----	----	
	1.250	14.080	2.557	21.01	U1/S3	0.0077	0.22	9.3	0.0236	34.36	
	14.080	17.920	14.080	3.63	U1/S3	0.0000	----	----	----	7.26	
	17.920	30.750	29.443	21.01	U1/S4	0.0077	0.22	9.3	0.0236	34.36	
	30.750	32.000	29.443	21.01	U1/S4	----	----	----	----	----	
4	0.000	1.250	2.557	20.87	U1/S4	----	----	----	----	----	
	1.250	14.080	2.557	20.87	U1/S4	0.0075	0.22	9.3	0.0236	34.36	
	14.080	17.920	17.920	3.86	U1/S5	0.0000	----	----	----	7.26	
	17.920	30.750	29.443	21.24	U1/S5	0.0080	0.22	9.3	0.0236	34.36	
	30.750	32.000	29.443	21.24	U1/S5	----	----	----	----	----	
5	0.000	1.250	2.557	22.96	U1/S5	----	----	----	----	----	
	1.250	14.151	2.557	22.96	U1/S5	0.0100	0.22	8.8	0.0249	35.44	
	14.151	21.880	21.880	6.91	U1/Odd	0.0000	----	----	----	7.26	
	21.880	30.917	29.609	18.57	U1/Odd	0.0048	0.22	8.7	0.0254	35.86 *8	
	30.917	32.000	29.609	18.57	U1/Odd	----	----	----	----	----	

Table 6 - Comparison of Hand Solution with spBeam Solution for One-Way Joist

Flexural Design						
Span Location	Design Moment (ft-kips)		Reinforcement Required for Flexure (in. ²)		Reinforcement Provided for Flexure (in. ²)	
	Hand	spBeam	Hand	spBeam	Hand	spBeam
Interior Negative	136.54	128.46	1.774	1.658	6-#5	6-#5
Positive	98.07	88.98	1.173	1.064	2-#7	2-#7
Shear Design						
Span Location	V _u (kips/ft)			ϕV _n (kips/ft)		
	Hand	spBeam		Hand	spBeam	
Interior Negative	23.82	22.96		34.34	35.44	

2.2.8. Conclusions and Observations

In this design example, the one-way joist system is modeled as a continuous T-beam representing single one-way joist. There is a good agreement between the hand solution and computer solution. Note that the coefficients traditionally used to determine moments do not address various types of support and geometry.

The maximum calculated total immediate (instantaneous) deflection $(DL + LL) = 0.316$ in., this value can be compared with maximum permissible calculated deflection limitation per project criteria in accordance to ACI 318-14. ACI 318-14 (Table 24.2.2)

In addition to deflection results, parametric studies can be performed in [spBeam](#) to optimize design and detailing results. With a minimum spacing of 1 in. between the 2-#7, two stirrups, and 1.50 in. cover on each side, a total width of 6.50 in. is required. The rib width at the bar level is 6.385 in. which is slightly less than required. For detailing purposes, one of the following options can be used:

1. Bottom bars can be bundled. This practice is often found in joist construction.
2. Stirrups can be rotated by a small angle to preserve the minimum spacing.
3. Bottom bars can be raised sufficiently to achieve the required width taking into the account the reduction into the moment capacity.
4. Other detailing options provided by the builder/formwork supply.

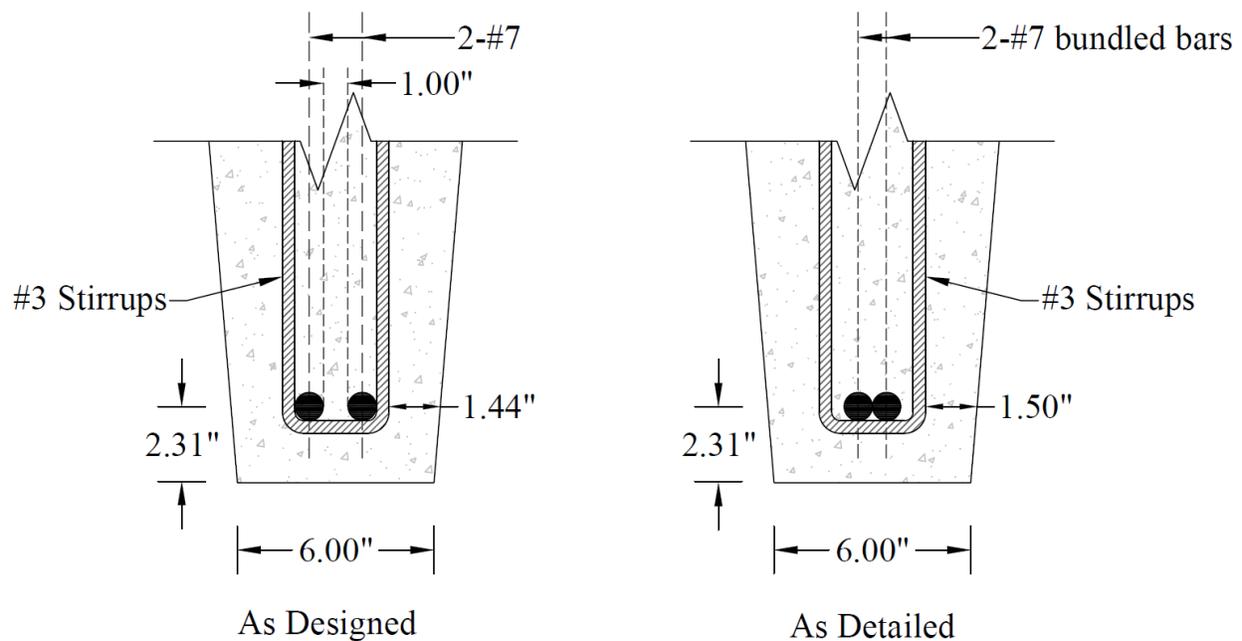


Figure 23 – Joist Cross-Section

2.3. Design of Beam along Grid B (Interior Frame)

In wide-module joist construction, the supporting beam, sometimes referred to as girder, depth is typically set to match the overall joist depth. Therefore, the beam depth is set to 21 in. This depth need to satisfy the minimum depth requirement of ACI 318-14 (Table 9.3.1.1) so that the deflection computations can be waived.

Using the minimum depth for non-prestressed beams in Table 9.3.1.1.

$$\text{End Span: } h = \frac{l}{18.5} = \frac{360}{18.5} = 19.46 \text{ in. (governs)} < 21 \text{ in.} \quad \text{ACI 318-14 (Table 9.3.1.1)}$$

$$\text{Interior Span: } h = \frac{l}{21} = \frac{360}{21} = 17.14 \text{ in.} \quad \text{ACI 318-14 (Table 9.3.1.1)}$$

Therefore, the preliminary beam depth satisfies the minimum depth requirement.

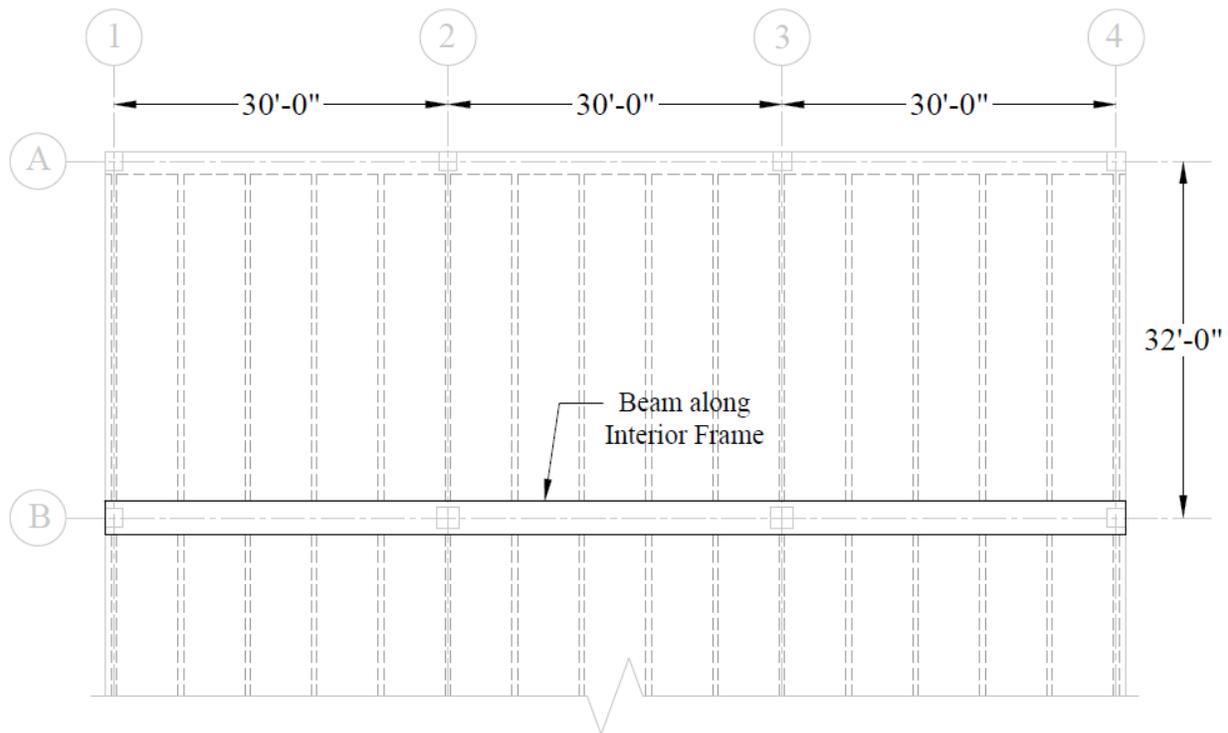


Figure 24 – Partial Plan View Showing Interior Beam along grid B

Beam (girder) design involves the following steps:

- 2.3.1. Determination of span loads
- 2.3.2. Determination of design moments and shears
- 2.3.3. Flexural Design

- 2.3.4. Shear Design
- 2.3.5. Deflections
- 2.3.6. Computer Program Solution
- 2.3.7. Summary and comparison of design results
- 2.3.8. Conclusions and observations

2.3.1. Determination of span loads

Dead Load:

Try 36 in. width for the beam (slightly larger than the column width that helps facilitate the forming, and reduces the beam longitudinal vs. column vertical bar interference)

$$\text{Joist \& Slab Weight} = \left[\frac{5}{12} + \left\{ \left(\left(\frac{(6+8.67)/2}{12} \right) \times \frac{16}{12} \right) / 6 \right\} \right] \times 0.15 \times \left(32 - \frac{36}{12} \right) = 2.40 \frac{\text{kips}}{\text{ft}}$$

$$\text{Beam Weight} = \left(\frac{21}{12} \times \frac{36}{12} \right) \times 0.15 = 0.79 \frac{\text{kips}}{\text{ft}}$$

$$\text{Superimposed Dead Load, } SDL = 0.02 \times 32 = 0.64 \frac{\text{kips}}{\text{ft}}$$

Live Load:

Check for live load reduction per ASCE/SEI 7-10

$$L = L_o \times \left(0.25 + \frac{15}{\sqrt{K_{LL} \times A_T}} \right) \quad \text{ASCE/SEI 7-10 (Eq. 4-1)}$$

Where:

L = Reduced design live load per ft² of area supported by the member

L_o = Unreduced design live load per ft² of area supported by the member = 80 psf

K_{LL} = Live load element factor = 2 for interior beams ASCE/SEI 7-10 (Table 4-2)

A_T = Tributary area = (30 × 32) = 960 ft²

$$L = 80 \times \left(0.25 + \frac{15}{\sqrt{2 \times 960}} \right) = 47.39 \text{ psf}$$

Which satisfies $0.50 \times L_o$ requirement for members supporting one floor. ASCE/SEI 7-10 (4.7.2)

$$L = \frac{47.39}{1,000} \times 32 = 1.52 \frac{\text{kips}}{\text{ft}}$$

Load Combination:

The following gravity load combinations are considered:

$$U = 1.40 \times D \qquad \qquad \qquad \text{ACI 318-14 (Eq. 5.3.1a)}$$

$$w_u = 1.40 \times (2.40 + 0.79 + 0.64) = 1.40 \times 3.83 = 5.36 \frac{\text{kips}}{\text{ft}}$$

$$U = 1.20 \times D + 1.60 \times L \qquad \qquad \qquad \text{ACI 318-14 (Eq. 5.3.1b)}$$

$$w_u = 1.20 \times (2.40 + 0.79 + 0.64) + 1.60 \times 1.52 = 1.20 \times 3.83 + 1.60 \times 1.52 = 7.02 \frac{\text{kips}}{\text{ft}}$$

The span loads are governed by the second load combination.

2.3.2. Determination of design moment and shear

The factored moment and shear can be determined using the simplified method if the requirements are satisfied: ACI 318-14 (6.5.1)

- ✓ Members are prismatic.
- ✓ Loads are uniformly distributed.
- ✓ $L \leq 3D$ ($1.52 \text{ kips/ft} \leq 3 \times 3.83 \text{ kips/ft}$)
- ✓ There are at least two spans.
- ✓ The longer of two adjacent spans does not exceed the shorter by more than 20 percent.

Thus, the approximate coefficients can be used. The factored moment and shear are determined and summarized in the following tables. ACI 318-14 (Table 6.5.2 and Table 6.5.3)

Table 7 - Interior Beam Design Moment Values		
Location		Design Moment Value
End Spans	Exterior Support Negative	$\frac{w_u \times l_n^2}{16} = \frac{7.02 \times 28.17^2}{16} = 348.25 \text{ ft-kips}$
	Mid-span	$\frac{w_u \times l_n^2}{14} = \frac{7.02 \times 28.17^2}{14} = 398.00 \text{ ft-kips}$
	Interior Support Negative	$\frac{w_u \times l_n^2}{10} = \frac{7.02 \times 28.08^2}{10} = 553.90 \text{ ft-kips}$
Interior Spans	Mid-span Positive	$\frac{w_u \times l_n^2}{16} = \frac{7.02 \times 28^2}{16} = 344.14 \text{ ft-kips}$
	Support Negative	$\frac{w_u \times l_n^2}{11} = \frac{7.02 \times 28^2}{11} = 500.56 \text{ ft-kips}$

Table 8 - Interior Beam Design Shear Values	
Location	Design Shear Value
End Span at Face of First Interior Support	$1.15 \times \frac{w_u \times l_n}{2} = 1.15 \times \frac{7.02 \times 28.17}{2} = 113.75 \text{ kips}$
At Face of all other Supports	$\frac{w_u \times l_n}{2} = \frac{7.02 \times 28.17}{2} = 98.91 \text{ kips}$

2.3.3. Flexural Design

For this interior beam, the end span moment values govern the design as tabulated in [Table 7](#).

Calculate the required reinforcement to resist the first interior support negative moment:

$$M_u = 553.90 \text{ ft-kips}$$

Use #8 bars with 1.50 in. concrete cover per [ACI 318-14 \(Table 20.6.1.3.1\)](#). To avoid interference with joist negative moment reinforcement, the clear cover to the girder top reinforcement is required to be increased by lowering the girder top reinforcement. The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, d , is calculated below:

$$d = 21 - \left(1.50 + \frac{3}{8} + \frac{5}{8} + 0.50 \times \frac{8}{8} \right) = 18.00 \text{ in.}$$

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section (jd). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.9, and jd will be taken equal to $0.919 \times d$ since we are designing for the negative moment in a rectangular beam (narrow compression zone). The assumptions will be verified once the area of steel is finalized.

$$\text{Assume } jd = 0.919 \times d = 0.919 \times 18.00 = 16.54 \text{ in.}$$

Interior beam width, $b = 36 \text{ in.}$

The required reinforcement at initial trial is calculated as follows:

$$A_s = \frac{M_u}{\phi \times f_y \times jd} = \frac{553.90 \times 12,000}{0.90 \times 60,000 \times 16.54} = 7.442 \text{ in.}^2$$

Recalculate 'a' for the actual $A_s = 7.442 \text{ in.}^2$:

$$a = \frac{A_s \times f_y}{0.85 \times f'_c \times b} = \frac{7.442 \times 60,000}{0.85 \times 5,000 \times 36} = 2.918 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{2.918}{0.85} = 3.433 \text{ in.}$$

$$\epsilon_t = \left(\frac{0.003}{c} \right) \times d_t - 0.003 = \left(\frac{0.003}{3.433} \right) \times 18.00 - 0.003 = 0.013 > 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi \times f_y \times \left(d - \frac{a}{2}\right)} = \frac{553.90 \times 12,000}{0.90 \times 60,000 \times \left(18.00 - \frac{2.918}{2}\right)} = 7.442 \text{ in.}^2$$

The minimum reinforcement shall not be less than

$$A_{s,\min} = \frac{3 \times \sqrt{f'_c}}{f_y} \times b_w \times d = \frac{3 \times \sqrt{5,000}}{60,000} \times 36 \times 18 = 2.29 \text{ in.}^2 \quad \text{\underline{ACI 318-14 (9.6.1.2(a))}}$$

And not less than

$$A_{s,\min} = \frac{200}{f_y} \times b_w \times d = \frac{200}{60,000} \times 36 \times 18 = 2.16 \text{ in.}^2 \quad \text{\underline{ACI 318-14 (9.6.1.2(b))}}$$

Provide 10-#8 bars: $A_{s,\text{prov}} = (10 \times 0.79) = 7.90 \text{ in.}^2 > 7.442 \text{ in.}^2$ **o.k.**

Maximum spacing allowed:

Check the requirement for distribution of flexural reinforcement to control flexural cracking:

$$s = 15 \times \left(\frac{40,000}{f_s}\right) - 2.50 \times c_c \leq 12 \times \left(\frac{40,000}{f_s}\right) \quad \text{\underline{ACI 318-14 (Table 24.3.2)}}$$

$$c_c = 21 - \left(18 + 0.50 \times \left(\frac{8}{8}\right)\right) = 2.5 \text{ in.}$$

$$\text{Use } f_s = \frac{2}{3} \times f_y = \frac{2}{3} \times 60,000 = 40,000 \text{ psi} \quad \text{\underline{ACI 318-14 (24.3.2.1)}}$$

$$s = 15 \times \left(\frac{40,000}{40,000}\right) - 2.50 \times 2.50 = 8.75 \text{ in. (governs)}$$

$$s = 12 \times \left(\frac{40,000}{40,000}\right) = 12 \text{ in.}$$

Spacing provided for 10-#8 bars

$$s_{\text{prov}} = \frac{(b_w - 2 \times d_s)}{\# \text{ of bars} - 1} = \frac{(36 - 2 \times 2.625)}{9} = 3.42 \text{ in.} < 8.75 \text{ in.} \quad \text{\underline{o.k.}}$$

Where $d_s = 2.625$ in. for #3 stirrup as shown in the following Figure. CRSI 2002 (Figure 12-9)

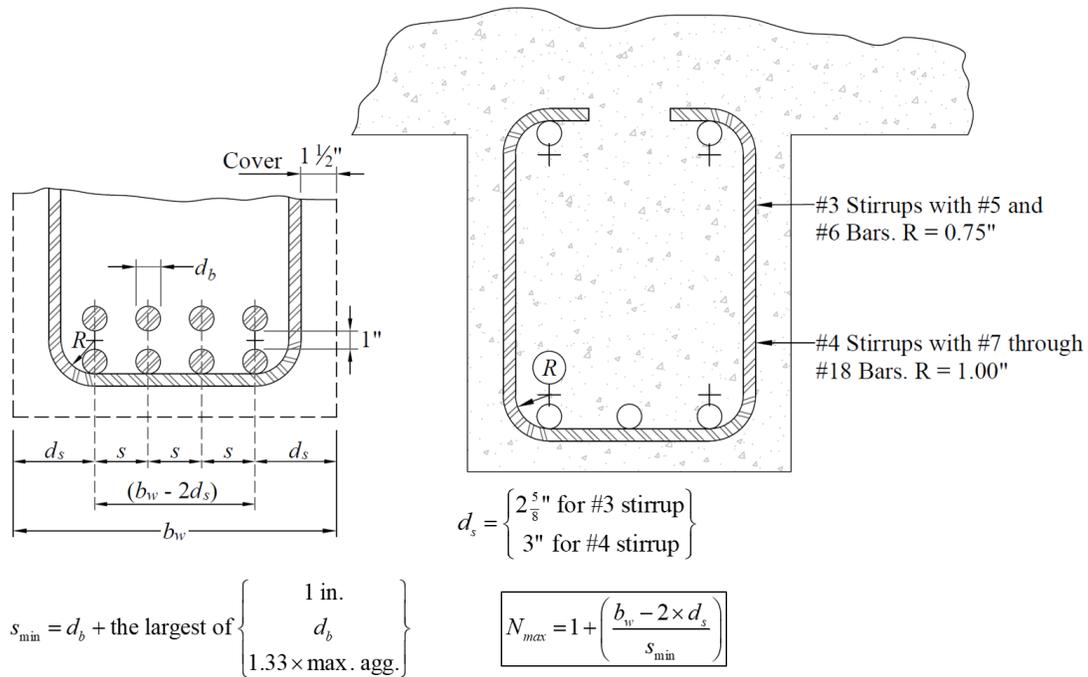


Figure 25 – Maximum Number of Bars in Beams

Check the spacing, s provided, is greater than the minimum center to center spacing, s_{min} where:

$$s_{min} = d_b + \text{the largest of } \begin{cases} 1 \text{ in.} \\ d_b \\ 1.33 \times \text{max. agg.} \end{cases}$$

CRSI 2002 (Figure 12-9)

Where maximum aggregate size is $\frac{3}{4}$ "

$$s_{min} = 1.00 + \max \begin{cases} 1.00 \\ 1.00 \\ 1.33 \times 0.75 = 1.00 \end{cases} = 1.00 + 1.00 = 2.00 \text{ in.}$$

Since the spacing provided is greater than 2.00 in. Therefore, 10-#8 bars are **o.k.**

All the values on Table 9 are calculated based on the procedure outlined above.

Table 9 - Reinforcing Design Summary for Interior Beam					
	End Span			Interior Span	
	Exterior Negative	Positive	Interior Negative	Positive	Negative
Design Moment, M_u (ft-kips)	348.25	398.00	553.90	344.14	500.56
Effective depth, d (in.)	18.00*	18.625**	18.00*	18.625**	18.00*
A_s req'd (in.²)	4.522	5.013	7.442	4.301	6.664
A_s min (in.²)	2.29	2.37	2.29	2.37	2.29
Reinforcement	6-#8	7-#8	10-#8	6-#8	9-#8

* The beam top bars are to be placed below the joist top bars.

** The beam bottom bars are to be placed at the bottom-most layer. The joist bottom bars, then, shall be spliced at joist-beam intersection.

2.3.4. Shear Design

From Table 8 above, the shear value in end span at face of first interior support governs.

$$V_u = 1.15 \times \frac{w_u \times l_n}{2} = 1.15 \times \frac{7.02 \times 28.17}{2} = 113.75 \text{ kips}$$

The design shear at a distance, d , away from the face of support,

$$V_u = 113.75 - 7.02 \times \frac{18}{12} = 103.21 \text{ kips}$$

Shear strength provided by concrete

$$\phi V_c = \phi \times (2 \times \sqrt{f'_c} \times b_w \times d)$$

ACI 318-14 (Eq. 22.5.5.1)

$$\phi V_c = 0.75 \times (2.00 \times 1.00 \times \sqrt{5,000} \times 36 \times 18) = 68,730.78 \text{ lb} = 68.73 \text{ kips}$$

Since $V_u > \frac{\phi V_c}{2}$, shear reinforcement is required.

Try #3, Grade 60 four-leg stirrups ($A_v = 0.44 \text{ in.}^2$) with a 90° hook.

The nominal shear strength required to be provided by shear reinforcement is

$$V_s = V_n - V_c = \frac{V_u}{\phi} - V_c = \left(\frac{103.21}{0.75} \right) - 91.64 = 45.98 \text{ kips}$$

Check whether V_s is less than $8 \times \sqrt{f'_c} \times b_w \times d$

If V_s is greater than $8 \times \sqrt{f'_c} \times b_w \times d$, then the cross-section has to be revised as ACI 318-14 limits the shear capacity to be provided by stirrups to $8 \times \sqrt{f'_c} \times b_w \times d$ ACI 318-14 (22.5.1.2)

$$8 \times \sqrt{f'_c} \times b_w \times d = 8 \times \sqrt{5,000} \times 36 \times 18 = 366,564.16 \text{ lb} = 366.56 \text{ kips}$$

Since V_s does not exceed $8 \times \sqrt{f'_c} \times b_w \times d$, the cross section is adequate.

Calculate the required stirrup spacing as

$$s_{req'd} = \frac{\phi \times A_v \times f_{yt} \times d}{V_u - \phi \times V_c} = \frac{0.75 \times 0.44 \times 60,000 \times 18}{103.21 - 68.73} = 10.34 \text{ in.} \quad \text{ACI 318-14 (22.5.10.5.3)}$$

Check whether the required spacing based on the shear demand meets the spacing limits for shear reinforcement per ACI 318-14 (9.7.6.2.2).

Check whether V_s is less than $4 \times \sqrt{f'_c} \times b_w \times d$

$$4 \times \sqrt{f'_c} \times b_w \times d = 4 \times \sqrt{5,000} \times 36 \times 18 = 183,282.08 \text{ lb} = 183.28 \text{ kips} > V_s = 45.98 \text{ kips}$$

Therefore, maximum stirrup spacing shall be the smallest of $d/2$ and 24 in. ACI 318-14 (Table 9.7.6.2.2)

$$s_{max} = \text{lesser of } \left[\frac{d}{2} \right] \text{ and } \left[24 \text{ in.} \right] = \text{lesser of } \left[\frac{18}{2} \right] \text{ and } \left[24 \text{ in.} \right] = \text{lesser of } \left[9 \text{ in.} \right] \text{ and } \left[24 \text{ in.} \right] = 9 \text{ in.}$$

This value governs over the required stirrup spacing of 10.34 in which was based on the demand. Note that since the stirrup spacing is governed by s_{max} , the size of the stirrup can be kept as #3. Selecting #4 stirrup size will produce capacity more than what is required and therefore, be uneconomical.

Check the maximum stirrup spacing based on minimum shear reinforcement

$$s_{max} \leq \frac{A_v \times f_{yt}}{0.75 \times \sqrt{f'_c} \times b_w} = \frac{0.44 \times 60,000}{0.75 \times \sqrt{5,000} \times 36} = 13.83 \text{ in. (does not govern)} \quad \text{ACI 318-14 (10.6.2.2(a))}$$

$$s_{max} \leq \frac{A_v \times f_{yt}}{50 \times b_w} = \frac{0.44 \times 60,000}{50 \times 36} = 14.67 \text{ in. (does not govern)} \quad \text{ACI 318-14 (10.6.2.2(b))}$$

Therefore, s_{max} value is governed by the spacing limit per ACI 318-14 (9.7.6.2.2), and is equal to 9 in.

Use #3 @ 8 in. stirrups

$$V_n = \frac{A_v \times f_y \times d}{s} + V_c$$

ACI 318-14 (22.5.1.1 and 22.5.10.5.3)

$$V_n = \frac{0.44 \times 60,000 \times 18}{8} + 91.64 = 59.40 + 91.64 = 151.04 \text{ kips}$$

o.k.

$$\phi V_n = 0.75 \times 151.04 = 113.28 \text{ kips} > V_u = 103.21 \text{ kips}$$

Compute where $\frac{V_u}{\phi}$ is equal to $\frac{V_c}{2}$, and the stirrups can be stopped

$$x = \frac{\left(\frac{V_u}{\phi}\right) - \left(\frac{V_c}{2}\right)}{\frac{V_u}{\phi}} \times \frac{l_n}{2} = \frac{\left(\frac{103.21}{0.75}\right) - \left(\frac{91.64}{2}\right)}{\left(\frac{103.21}{0.75}\right)} \times \frac{28.17 \times 12}{2} = 112.73 \text{ in.}$$

At interior end of the exterior span, use 16-#3 @ 8 in o.c., Place 1st stirrup 2 in. from the face of the column.

2.3.5. Deflections

Since the preliminary beam depth met minimum depth requirement, the deflection calculations are not required. A lesser depth maybe possible and consequently cost savings can be achieved through deflection computations. Deflection values are calculated and provided for every model created by [spBeam](#) Program and can be used by the engineer to make additional optimization decisions.

2.3.6. Computer Program Solution

[spBeam](#) Program can be utilized to analyze and design the interior continuous beam along grid B. The beam is modeled as a three span continuous rectangular beam.

The program calculates the internal forces (shear force and bending moment), moment and shear capacities, immediate and long-term deflection results, and required flexural reinforcement. The graphical and text results are provided here for both input and output of the [spBeam](#) model.

The beam is modeled as a 36 in. by 21 in. deep rectangular longitudinal beam with column supports. The supports can be modeled as pinned, fixed, or using actual geometric properties of the beam-column joint. A value of 100 is used in this model for column stiffness share, indicating the actual column stiffness. When the percentage lies between zero and 100%, the joint stiffness contribution by the column is multiplied by that percentage. The default value is 100%.

Support Data

Columns | Column Capitals | Transverse Beams | Boundary Conditions

Support: 1

Stiffness share %: 100

Above: Height (ft) 12, c1 (in) 20, c2 (in) 20

Below: Height (ft) 12, c1 (in) 20, c2 (in) 20

Modify Copy...

Sup. No	Stiff%	HtA	c1A	c2A	HtB	c1B	c2B
1	100	12	20	20	12	20	20
2	100	12	24	24	12	24	24
3	100	12	24	24	12	24	24
4	100	12	20	20	12	20	20

OK Cancel

Figure 26 – spBeam Model – Support Data

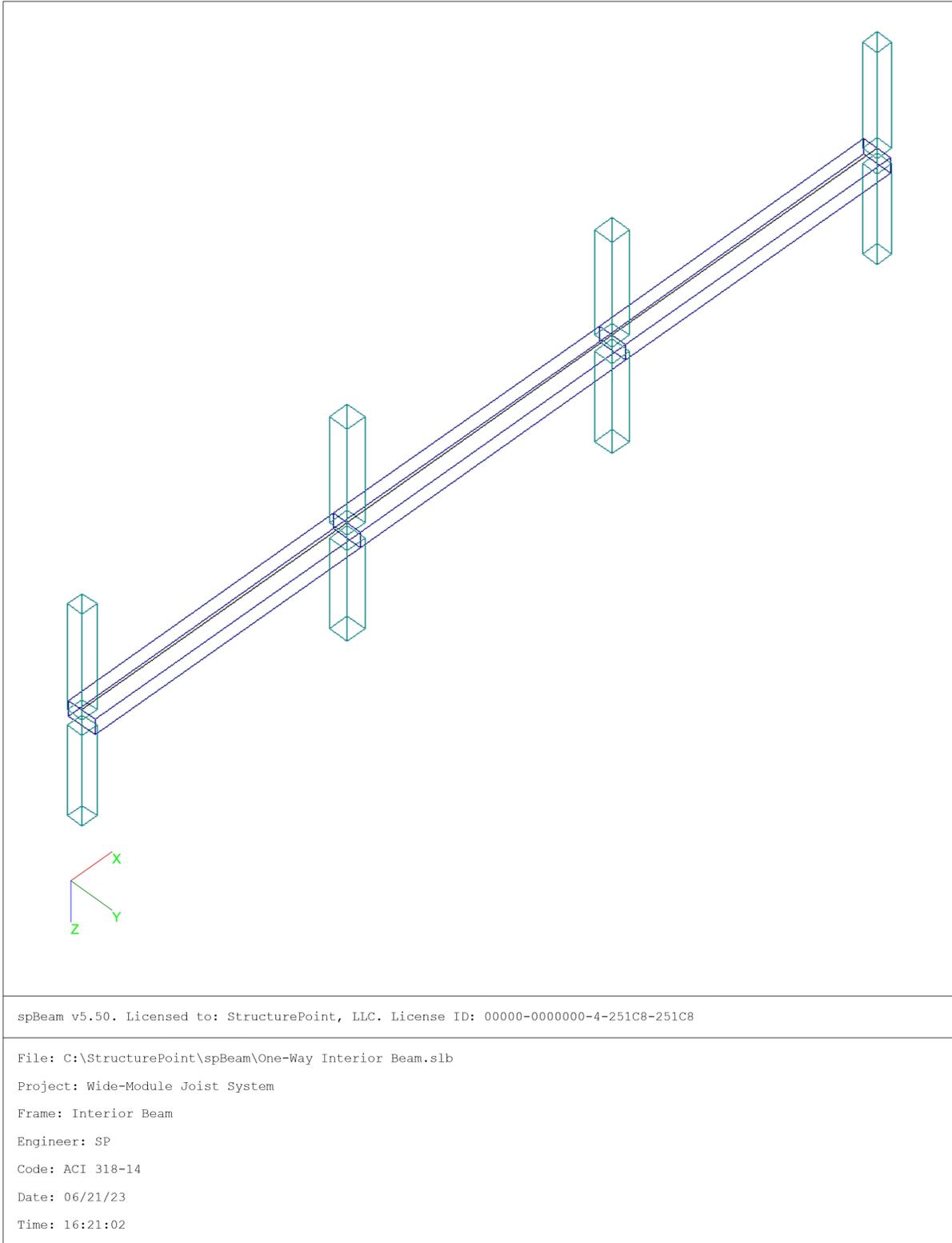


Figure 27 – spBeam Model – Isometric View – Interior Continuous Beam along Grid B



Figure 28 – spBeam Model – Loads (Including Live Load Patterning)

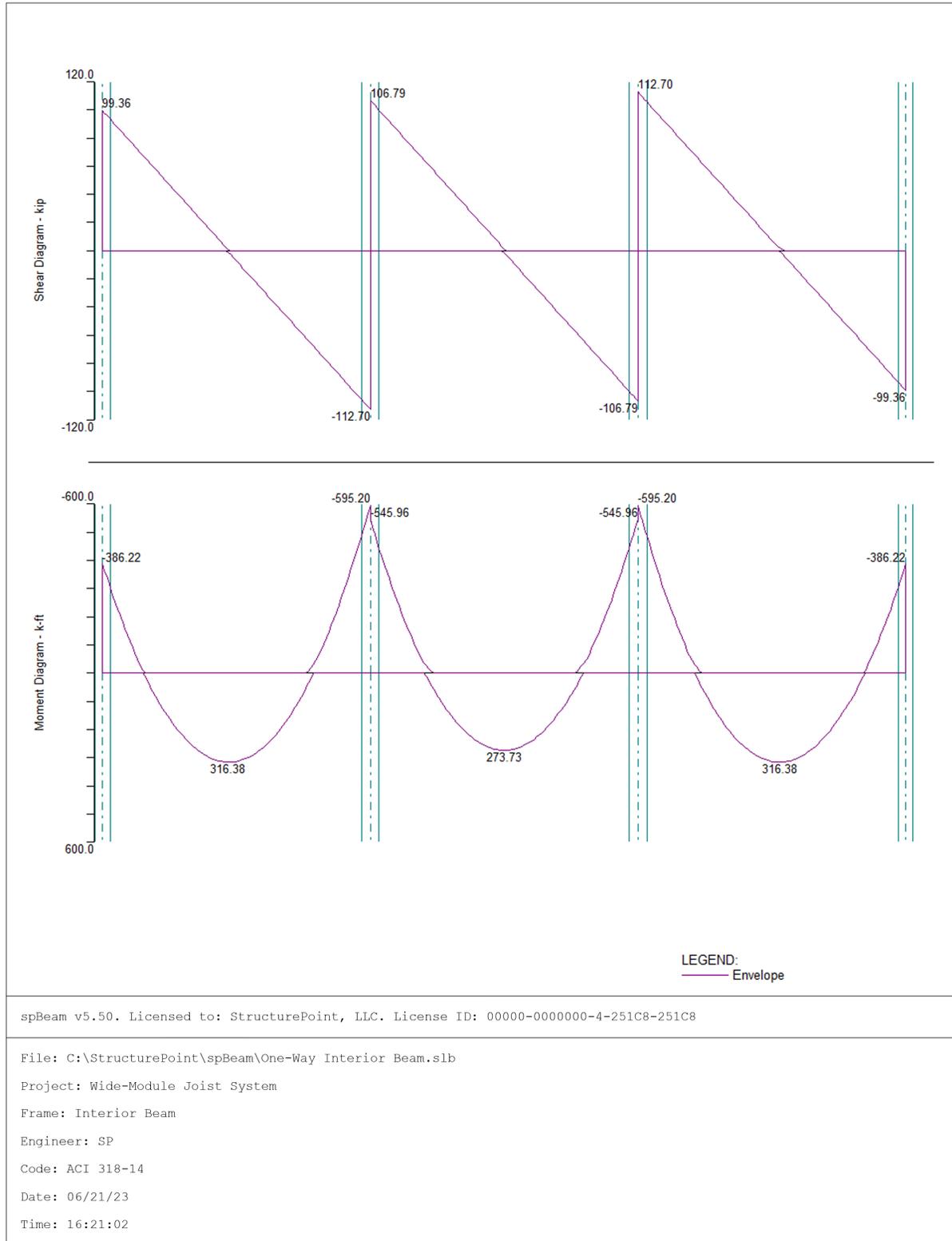


Figure 29 – spBeam Model – Internal Forces (Shear Force Diagram and Bending Moment Diagram)

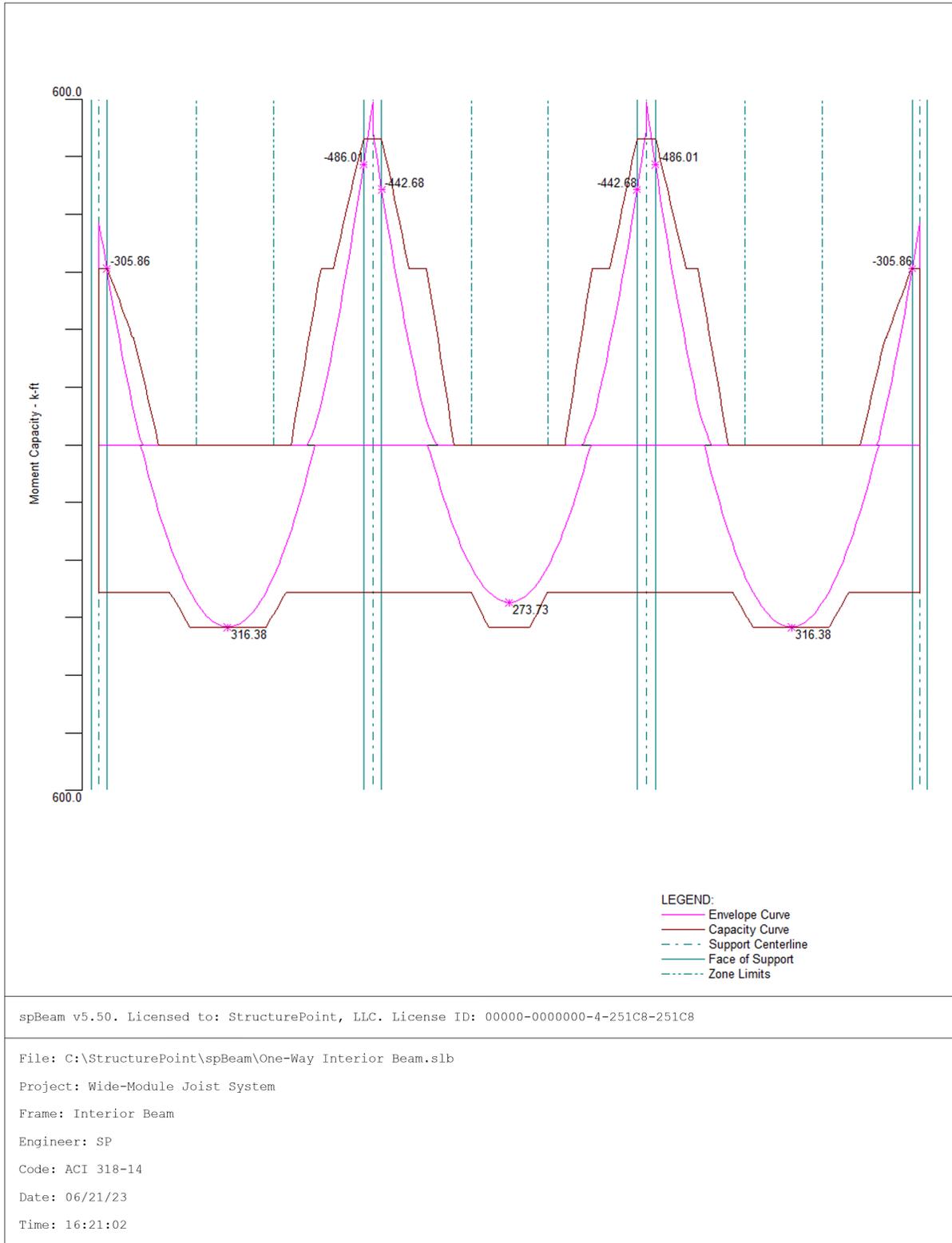


Figure 30 – spBeam Model – Moment Capacity Diagram

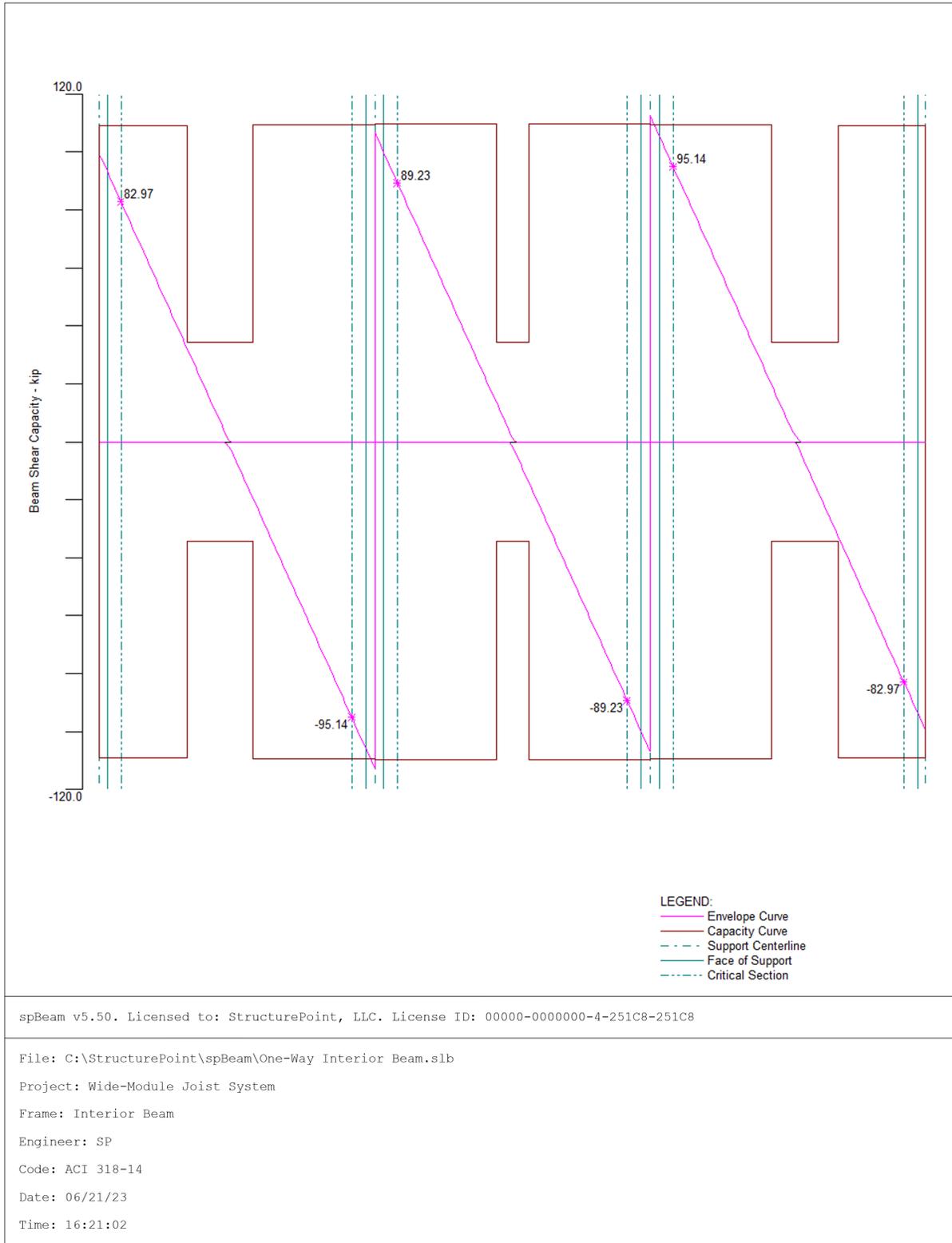


Figure 31 – spBeam Model – Shear Capacity Diagram

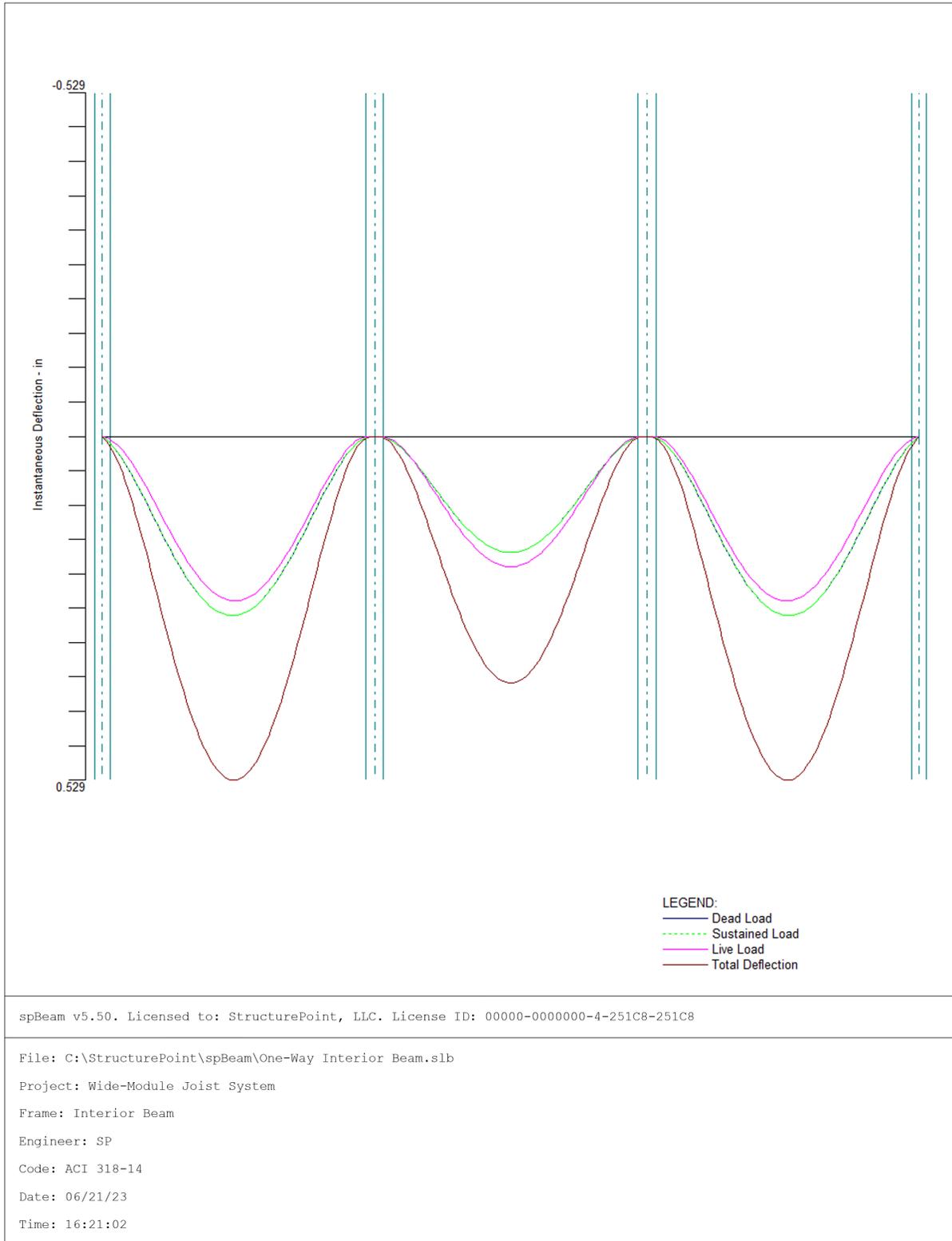


Figure 32 – spBeam Model – Immediate Deflection Diagram

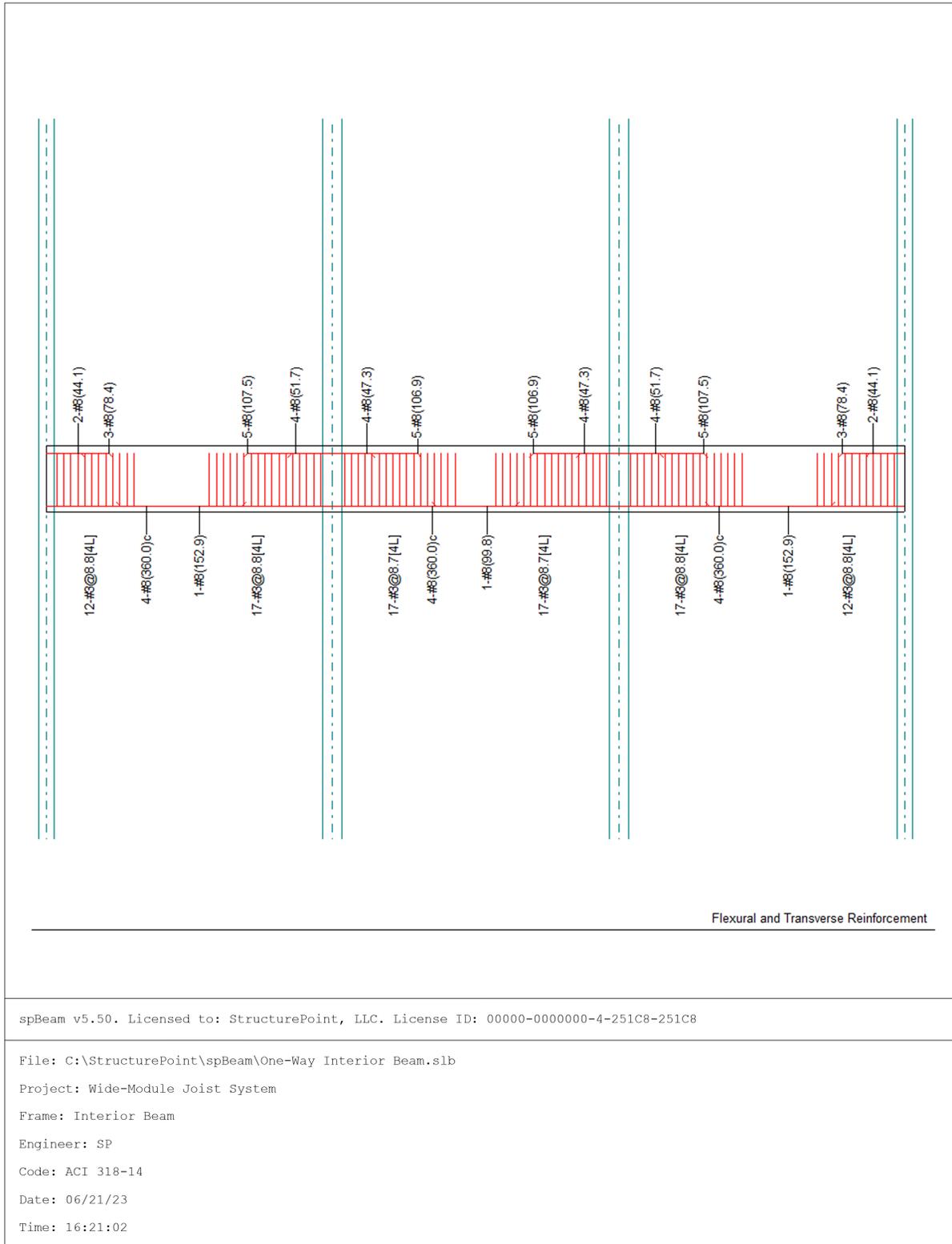


Figure 33 – spBeam Model – Reinforcement Diagram

2.3.7. Summary and Comparison of Results

1. Design Results

1.1. Top Reinforcement

Span Zone	Width ft	M _{max} k-ft	X _{max} ft	A _{s,min} in ²	A _{s,max} in ²	A _{s,req} in ²	Sp _{Prov} in	Bars
1 Left	3.00	305.86	0.833	2.291	13.770	3.946	7.578	5-#8
Midspan	3.00	0.00	14.917	0.000	13.770	0.000	0.000	---
Right	3.00	486.01	29.000	2.291	13.770	6.454	3.789	9-#8
2 Left	3.00	442.68	1.000	2.291	13.770	5.836	3.789	9-#8
Midspan	3.00	0.00	15.000	0.000	13.770	0.000	0.000	---
Right	3.00	442.68	29.000	2.291	13.770	5.836	3.789	9-#8
3 Left	3.00	486.01	1.000	2.291	13.770	6.454	3.789	9-#8
Midspan	3.00	0.00	15.083	0.000	13.770	0.000	0.000	---
Right	3.00	305.86	29.167	2.291	13.770	3.946	7.578	5-#8

1.2. Bottom Reinforcement

Span	Width ft	M _{max} k-ft	X _{max} ft	A _{s,min} in ²	A _{s,max} in ²	A _{s,req} in ²	Sp _{Prov} in	Bars
1	3.00	316.38	14.072	2.371	14.248	3.938	7.578	5-#8
2	3.00	273.73	15.000	2.371	14.248	3.387	7.578	5-#8
3	3.00	316.38	15.928	2.371	14.248	3.938	7.578	5-#8

1.3. Longitudinal Beam Transverse Reinforcement Demand and Capacity

1.3.1. Beam Transverse Reinforcement Capacity

Notes:

*8 - Minimum transverse (stirrup) reinforcement governs.

Span	Start ft	End ft	Required				Provided				
			X _u ft	V _u kip	Comb/Patt	A _v /s in ² /in	A _v in ²	Sp in	A _v /s in ² /in	ΦV _n kip	
1	0.000	1.083	2.333	82.97	U1/Odd	-----	-----	-----	-----	-----	
	1.083	9.524	2.333	82.97	U1/Odd	0.0176	0.44	8.8	0.0500	109.20 *8	
	9.524	16.714	9.524	32.45	U1/Odd	0.0000	-----	-----	-----	34.37	
	16.714	28.750	27.500	95.14	U1/S2	0.0326	0.44	8.8	0.0503	109.45	
	28.750	30.000	27.500	95.14	U1/S2	-----	-----	-----	-----	-----	
2	0.000	1.250	2.500	89.23	U1/S2	-----	-----	-----	-----	-----	
	1.250	13.214	2.500	89.23	U1/S2	0.0253	0.44	8.7	0.0506	109.69 *8	
	13.214	16.786	13.214	13.96	U1/S2	0.0000	-----	-----	-----	34.37	
	16.786	28.750	27.500	89.23	U1/S3	0.0253	0.44	8.7	0.0506	109.69 *8	
	28.750	30.000	27.500	89.23	U1/S3	-----	-----	-----	-----	-----	
3	0.000	1.250	2.500	95.14	U1/S3	-----	-----	-----	-----	-----	
	1.250	13.286	2.500	95.14	U1/S3	0.0326	0.44	8.8	0.0503	109.45	
	13.286	20.476	20.476	32.45	U1/Odd	0.0000	-----	-----	-----	34.37	
	20.476	28.917	27.667	82.97	U1/Odd	0.0176	0.44	8.8	0.0500	109.20 *8	
	28.917	30.000	27.667	82.97	U1/Odd	-----	-----	-----	-----	-----	

Table 10 - Comparison of Hand Solution with spBeam Solution for One-Way Interior Beam						
Flexural Design						
Span Location	Design Moment (ft-kips)		Reinforcement Required for Flexure (in. ²)		Reinforcement Provided for Flexure (in. ²)	
	Hand	spBeam	Hand	spBeam	Hand	spBeam
Interior Negative	553.90	486.01	7.442	6.454	10-#8	9-#8
Positive	344.14	316.28	4.301	3.938	6-#8	5-#8
Shear Design						
Span Location	V_u (kips/ft)			ϕV_n (kips/ft)		
	Hand	spBeam		Hand	spBeam	
Interior Negative	103.21	95.14		113.28	109.45	

2.3.8. Conclusions and Observations

In this design example, the interior beam is modeled as a continuous rectangular longitudinal beam. There is a good agreement between the hand solution and computer solution. Note that the coefficients traditionally used to determine moments do not address various types of support and geometry.

The maximum calculated total immediate (instantaneous) deflection (DL + LL) = 0.529 in., this value can be compared with maximum permissible calculated deflection limitation per project criteria in accordance to ACI 318-14. ACI 318-14 (Table 24.2.2)

In addition to deflection results, parametric studies can be performed in [spBeam](#) to optimize design and detailing results.

The reinforcement diagram ([Figure 33](#)) shows the minimum length required (including the development length) for flexural design. The bars can be extended and detailed to provide the required support for shear stirrups.

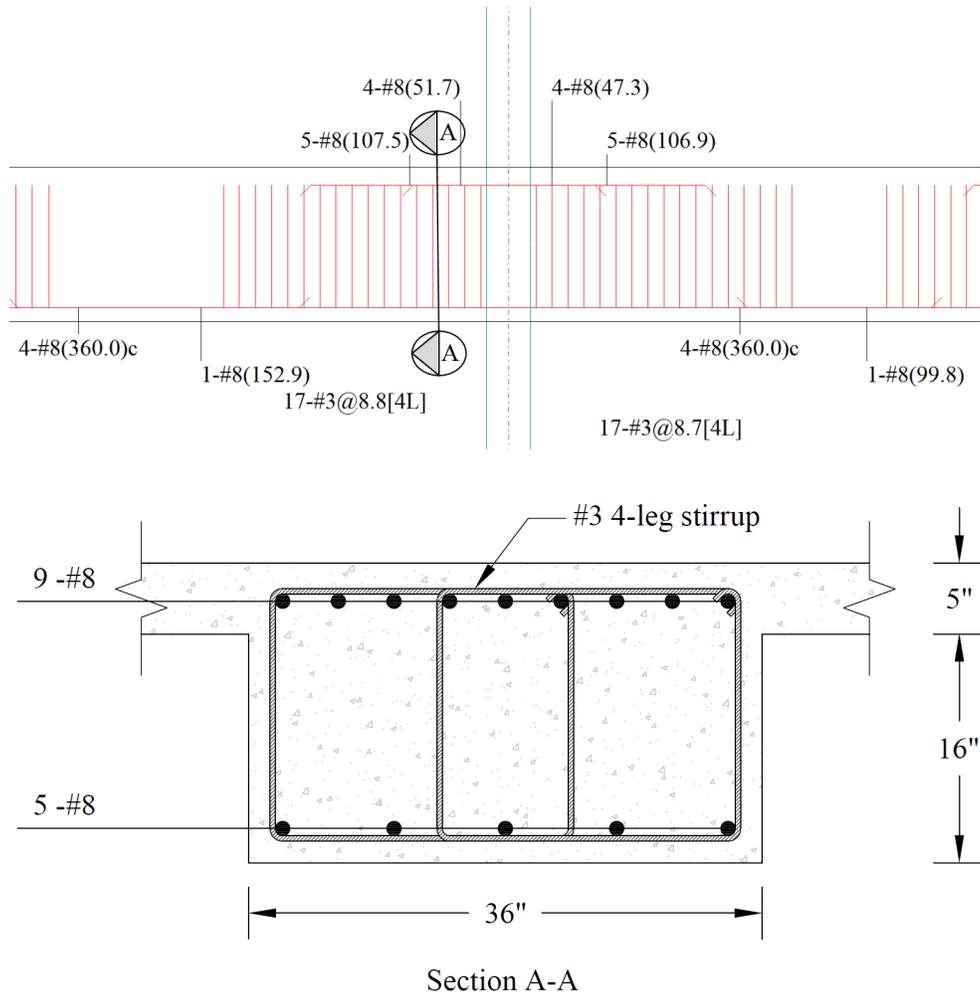


Figure 34 – Interior Beam Cross-Section (Near the First Interior Support)

2.4. Design of Beam along Grid A (Exterior Frame)

In the wide-module joist construction, the supporting beam depths shall be same as the overall joist depth. Therefore, the beam depth is set to 21 in. This beam depth need to satisfy the minimum depth requirement of ACI 318-14 (Table 9.3.1.1) so that the deflection computations can be waived. The beams of the exterior frame shall be designed and detailed for the combined effects of flexure, shear, and torsion according to ACI 318.

Using the minimum depth for non-prestressed beams in Table 9.3.1.1.

$$\text{End Span: } h = \frac{l}{18.5} = \frac{360}{18.5} = 19.46 \text{ in. (governs)} < 21 \text{ in.} \quad \text{ACI 318-14 (Table 9.3.1.1)}$$

$$\text{Interior Span: } h = \frac{l}{21} = \frac{360}{21} = 17.14 \text{ in.} \quad \text{ACI 318-14 (Table 9.3.1.1)}$$

Therefore, the preliminary beam depth satisfies the minimum depth requirement.

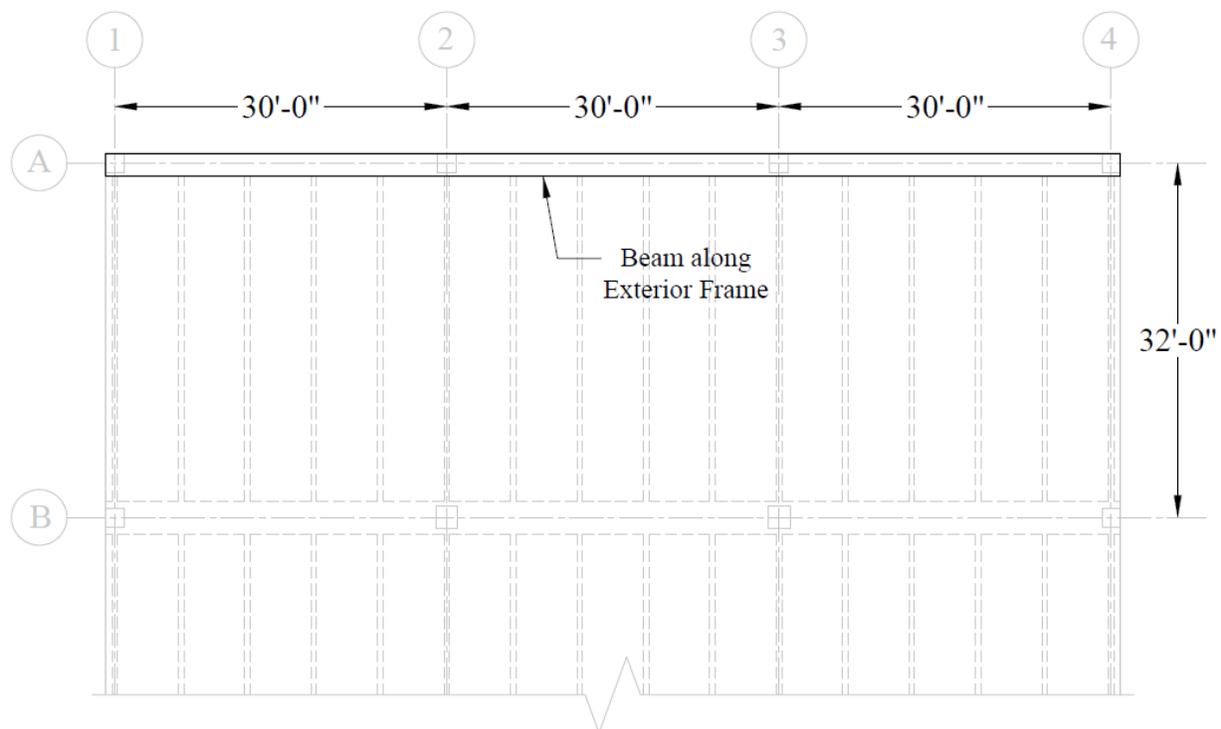


Figure 35 – Partial Plan View Showing Exterior Beam along grid A

Beam (girder) design involves the following steps:

- 2.4.1. Determination of span loads
- 2.4.2. Determination of design moment, shear, and torsion
- 2.4.3. Flexural and torsion design

- 2.4.4. Shear and torsion design
- 2.4.5. Deflections
- 2.4.6. Computer program solution
- 2.4.7. Summary and comparison of design results
- 2.4.8. Conclusions and observations

2.4.1. Determination of span loads

Dead Load:

Try 24 in. width for the beam (slightly larger than the column width that helps facilitate the forming, and reduces the beam longitudinal vs. column vertical bar interference)

$$\text{Joist \& Slab Weight} = \left[\frac{5}{12} + \left\{ \left(\left(\frac{(6+8.67)/2}{12} \right) \times \frac{16}{12} \right) / 6 \right\} \right] \times 0.15 \times \left(16 - \frac{24-20/2}{12} \right) = 1.23 \frac{\text{kips}}{\text{ft}}$$

$$\text{Beam Weight} = \left(\frac{21}{12} \times \frac{24}{12} \right) \times 0.15 = 0.53 \frac{\text{kips}}{\text{ft}}$$

$$\text{Superimposed Dead Load } SDL = 0.02 \times (16 + 10/12) = 0.34 \frac{\text{kips}}{\text{ft}}$$

Live Load:

Check for live load reduction per ASCE/SEI 7-10

$$L = L_o \times \left(0.25 + \frac{15}{\sqrt{K_{LL} \times A_T}} \right) \quad \text{ASCE/SEI 7-10 (Eq. 4-1)}$$

Where:

L = Reduced design live load per ft² of area supported by the member

L_o = Unreduced design live load per ft² of area supported by the member = 80 psf

K_{LL} = 2 (edge beams without cantilever slabs) ASCE/SEI 7-10 (Table 4-2)

A_T = Tributary area = $(30 \times (16 - 10/12)) = 505 \text{ ft}^2$

$$L = 80 \times \left(0.25 + \frac{15}{\sqrt{2 \times 505}} \right) = 57.76 \text{ psf}$$

Which satisfies $0.50 \times L_o$ requirement for members supporting one floor. ASCE/SEI 7-10 (4.7.2)

$$L = \frac{57.76}{1,000} \times \left(16 + \frac{10}{12}\right) = 0.97 \frac{\text{kips}}{\text{ft}}$$

Load Combination:

The following gravity load combinations are considered:

$$U = 1.40 \times D \quad \text{ACI 318-14 (Eq. 5.3.1a)}$$

$$w_u = 1.40 \times (1.23 + 0.53 + 0.34) = 1.40 \times 2.10 = 2.93 \frac{\text{kips}}{\text{ft}}$$

$$U = 1.20 \times D + 1.60 \times L \quad \text{ACI 318-14 (Eq. 5.3.1b)}$$

$$w_u = 1.20 \times (1.23 + 0.53 + 0.34) + 1.60 \times 0.97 = 1.20 \times 2.10 + 1.60 \times 0.97 = 4.06 \frac{\text{kips}}{\text{ft}}$$

The span loads are governed by the second load combination.

For factored torsional moment calculations, the beam self-weight is not included since it is applied along the beam section centerline. And the moment arm is the distance from the midspan to the centerline of the exterior beam section = $16/2 - (24/2 - 20/2)/12 = 7.83$ ft.

Thus, the following load combinations are used for the calculation of the factored torsional moment:

$$U = 1.40 \times D \quad \text{ACI 318-14 (Eq. 5.3.1a)}$$

$$T_u = (1.40 \times (1.23 + 0.34)) \times 7.83 = 1.40 \times 1.57 \times 7.83 = 17.17 \frac{\text{ft-kips}}{\text{ft}}$$

$$U = 1.20 \times D + 1.60 \times L \quad \text{ACI 318-14 (Eq. 5.3.1b)}$$

$$T_u = (1.20 \times (1.24 + 0.34) + 1.60 \times 0.97) \times 7.83 = (1.20 \times 1.57 + 1.60 \times 0.97) \times 7.83 = 26.91 \frac{\text{ft-kips}}{\text{ft}}$$

The span factored torsional moments are governed by the second load combination.

2.4.2. Determination of design flexural moment, shear, and torsional moment

The factored moment and shear can be determined using the simplified method if the requirements are satisfied: **ACI 318-14 (6.5.1)**

- ✓ Members are prismatic.
- ✓ Loads are uniformly distributed.
- ✓ $L \leq 3D$ ($0.97 \text{ kips/ft} \leq 3 \times 2.10 \text{ kips/ft}$)
- ✓ There are at least two spans.
- ✓ The longer of two adjacent spans does not exceed the shorter by more than 20 percent.

Thus, the approximate coefficients can be used. The factored moments and shears are determined and summarized in the following tables. **ACI 318-14 (Table 6.5.2 and Table 6.5.3)**

Table 11 - Exterior Beam Design Flexural Moment Values		
Location		Design Flexural Moment Value
End Spans	Exterior Support Negative	$\frac{w_u \times l_n^2}{16} = \frac{4.06 \times 28.33^2}{16} = 203.95 \text{ ft-kips}$
	Mid-span	$\frac{w_u \times l_n^2}{14} = \frac{4.06 \times 28.33^2}{14} = 233.08 \text{ ft-kips}$
	Interior Support Negative	$\frac{w_u \times l_n^2}{10} = \frac{4.06 \times 28.33^2}{10} = 326.31 \text{ ft-kips}$
Interior Spans	Mid-span Positive	$\frac{w_u \times l_n^2}{16} = \frac{4.06 \times 28.33^2}{16} = 203.95 \text{ ft-kips}$
	Support Negative	$\frac{w_u \times l_n^2}{11} = \frac{4.06 \times 28.33^2}{11} = 296.65 \text{ ft-kips}$

Table 12 - Exterior Design Shear Values	
Location	Design Shear Value
End Span at Face of First Interior Support	$1.15 \times \frac{w_u \times l_n}{2} = 1.15 \times \frac{4.06 \times 28.33}{2} = 66.22 \text{ kips}$
At Face of all other Supports	$\frac{w_u \times l_n}{2} = \frac{4.06 \times 28.33}{2} = 57.58 \text{ kips}$

Any structural analysis method can be used to calculate the torsional. The following table shows the torsional moment values at the centerline of the supports:

Table 13 - Exterior Design Torsional Moment Values	
Location	Design Torsional Moment Value
At the centerline of all Supports	$\frac{w_u \times l}{2} = \frac{26.91 \times 30}{2} = 403.59 \text{ ft-kips}$

2.4.3. Flexural, Shear, and Torsion Design

For this exterior beam, the end span moment values govern the design as tabulated in [Table 11](#).

Calculate the required reinforcement to resist the first interior support negative moment:

$$M_u = 326.31 \text{ ft-kips}$$

Use #8 bars with 1.50 in. concrete cover per [ACI 318-14 \(Table 20.6.1.3.1\)](#). To avoid interference with joist negative moment reinforcement, the clear cover to the girder top reinforcement is required to be increased by lowering the girder top reinforcement. The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, d , is calculated below:

$$d = 21 - \left(1.50 + \frac{3}{8} + \frac{5}{8} + 0.50 \times \frac{8}{8} \right) = 18.00 \text{ in.}$$

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section (jd). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.9, and jd will be taken equal to $0.929 \times d$ since we are designing for the negative moment in a rectangular beam (narrow compression zone). The assumptions will be verified once the area of steel is finalized.

$$\text{Assume } jd = 0.929 \times d = 0.929 \times 18.00 = 16.73 \text{ in.}$$

Exterior beam width, $b = 24$ in.

The required reinforcement at initial trial is calculated as follows:

$$A_s = \frac{M_u}{\phi \times f_y \times jd} = \frac{326.31 \times 12,000}{0.90 \times 60,000 \times 16.73} = 4.336 \text{ in.}^2$$

Recalculate 'a' for the actual $A_s = 4.336 \text{ in.}^2$:

$$a = \frac{A_s \times f_y}{0.85 \times f'_c \times b} = \frac{4.336 \times 60,000}{0.85 \times 5,000 \times 24} = 2.550 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{2.550}{0.85} = 3.000 \text{ in.}$$

$$\epsilon_t = \left(\frac{0.003}{c} \right) \times d_t - 0.003 = \left(\frac{0.003}{3.000} \right) \times 18.00 - 0.003 = 0.015 > 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi \times f_y \times \left(d - \frac{a}{2}\right)} = \frac{326.31 \times 12,000}{0.90 \times 60,000 \times \left(18.00 - \frac{2.550}{2}\right)} = 4.336 \text{ in.}^2$$

The minimum reinforcement shall not be less than

$$A_{s,\min} = \frac{3 \times \sqrt{f'_c}}{f_y} \times b_w \times d = \frac{3 \times \sqrt{5,000}}{60,000} \times 24 \times 18 = 1.53 \text{ in.}^2 \quad \text{ACI 318-14 (9.6.1.2(a))}$$

And not less than

$$A_{s,\min} = \frac{200}{f_y} \times b_w \times d = \frac{200}{60,000} \times 24 \times 18 = 1.44 \text{ in.}^2 \quad \text{ACI 318-14 (9.6.1.2(b))}$$

All the values on the following table are calculated based on the procedure outlined above.

Table 14 - Reinforcing Design Summary (Flexure only) for Exterior Beam					
	End Span			Interior Span	
	Exterior Negative	Positive	Interior Negative	Positive	Negative
Design Moment, M_u (ft-kips)	203.95	233.08	326.31	203.65	296.65
Effective depth, d (in.)	18.00*	18.625**	18.00*	18.625**	18.00*
$A_{s, \text{req'd}}$ (in.²)	2.631	2.915	4.336	2.535	3.912
$A_{s, \min}$ (in.²)	1.53	1.58	1.53	1.58	1.53
* The beam top bars are to be placed below the joist top bars.					
** The beam bottom bars are to be placed at the bottom-most layer. The joist bottom bars, then, shall be spliced at joist-beam intersection.					

Torsion requirements for longitudinal steel have to be determined and combined with reinforcement area required for flexure.

Calculate the required reinforcement to resist torsion:

Check if torsional effects can be neglected:

If $T_u < \phi T_{th}$, it shall be permitted to neglect torsional effects.

ACI 318-14 (22.7.1.1)

Where:

$$T_u = 403.59 \text{ ft-kips}$$

$$\phi T_{th} = 12.47 \text{ ft-kips} = \text{Threshold torsion (the calculation of } \phi T_{th} \text{ is shown in the next section)}$$

Since $T_u > \phi T_{th}$, the torsional effects must be considered.

Check if the factored design torsion can be reduced:

It is permitted to reduce T_u to ϕT_{cr} ; due to redistribution of internal forces after torsional cracking; if the exterior continuous beam meet the following requirements: **ACI 318-14 (22.7.3.2)**

1. The beam is statically indeterminate (continuous beam).
2. $T_u \geq \phi T_{cr}$.

To check the second condition, ϕT_{cr} need to be calculated as follows:

Since the beams are cast monolithically with slab and joists, A_{cp} (area enclosed by outside perimeter of concrete cross section) and p_{cp} (outside perimeter of concrete cross section) for the beam can include a portion of the adjoining slab. The effective width, b_e , of the overhanging flange must conform to **ACI 318-14 (8.4.1.8)**:

$$b_e = h - h_f = 21 - 5 = 16 \text{ in. (governs)}$$

$$b_e = 4 \times h_f = 4 \times 5 = 20 \text{ in.}$$

$$A_{cp} = (21 \times 24) + (16 \times 5) = 584 \text{ in.}^2$$

$$P_{cp} = 2 \times (21 + 24 + 16) = 122 \text{ in.}$$

$$\frac{A_{cp}^2}{P_{cp}} = \frac{(584)^2}{122} = 2795.54 \text{ in.}^3$$

The torsional properties of the beam ignoring the overhanging flange are the following:

$$A_{cp} = (21 \times 24) = 504 \text{ in.}^2$$

$$P_{cp} = 2 \times (21 + 24) = 90 \text{ in.}$$

$$\frac{A_{cp}^2}{P_{cp}} = \frac{(504)^2}{90} = 2822.40 \text{ in.}^3 > 2795.54 \text{ in.}^3$$

Therefore, consider the rectangular section.

ACI 318-14 (9.2.4.4(b))

$$\phi T_{cr} = \phi \times 4 \times \lambda \times \sqrt{f'_c} \times \left(\frac{A_{cp}^2}{P_{cp}} \right)$$

ACI 318-14 (Table 22.7.5.1(a))

$$\phi T_{cr} = 0.75 \times 4.00 \times 1.00 \times \sqrt{5,000} \times 2822.40 = 598,721.45 \text{ in-lb} = 49.89 \text{ ft-kips}$$

$$\phi T_{th} = \phi \times \lambda \times \sqrt{f'_c} \times \left(\frac{A_{cp}^2}{P_{cp}} \right) = \frac{\phi T_{cr}}{4} = \frac{49.89}{4} = 12.47 \text{ ft-kips}$$

Checking the second condition of ACI 318-14 (22.7.3.2):

$$T_u = 406.59 \text{ ft-kips} > \phi T_{cr} = 49.89 \text{ ft-kips}$$

Thus, ACI 318-14 permits to reduce T_u to ϕT_{cr} .

$$T_u = \phi T_{cr} = 49.89 \text{ ft-kips}$$

ACI 318-14 (22.7.3.2)

It is assumed that the torsional loading on the beam is uniformly distributed along the span.

Determine the adequacy of cross-sectional dimensions for the torsion:

For solid sections, the limit on shear and torsion is given by:

$$\sqrt{\left(\frac{V_u}{b_w \times d} \right)^2 + \left(\frac{T_u \times p_h}{1.70 \times A_{oh}^2} \right)^2} \leq \phi \times \left(\frac{V_c}{b_w \times d} + 8 \times \sqrt{f'_c} \right)$$

ACI 318-14 (22.7.7.1)

Where:

A_{oh} = area enclosed by centerline of outermost closed transverse torsional reinforcement.

p_h = perimeter of centerline of outermost closed transverse torsional reinforcement.

Using $d = 18.00$ in., the factored shear force at the critical section located at a distance d from the face of the support is:

$$V_u = 66.22 - 4.06 \times \frac{18.00}{12} = 60.13 \text{ kips}$$

Also, the nominal shear strength provided by the concrete is:

$$V_c = 2 \times \lambda \times \sqrt{f'_c} \times b_w \times d \quad \text{ACI 318-14 (Eq. 22.5.5.1)}$$

Using a 1.50-in. clear cover to #3 closed stirrups at bottom and 2.125 in clear cover to #3 closed stirrups at top.

$$A_{oh} = [21 - ((2.125 + 1.50) + 0.375)] \times [24 - ((1.50 + 1.50) + 0.375)] = 350.63 \text{ in}^2$$

$$p_h = 2 \times \left\{ [21 - ((2.125 + 1.50) + 0.375)] + [24 - ((1.50 + 1.50) + 0.375)] \right\} = 75.25 \text{ in.}$$

$$\sqrt{\left(\frac{60,125.43}{24 \times 18}\right)^2 + \left(\frac{49.89 \times 12,000 \times 75.25}{1.70 \times 350.63^2}\right)^2} = 256.60 \text{ psi}$$

$$< 0.75 \times \left[\frac{2 \times \sqrt{5,000} \times 24 \times 18}{24 \times 18} + 8 \times \sqrt{5,000} \right] = 530.33 \text{ kips}$$

Therefore, the section is adequate.

Determine the transverse reinforcement required for torsion:

$$\frac{A_t}{s} = \frac{T_u}{\phi \times 2 \times A_o \times f_{yt} \times \cot(\theta)} \quad \text{ACI 318-14 (Eq. 22.7.6.1a)}$$

Where:

$$A_o = 0.85 \times A_{oh} = 0.85 \times 350.63 = 298.03 \text{ in.}^2 \quad \text{ACI 318-14 (22.7.6.1.1)}$$

$$\theta = 45^\circ \quad \text{ACI 318-14 (22.7.6.1.2(a))}$$

Therefore,

$$\frac{A_t}{s} = \frac{49.89 \times 12,000}{0.75 \times 2 \times 298.03 \times 60,000 \times \cot(45^\circ)} = 0.0223 \text{ in}^2 / \text{in per leg}$$

Determine the transverse reinforcement required for shear:

From Table 12 above, the maximum shear value occurs at the face of the first interior support in the end span.

$$1.15 \times \frac{w_u \times l_n}{2} = 1.15 \times \frac{4.06 \times 28.33}{2} = 66.22 \text{ kips}$$

The design shear at a distance, d , away from the face of support,

$$V_u = 66.22 - 4.06 \times \frac{18}{12} = 60.13 \text{ kips}$$

Shear strength provided by concrete

$$\phi V_c = \phi \times (2 \times \sqrt{f'_c} \times b_w \times d) \quad \text{ACI 318-14 (Eq. 22.5.5.1)}$$

$$\phi V_c = 0.75 \times (2.00 \times 1.00 \times \sqrt{5,000} \times 24 \times 18) = 45,820.52 \text{ lb} = 45.82 \text{ kips}$$

Since $V_u > \frac{\phi V_c}{2}$, shear reinforcement is required.

The nominal shear strength required to be provided by shear reinforcement is

$$V_s = V_u - V_c = \frac{V_u}{\phi} - V_c = \left(\frac{60.13}{0.75} \right) - 61.09 = 19.07 \text{ kips}$$

Check whether V_s is less than $8 \times \sqrt{f'_c} \times b_w \times d$

If V_s is greater than $8 \times \sqrt{f'_c} \times b_w \times d$, then the cross-section has to be revised as ACI 318-14 limits the shear capacity to be provided by stirrups to $8 \times \sqrt{f'_c} \times b_w \times d$ ACI 318-14 (22.5.1.2)

$$8 \times \sqrt{f'_c} \times b_w \times d = 8 \times \sqrt{5,000} \times 24 \times 18 = 244,376.10 \text{ lb} = 244.38 \text{ kips}$$

Since V_s does not exceed $8 \times \sqrt{f'_c} \times b_w \times d$, the cross section is adequate.

Calculate the required transverse reinforcement for shear as

$$\frac{A_v}{s} = \frac{V_u - \phi V_c}{\phi \times f_{yt} \times d} = \frac{60.13 - 45.82}{0.75 \times 60,000 \times 18} = 0.0177 \frac{\text{in.}^2}{\text{in}} \quad \text{ACI 318-14 (22.5.10.5.3)}$$

Calculate total required transverse reinforcement for combined shear and torsion:

$$\frac{A_v}{s} + 2 \times \frac{A_t}{s} = 0.0177 + 2 \times 0.0223 \frac{\text{in.}^2/\text{in.}}{\text{leg}} = 0.0623 \text{ in.}^2/\text{in.}$$

Minimum transverse reinforcement for shear and torsion is calculated as follows:

$$\frac{(A_v + 2 \times A_t)_{\min}}{s} = \text{greater of} \left[\begin{array}{l} 0.75 \times \sqrt{f'_c} \times \left(\frac{b_w}{f_{yt}} \right) \\ 50 \times \left(\frac{b_w}{f_{yt}} \right) \end{array} \right] \quad \text{ACI 318-14 (9.6.4.2)}$$

$$\frac{(A_v + 2 \times A_t)_{\min}}{s} = \text{greater of} \left[\begin{array}{l} 0.75 \times \sqrt{5,000} \times \left(\frac{24}{60,000} \right) \\ 50 \times \left(\frac{24}{60,000} \right) \end{array} \right] = \left[\begin{array}{l} 0.0212 \\ 0.0200 \end{array} \right] = 0.0212 \text{ in.}^2/\text{in} < 0.0623 \text{ in.}^2/\text{in}$$

Then, provide $\frac{A_v}{s} + 2 \times \frac{A_t}{s} = 0.0623 \text{ in.}^2/\text{in.}$

Calculate the required spacing:

Maximum spacing of transverse torsion reinforcement:

$$s_{\max} = \text{lesser of} \left[\begin{array}{l} p_h / 8 \\ 12 \text{ in.} \end{array} \right] \quad \text{ACI 318-14 (9.7.6.3.3)}$$

$$s_{\max} = \text{lesser of} \left[\begin{array}{l} 75.25 / 8 \\ 12 \text{ in.} \end{array} \right] = \text{lesser of} \left[\begin{array}{l} 9.41 \text{ in.} \\ 12 \text{ in.} \end{array} \right] = 9.41 \text{ in.}$$

Maximum spacing of transverse shear reinforcement:

Check whether the required spacing based on the shear demand meets the spacing limits for shear reinforcement per ACI 318-14 (9.7.6.2.2).

Check whether V_s is less than $4 \times \sqrt{f'_c} \times b_w \times d$

$$4 \times \sqrt{f'_c} \times b_w \times d = 4 \times \sqrt{5,000} \times 24 \times 18 = 122,188.05 \text{ lb} = 122.19 \text{ kips} > V_s = 19.07 \text{ kips}$$

Therefore, maximum stirrup spacing shall be the smallest of $d/2$ and 24 in.

$$s_{\max} = \text{lesser of } \left[\begin{array}{l} d/2 \\ 24 \text{ in.} \end{array} \right] \quad \text{ACI 318-14 (Table 9.7.6.2.2)}$$

$$s_{\max} = \text{lesser of } \left[\begin{array}{l} 18/2 \\ 24 \text{ in.} \end{array} \right] = \text{lesser of } \left[\begin{array}{l} 9 \text{ in.} \\ 24 \text{ in.} \end{array} \right] = 9 \text{ in. (governs)}$$

Using a bundle of 2-#3 closed stirrups with 2 legs (area per leg = 0.22 in.²), the required spacing, s , at the critical section is:

$$s = 0.44 / 0.0623 = 7.06 \text{ in.} < 9.00 \text{ in.}$$

Provide a bundle of 2-#3 closed stirrups with 2 legs spaced at 7 in. on center. #3 bars are selected for consistency with the transverse reinforcement size used for the joist and interior beam. The stirrups are bundled at outer legs to maintain A_o value in calculation of A_v/s for torsion. The bundle of 2-#3 bars are defined as user-defined reinforcement of size #2 in [spBeam](#) Program.

The designer may choose to utilize #4 closed stirrups with 2 legs at 6 in. on center alternatively.

In view of the shear and torsion distribution along the span length, this same reinforcement and spacing can be provided throughout the span length.

Calculate the additional required longitudinal reinforcement for torsion:

$$A_t = \frac{T_u \times p_h}{\phi \times 2 \times A_o \times f_y \times \cot(\theta)} \quad \text{ACI 318-14 (Eq. 22.7.6.1b)}$$

Where:

$$A_o = 0.85 \times A_{oh} = 0.85 \times 350.63 = 298.03 \text{ in.}^2 \quad \text{ACI 318-14 (22.7.6.1.1)}$$

$$\theta = 45^\circ \quad \text{ACI 318-14 (22.7.6.1.2(a))}$$

Therefore,

$$A_t = \frac{49.89 \times 12,000 \times 75.25}{0.75 \times 2.00 \times 298.03 \times 60,000 \times \cot(45^\circ)} = 1.680 \text{ in.}^2$$

The minimum total area of longitudinal torsional reinforcement:

$$A_{l,\min} = \text{lesser of } \left[\begin{array}{l} \frac{5 \times \sqrt{f'_c} \times A_{cp}}{f_y} - \left(\frac{A_t}{s} \right) \times p_h \times \frac{f_{yt}}{f_y} \\ \frac{5 \times \sqrt{f'_c} \times A_{cp}}{f_y} - \left(\frac{25 \times b_w}{f_{yt}} \right) \times p_h \times \frac{f_{yt}}{f_y} \end{array} \right] \quad \text{ACI 318-14 (9.6.4.3)}$$

$$A_{l,min} = \text{lesser of } \left[\begin{array}{l} \frac{5 \times \sqrt{5,000} \times 504}{60,000} - \left(0.022 \times 75.25 \times \frac{60,000}{60,000} \right) \\ \frac{5 \times \sqrt{5,000} \times 504}{60,000} - \left(\frac{25 \times 24}{60,000} \times 75.25 \times \frac{60,000}{60,000} \right) \end{array} \right] = \left[\begin{array}{l} 1.29 \\ 2.22 \end{array} \right] = 1.29 \text{ in.}^2$$

Since $A_l > A_{l,min}$, use $A_l = 1.680 \text{ in.}^2$

The longitudinal reinforcement is to be distributed around the perimeter of the stirrups, with a maximum spacing of 12 in. There shall be at least one longitudinal bar in each corner of the stirrups.

ACI 318-14 (9.7.5.1)

Longitudinal bars shall have a diameter at least 0.042 times the stirrup spacing, but not less than 3/8 in.

ACI 318-14 (9.7.5.2)

To meet the maximum spacing requirement, a bar has to be provided between corner bars at all four sides. This configuration leads to eight-bars; three at top, three at bottom, and one at each side. Therefore, the reinforcement area per bar is $A_s = 1.680/8 = 0.21 \text{ in.}^2$

Then, use #5 bars for longitudinal bars which also meets minimum bar diameter requirement of 3/8 in. A_l shall be provided in addition to the required flexural reinforcement at the negative moment regions (support-top) and positive moment region (mid-span-bottom). At mid-span-top region where flexural reinforcement is not required for flexure, 3-#5 bars shall be provided. Class B lap splice is to be provided.

Table 15 - Reinforcing Design Summary (Flexure + Torsion)					
	End Span			Interior Span	
	Top Reinforcing for Exterior Negative Moment	Bottom Reinforcing for Positive Moment	Top Reinforcing for Interior Negative Moment	Bottom Reinforcing for Positive Moment	Top Reinforcing for Interior Negative Moment
Required Longitudinal Reinforcement (in. ²)	2.631	2.915	4.336	2.535	3.912
Required Torsional Longitudinal Reinforcement (in. ²)	$1.680 \times (3/8) = 0.63$	$1.680 \times (3/8) = 0.63$	$1.680 \times (3/8) = 0.63$	$1.680 \times (3/8) = 0.63$	$1.680 \times (3/8) = 0.63$
Required Total Longitudinal Reinforcement (in. ²)	3.261	3.545	4.966	3.165	4.542
Reinforcement	5-#8	5-#8	7-#8	4-#8	6-#8

Maximum spacing allowed:

Check the requirement for distribution of flexural reinforcement to control flexural cracking:

$$s = 15 \times \left(\frac{40,000}{f_s} \right) - 2.50 \times c_c \leq 12 \times \left(\frac{40,000}{f_s} \right) \quad \text{ACI 318-14 (Table 24.3.2)}$$

$$c_c = 21 - \left(18 + 0.50 \times \left(\frac{8}{8} \right) \right) = 2.50 \text{ in.}$$

$$\text{Use } f_s = \frac{2}{3} \times f_y = \frac{2}{3} \times 60,000 = 40,000 \text{ psi} \quad \text{ACI 318-14 (24.3.2.1)}$$

$$s = 15 \times \left(\frac{40,000}{40,000} \right) - 2.50 \times 2.50 = 8.75 \text{ in. (governs)}$$

$$s = 12 \times \left(\frac{40,000}{40,000} \right) = 12 \text{ in.}$$

Spacing provided for 4-#8 bars

$$s_{prov} = \frac{(b_w - 2 \times d_s)}{\# \text{ of bars} - 1} = \frac{(24 - 2 \times 2.625)}{3} = 6.25 \text{ in.} < 8.75 \text{ in.} \quad \text{o.k.}$$

Where $d_s = 2.625$ in. for #3 stirrup.

CRSI 2002 (Figure 12-9)

Check the spacing, s provided, is greater than the minimum center to center spacing, s_{min} where:

$$s_{min} = d_b + \max \left\{ \begin{array}{l} 1 \\ d_b \\ 1.33 \times \text{max .agg.} \end{array} \right\} \quad \text{CRSI 2002 (Figure 12-9)}$$

Where maximum aggregate size is $\frac{3}{4}$ "

$$s_{min} = 1.00 + \max \left\{ \begin{array}{l} 1.00 \\ 1.00 \\ 1.33 \times 0.75 = 1.00 \end{array} \right\} = 1.00 + 1.00 = 2 \text{ in.}$$

Spacing provided for 7-#8 bars

$$s_{prov} = \frac{(b_w - 2 \times d_s)}{\# \text{ of bars} - 1} = \frac{(24 - 2 \times 2.625)}{6} = 3.13 \text{ in.} > 2.00 \text{ in.} \quad \text{o.k.}$$

Therefore, the reinforcement selections in the previous table meet the spacing requirements.

2.4.4. Deflections

Since the preliminary beam depth met minimum depth requirement, the deflection calculations are not required. A lesser depth maybe possible and consequently cost savings can be achieved through deflection computations. Deflection values are calculated and provided for every model created by [spBeam](#) Program and can be used by the engineer to make additional optimization decisions.

2.4.5. Computer Program Solution

[spBeam](#) Program can be utilized to analyze and design the exterior continuous beam along grid A. The beam is modeled as a three span continuous rectangular beam.

The program calculates the internal forces (shear force and bending moment), moment and shear capacities, immediate and long-term deflection results, and required flexural reinforcement. The graphical and text results are provided here for both input and output of the [spBeam](#) model.

The beam is modeled as a 24 in. by 21 in. deep rectangular longitudinal beam with column supports at 100% stiffness share.

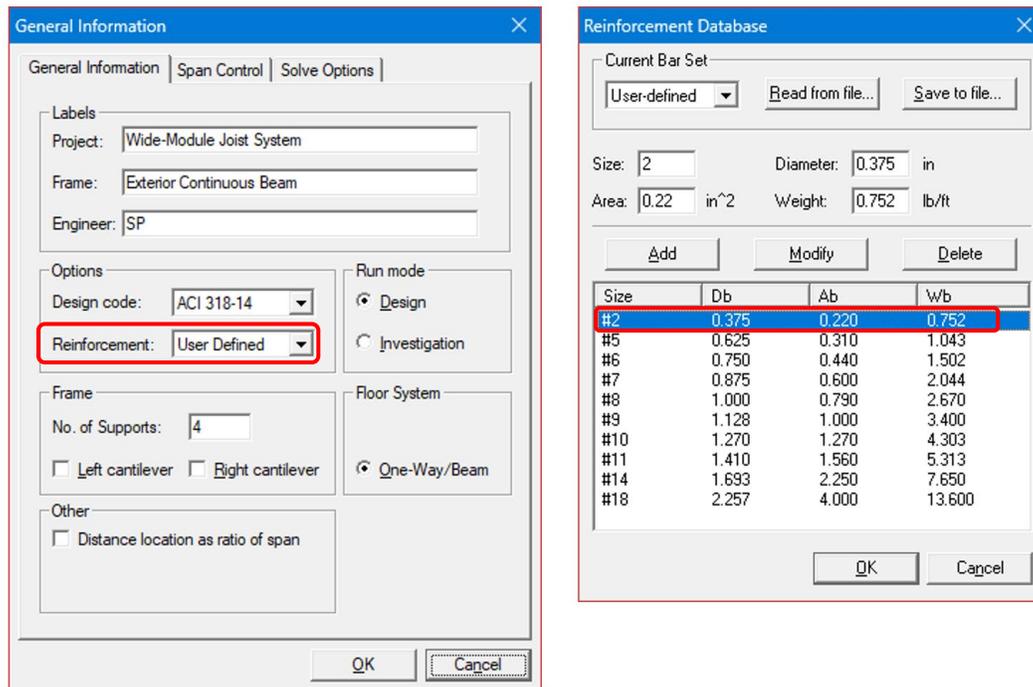


Figure 36 – [spBeam](#) Reinforcement (User-Defined Bar Set)

The reinforcement database is selected as User-defined in order to define a bundle of 2-#3 bars as #2 with cross-sectional area of 0.22 in². (Different than #2 defined earlier for welded wire).

Torsion analysis was engaged using the torsion analysis and design check box located on the solve options tab in the input dialog box. The design for torsion is based on a thin-walled tube, space truss analogy. [spBeam](#) allows both equilibrium and compatibility torsion conditions. In the equilibrium mode, which is assumed by default, unreduced total value of the torsional design moment is used in the design. In the compatibility mode, factored torsional moments that exceed cracking moment T_{cr} are reduced to the value of T_{cr} . However, it is user's responsibility to determine which mode is appropriate and the program does not perform any redistribution of internal forces if compatibility torsion is selected. In this model, the following solve options were used.

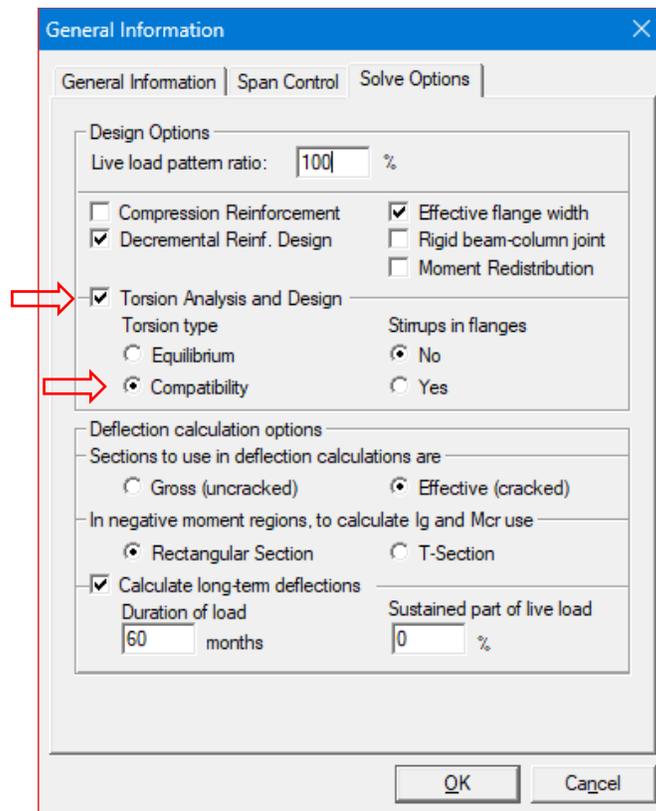


Figure 37 – [spBeam](#) Solve Options

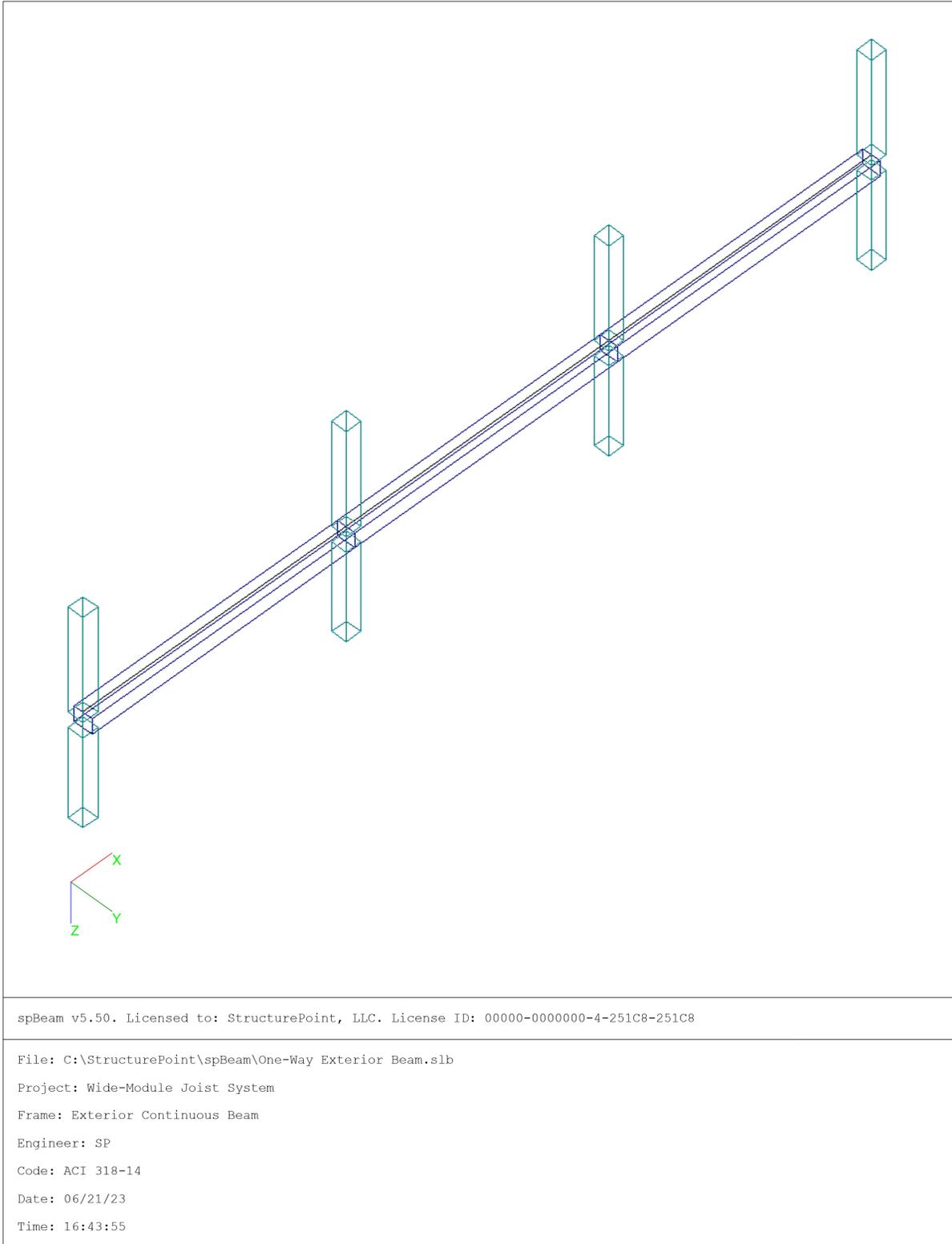


Figure 38 – spBeam Model – Isometric View – Exterior Continuous Beam along Grid A



Figure 39 – spBeam Model – Loads (Including Live Load Patterning)

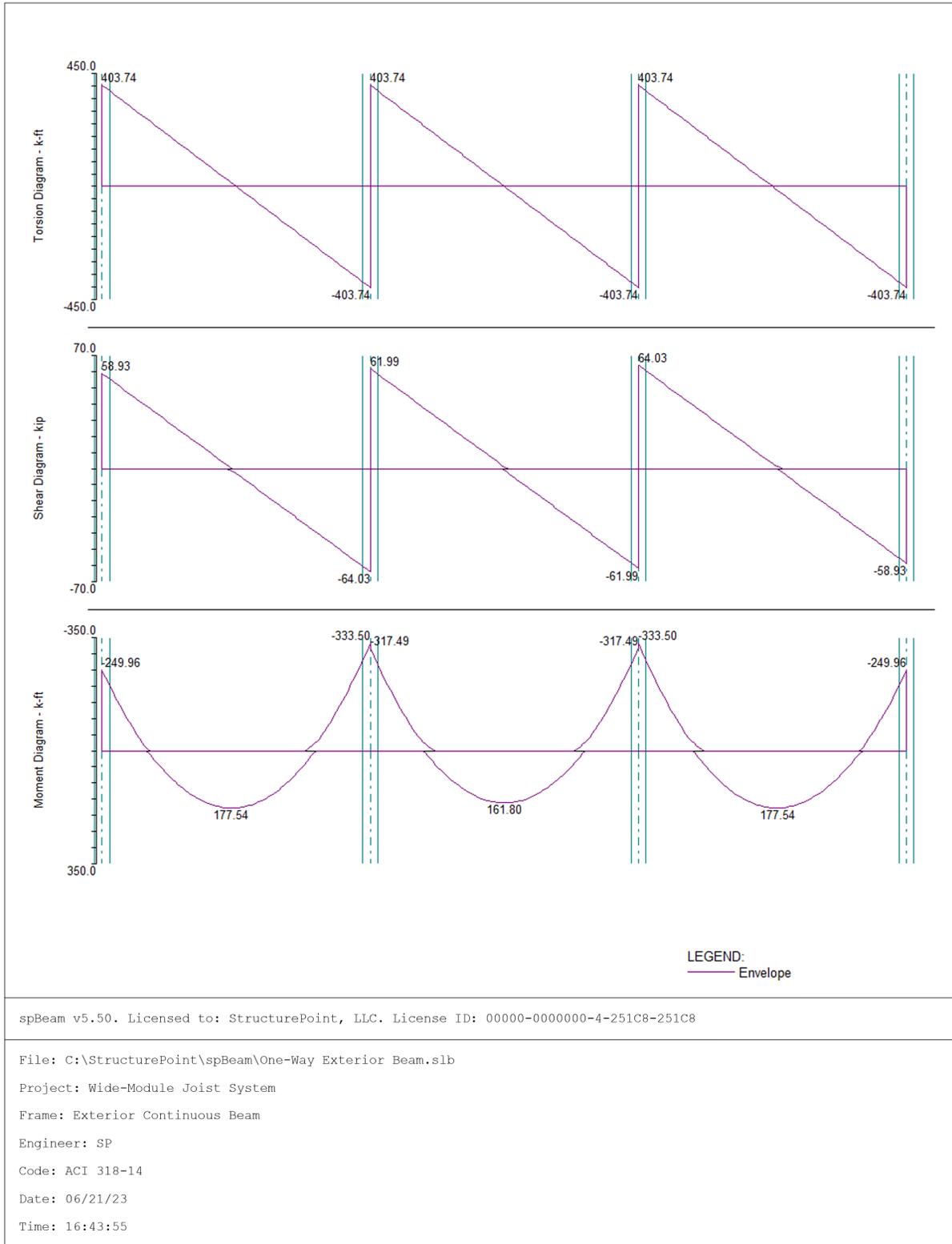


Figure 40 – spBeam Model – Internal Forces (Shear Force Diagram and Bending Moment Diagram)

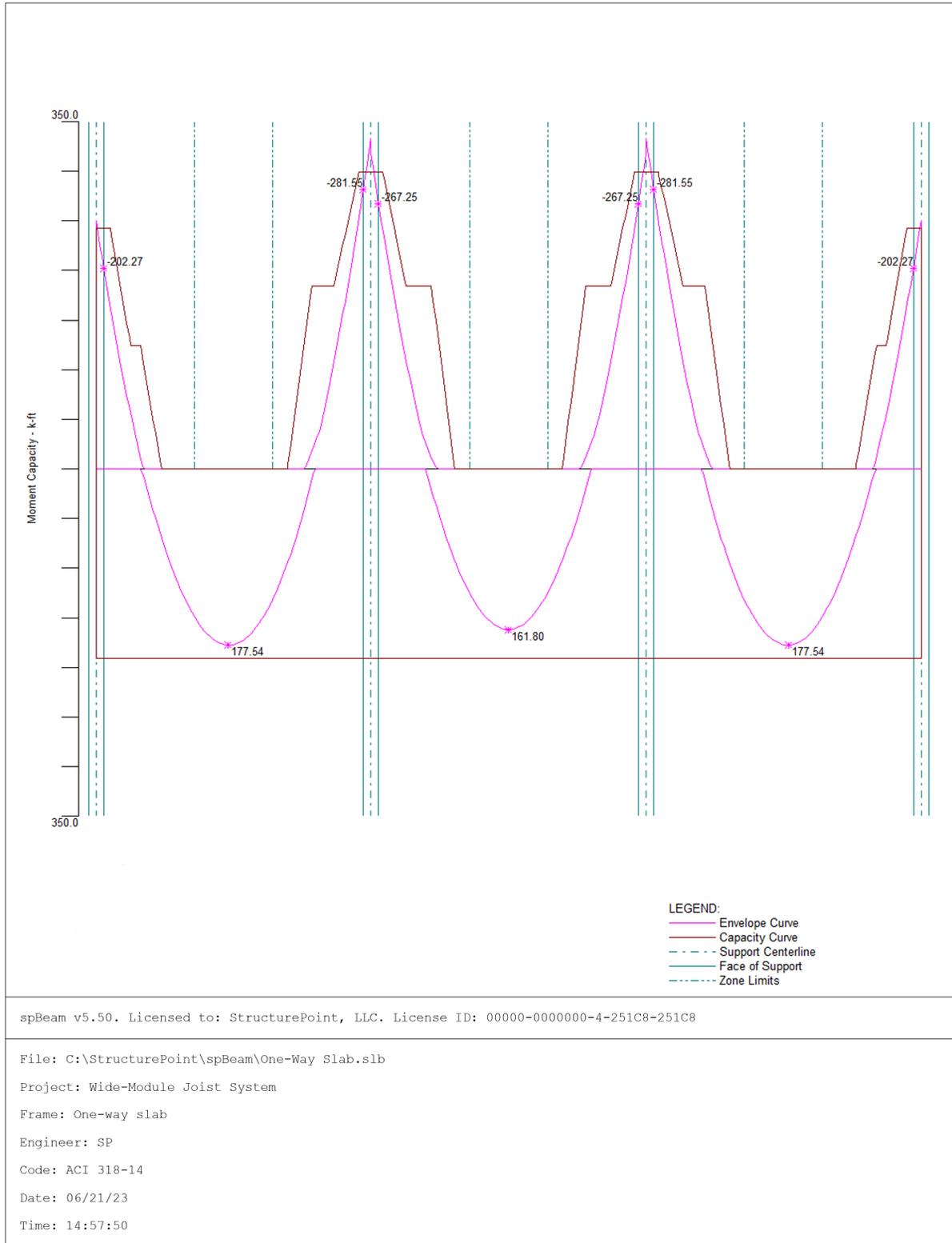


Figure 41 – spBeam Model – Moment Capacity Diagram

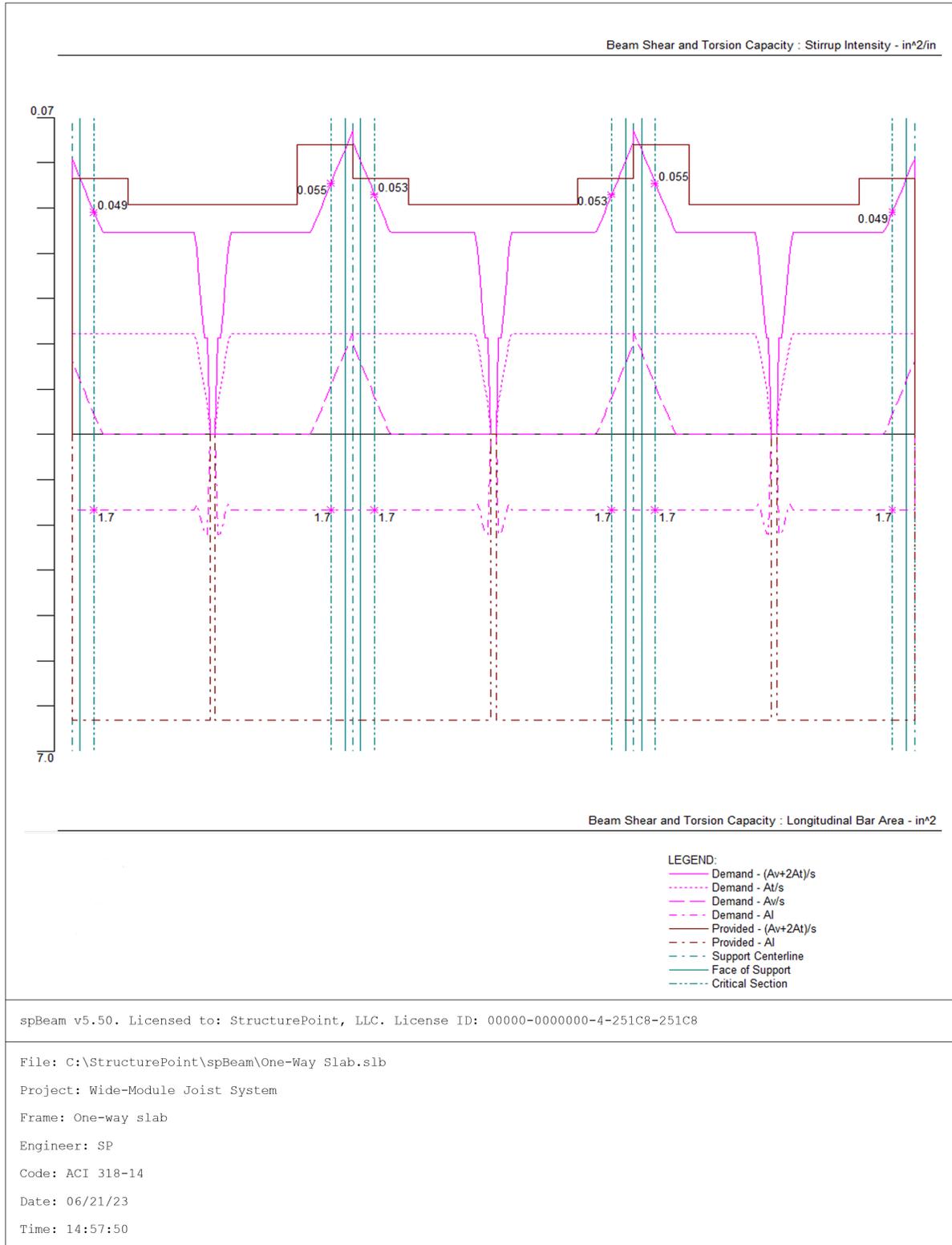


Figure 42 – spBeam Model – Shear Capacity Diagram

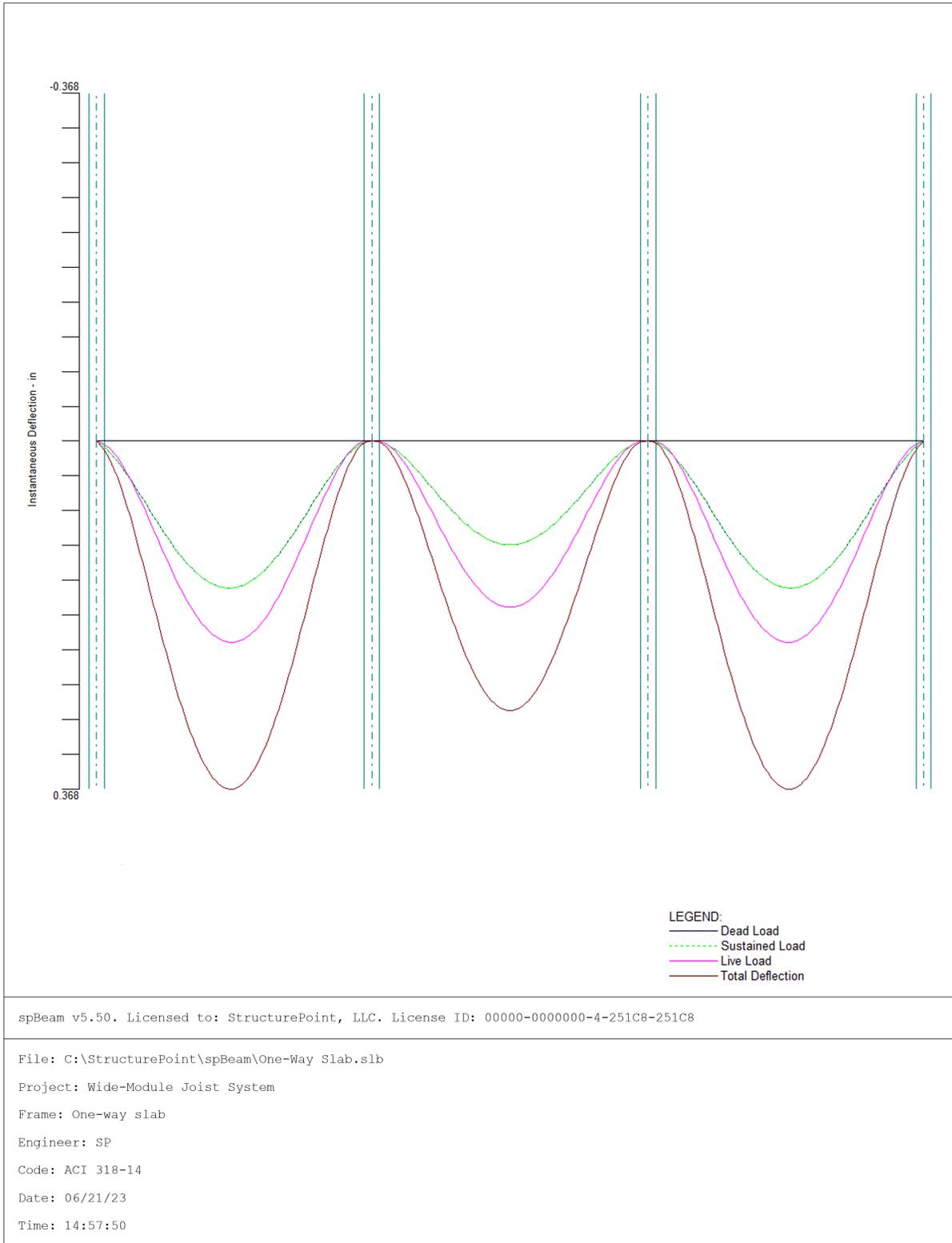


Figure 43 – spBeam Model – Immediate Deflection Diagram

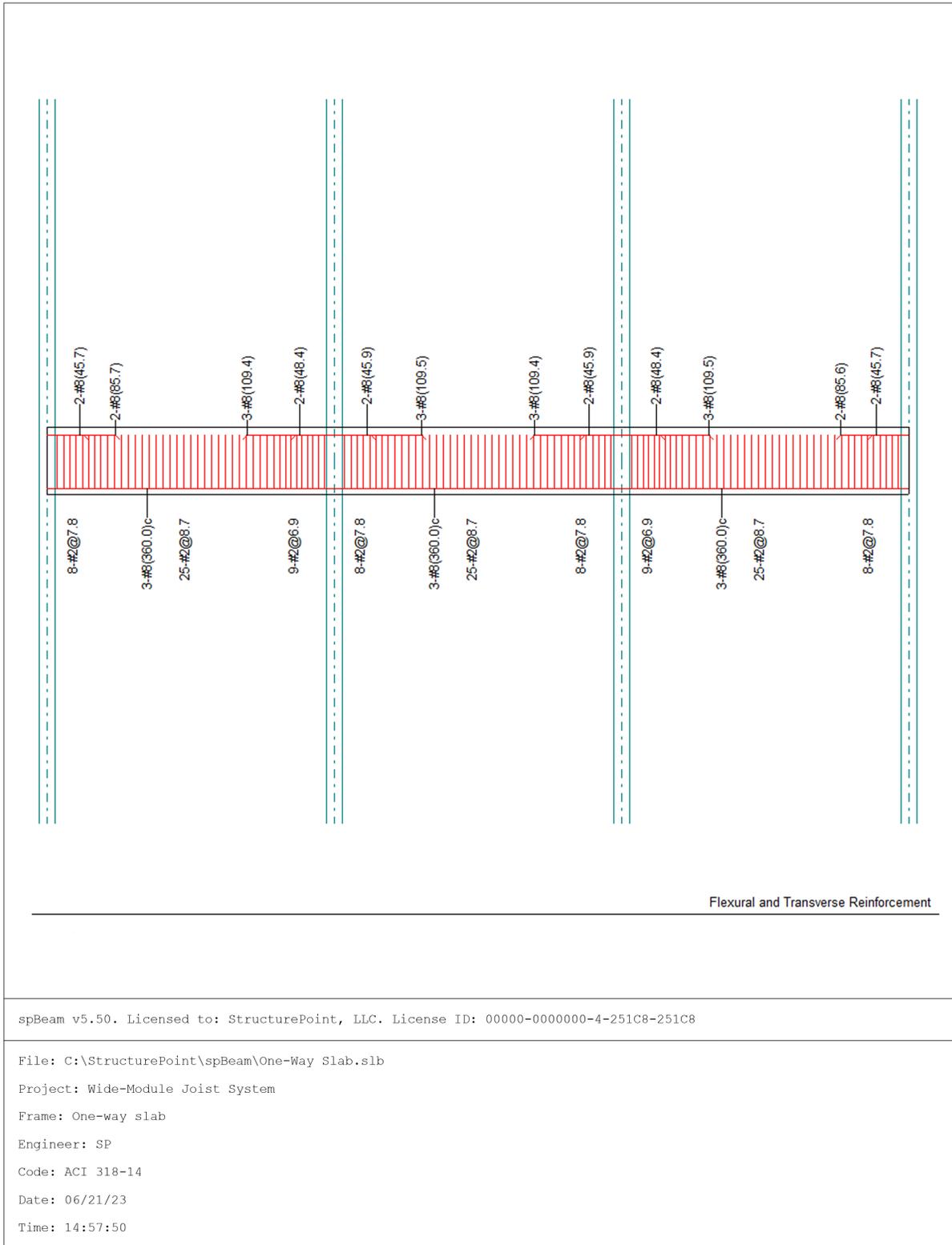


Figure 44 – spBeam Model – Reinforcement Diagram

2.4.6. Summary and Comparison of Results

1. Design Results

1.1. Top Reinforcement

Span Zone	Width ft	M _{max} k-ft	X _{max} ft	A _{s,min} in ²	A _{s,max} in ²	A _{s,req} in ²	Sp _{Prov} in	Bars
1 Left	2.00	202.27	0.833	1.527	9.180	2.608	6.368	4-#8
Midspan	2.00	0.00	15.000	0.000	9.180	0.000	0.000	---
Right	2.00	281.55	29.167	1.527	9.180	3.700	4.776	5-#8
2 Left	2.00	267.25	0.833	1.527	9.180	3.499	4.776	5-#8
Midspan	2.00	0.00	15.000	0.000	9.180	0.000	0.000	---
Right	2.00	267.25	29.167	1.527	9.180	3.499	4.776	5-#8
3 Left	2.00	281.55	0.833	1.527	9.180	3.700	4.776	5-#8
Midspan	2.00	0.00	15.000	0.000	9.180	0.000	0.000	---
Right	2.00	202.27	29.167	1.527	9.180	2.608	6.368	4-#8

1.2. Bottom Reinforcement

Span	Width ft	M _{max} k-ft	X _{max} ft	A _{s,min} in ²	A _{s,max} in ²	A _{s,req} in ²	Sp _{Prov} in	Bars
1	2.00	177.54	14.433	1.580	9.499	2.194	9.552	3-#8
2	2.00	161.80	15.000	1.580	9.499	1.993	9.552	3-#8
3	2.00	177.54	15.567	1.580	9.499	2.194	9.552	3-#8

1.3. Longitudinal Beam Shear and Torsion Reinforcement Required

1.3.1. Transverse Reinforcement Demand

Notes:

*4 - Design torsional moment reduced to PhiTcr due to compatibility torsion.

Span	Start ft	End ft	Required								Demand A _{(V+2T)/s} in ² /in
			X _u ft	V _u kip	T _u k-ft	v _r ksi	Comb/Patt	A _v /s in ² /in	A _t /s in ² /in	A _{(V+2T)/s} in ² /in	
1	1.083	5.952	2.33	49.45	49.89	0.244	U2/Odd	0.0045	0.0223	0.0491	0.0491 *4
	5.952	9.571	5.95	24.31	49.89	0.223	U1/All	0.0000	0.0223	0.0446	0.0446 *4
	9.571	13.190	9.57	13.72	49.89	0.218	U1/All	0.0000	0.0223	0.0446	0.0446 *4
	13.190	16.810	16.81	10.37	48.71	0.212	U2/All	0.0000	0.0218	0.0436	0.0436
	16.810	20.429	20.43	18.07	49.89	0.220	U1/All	0.0000	0.0223	0.0446	0.0446 *4
	20.429	24.048	24.05	28.66	49.89	0.226	U1/All	0.0000	0.0223	0.0446	0.0446 *4
	24.048	28.917	27.67	54.56	49.89	0.250	U2/S2	0.0108	0.0223	0.0554	0.0554 *4
2	1.083	5.952	2.33	52.51	49.89	0.247	U2/S2	0.0083	0.0223	0.0529	0.0529 *4
	5.952	9.571	5.95	26.49	49.89	0.224	U1/All	0.0000	0.0223	0.0446	0.0446 *4
	9.571	13.190	9.57	15.89	49.89	0.219	U1/All	0.0000	0.0223	0.0446	0.0446 *4
	13.190	16.810	16.81	7.35	48.71	0.211	U2/All	0.0000	0.0218	0.0436	0.0436
	16.810	20.429	20.43	15.89	49.89	0.219	U1/All	0.0000	0.0223	0.0446	0.0446 *4
	20.429	24.048	24.05	26.49	49.89	0.224	U1/All	0.0000	0.0223	0.0446	0.0446 *4
	24.048	28.917	27.67	52.51	49.89	0.247	U2/S3	0.0083	0.0223	0.0529	0.0529 *4
3	1.083	5.952	2.33	54.56	49.89	0.250	U2/S3	0.0108	0.0223	0.0554	0.0554 *4
	5.952	9.571	5.95	28.66	49.89	0.226	U1/All	0.0000	0.0223	0.0446	0.0446 *4
	9.571	13.190	9.57	18.07	49.89	0.220	U1/All	0.0000	0.0223	0.0446	0.0446 *4
	13.190	16.810	16.81	4.33	48.71	0.211	U2/All	0.0000	0.0218	0.0436	0.0436
	16.810	20.429	20.43	13.72	49.89	0.218	U1/All	0.0000	0.0223	0.0446	0.0446 *4
	20.429	24.048	24.05	24.31	49.89	0.223	U1/All	0.0000	0.0223	0.0446	0.0446 *4
	24.048	28.917	27.67	49.45	49.89	0.244	U2/Odd	0.0045	0.0223	0.0491	0.0491 *4

1.3.2. Required Longitudinal Reinforcement

Notes:

*4 - Design torsional moment reduced to ΦT_{cr} due to compatibility torsion.

*5 - Minimum longitudinal reinforcement required.

Span	Start ft	End ft	X_u ft	T_u k-ft	Comb/Patt	A_l in ²
1	1.083	5.952	2.33	49.89	U1/All	1.680 *4
	5.952	9.571	5.95	49.89	U1/All	1.680 *4
	9.571	13.190	9.57	49.89	U1/All	1.680 *4
	13.190	16.810	14.43	15.25	U2/All	2.217 *5
	16.810	20.429	16.98	49.89	U2/All	1.680 *4
	20.429	24.048	20.43	49.89	U1/All	1.680 *4
	24.048	28.917	24.05	49.89	U1/All	1.680 *4
2	1.083	5.952	2.33	49.89	U1/All	1.680 *4
	5.952	9.571	5.95	49.89	U1/All	1.680 *4
	9.571	13.190	9.57	49.89	U1/All	1.680 *4
	13.190	16.810	14.43	15.25	U2/All	2.217 *5
	16.810	20.429	16.98	49.89	U2/All	1.680 *4
	20.429	24.048	20.43	49.89	U1/All	1.680 *4
	24.048	28.917	24.05	49.89	U1/All	1.680 *4
3	1.083	5.952	2.33	49.89	U1/All	1.680 *4
	5.952	9.571	5.95	49.89	U1/All	1.680 *4
	9.571	13.190	9.57	49.89	U1/All	1.680 *4
	13.190	16.810	14.43	15.25	U2/All	2.217 *5
	16.810	20.429	16.98	49.89	U2/All	1.680 *4
	20.429	24.048	20.43	49.89	U1/All	1.680 *4
	24.048	28.917	24.05	49.89	U1/All	1.680 *4

Span	M_u (ft-kips)		T_u (ft-kips)		A_v/s (in. ² /in. per leg)	
End Span	Hand	spBeam	Hand	spBeam	Hand	spBeam
Interior Negative	326.31	281.55	49.89	49.89	0.0223	0.0223
Span	A_v/s (in. ² /in.)		$(A_v+2A_t)/s$ (in. ² /in.)		A_l (in. ²)	
End Span	Hand	spBeam	Hand	spBeam	Hand	spBeam
Interior Negative	0.0177	0.0108	0.0623	0.0554	1.680	1.680

Span Location	Required Reinforcement Area for Flexure + Torsion (in. ²)		Reinforcement Provided for Flexure + Torsion	
End Span	Hand	spBeam	Hand	spBeam
Interior Negative	4.966	$3.700 + 1.68 \times (3/8) = 4.330$	Top Bar: 7-#8	Top Bar: 5-#8 + 3-#8 = 8-#8
Positive	3.545	$2.194 + 1.68 \times (3/8) = 2.824$	Bottom Bar: 5-#8	Bottom Bar: 3-#8 + 3-#8 = 6-#8

2.4.7. Conclusions and Observations

In this design example, the exterior beam is modeled as a continuous rectangular longitudinal beam. There is a good agreement between the hand solution and computer solution. Note that the coefficients traditionally used to determine moments do not address various types of support and geometry.

The maximum calculated total immediate (instantaneous) deflection $(DL + LL) = 0.368$ in., this value can be compared with maximum permissible calculated deflection limitation per project criteria in accordance to ACI 318-14. ACI 318-14 (Table 24.2.2)

In addition to deflection results, parametric studies can be performed in [spBeam](#) to optimize design and detailing results.

The following observations can be made regarding the reinforcement diagram ([Figure 44](#)):

- The longitudinal reinforcement for the flexural design only and does not include (A_t).
- A_t is shown in the text output report (Design Results) because it has to be detailed and distributed to meet the torsional spacing requirements:
 - At least one longitudinal bar must be present at each corner of the stirrups.
 - And a bar has to be provided between corner bars at all four sides to meet the maximum spacing requirement (max 12 in.).
 - This configuration leads to eight-bars in this example: three at top, three at bottom, and one at each side.
 - Then, for the top bar for the interior negative section for the end span we need:
 - 1) The longitudinal reinforcement for the flexural design 5-#8 is required for flexure.
 - 2) The longitudinal reinforcement for the torsional design 3-#8 (3 top bars of the 8 bars distributed around the perimeter of the stirrups). See [Table 17](#).
 - Also, for the bottom bar for the positive section for the end span we need:
 - 3) The longitudinal reinforcement for the flexural design 3-#8 is required for flexure.
 - 4) The longitudinal reinforcement for the torsional design 3-#8 (3 bottom bars of the 8 bars distributed around the perimeter of the stirrups). See [Table 17](#).
- Transverse reinforcement shown reflects the total size and quantity of stirrups to resist the combined effects of shear and torsion.
- Top bars minimum length required (including the development length) for flexural design is shown. The bars can be extended and detailed to provide the required support for shear stirrups.

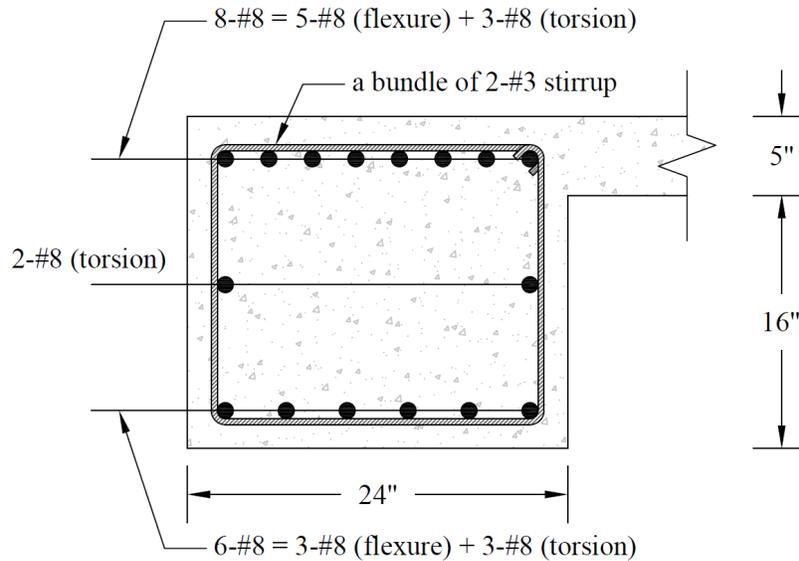


Figure 45 – Exterior Beam Cross-Section

2.5. Design of Interior, Edge, and Corner Columns

This section includes the design of interior, edge, and corner columns using [spColumn](#) software. The preliminary dimensions for these columns were calculated previously in [Section 1.2](#).

2.5.1. Determination of factored loads

Interior Column:

Total Factored Load on 1st story interior column (@ 1st interior support) are reorganized based on the calculations on [Section 1.2](#) as follows:

$$P_D = \text{Total service dead load} = 1.15 \times \left[(4 \times (82.88 + 20) + 1 \times 82.88) \times 960 \right] / 1000 + 36.00 = 581.79 \text{ kips}$$

$$P_L = \text{Total service live load} = 1.15 \times \left[(4 \times (39.36) + 1 \times 12) \times 960 \right] / 1000 = 187.08 \text{ kips}$$

$$P_u = 1.20 \times P_D + 1.60 \times P_L = 1.20 \times 581.79 + 1.60 \times 187.08 = 997.48 \text{ kips}$$

$M_{u,x}$ = unbalance flexural moment at the Interior support for the interior beam (see [Figure 29](#))

$$= 595.20 - 545.96 = 49.24 \text{ ft-kips}$$

$M_{u,y}$ = unbalance flexural moment at the interior support for the joist (see [Figure 18](#))

$$= 154.53 - 141.78 = 12.75 \text{ ft-kips}$$

The factored loads are then input into [spColumn](#) to construct the axial load – moment interaction diagram as shown in the sample input below.

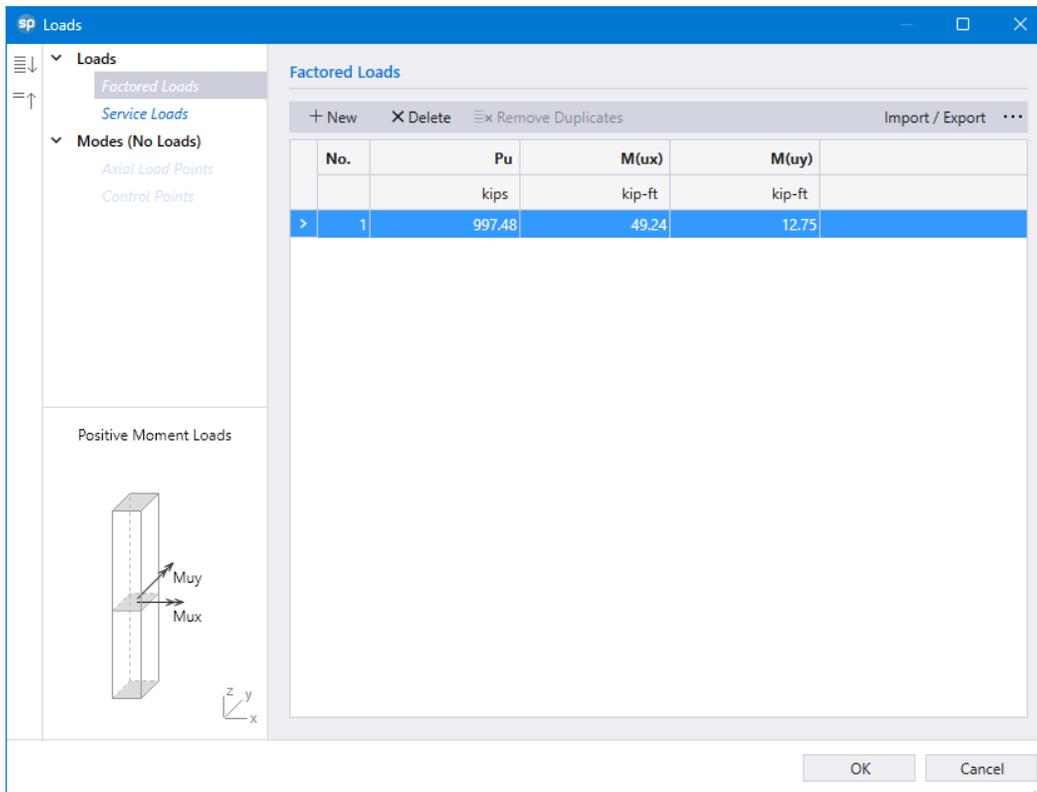


Figure 46 – [spColumn](#) Input Loads

Edge (Exterior) Column:

Total Factored Load on 1st story edge column (@ 1st interior support) are reorganized based on the calculations on [Section 1.2](#) as follows:

$$P_D = \text{Total service dead load} = 1.15 \times \left[(4 \times (82.88 + 20) + 1 \times 82.88) \times 480 \right] / 1000 + 25.00 = 297.90 \text{ kips}$$

$$P_L = \text{Total service live load} = 1.15 \times \left[(4 \times (47.39) + 1 \times 14.40) \times 480 \right] / 1000 = 112.58 \text{ kips}$$

$$P_u = 1.20 \times P_D + 1.60 \times P_L = 1.20 \times 297.70 + 1.60 \times 112.58 = 537.60 \text{ kips}$$

$$M_{u,x} = \text{unbalance flexural moment at the Interior support (see [Figure 40](#))}$$

$$= 333.50 - 317.49 = 16.01 \text{ ft-kips}$$

$$M_{u,y} = \text{torsional moment at the Interior support for the exterior beam (see [Section 2.4.3](#)) + unbalance flexural moment at exterior support for the joist (see [Figure 18](#))}$$

$$= 2 \times T_u + 73.96 = 2 \times 49.89 + 73.96 = 173.75 \text{ ft-kips}$$

Corner Column:

Total Factored Load on 1st story corner column (@ exterior support) are reorganized based on the calculations on [Section 1.2](#) as follows:

$$P_D = \text{Total service dead load} = 1.15 \times \left[(4 \times (82.88 + 20) + 1 \times 82.88) \times 240 \right] / 1000 + 25.00 = 161.45 \text{ kips}$$

$$P_L = \text{Total service live load} = 1.15 \times \left[(4 \times (58.76) + 1 \times 19.20) \times 240 \right] / 1000 = 70.14 \text{ kips}$$

$$P_u = 1.20 \times P_D + 1.60 \times P_L = 1.20 \times 161.45 + 1.60 \times 70.14 = 305.96 \text{ kips}$$

$$M_{u,x} = \text{unbalance flexural moment at the Exterior support (see [Figure 40](#))} = 249.96 \text{ ft-kips}$$

$$M_{u,y} = \text{torsional moment at the Exterior support for the exterior beam (see [Section 2.4.3](#)) + unbalance flexural moment at exterior support for the joist (see [Figure 18](#))}$$

$$= T_u + 73.96 = 49.89 + 73.96 = 123.85 \text{ ft-kips}$$

2.5.2. Column Capacity Diagram (Axial-Moment Interaction)

The axial force and moment interaction diagram is constructed using [spColumn](#) based on geometry and load input shown below:

Interior Column:

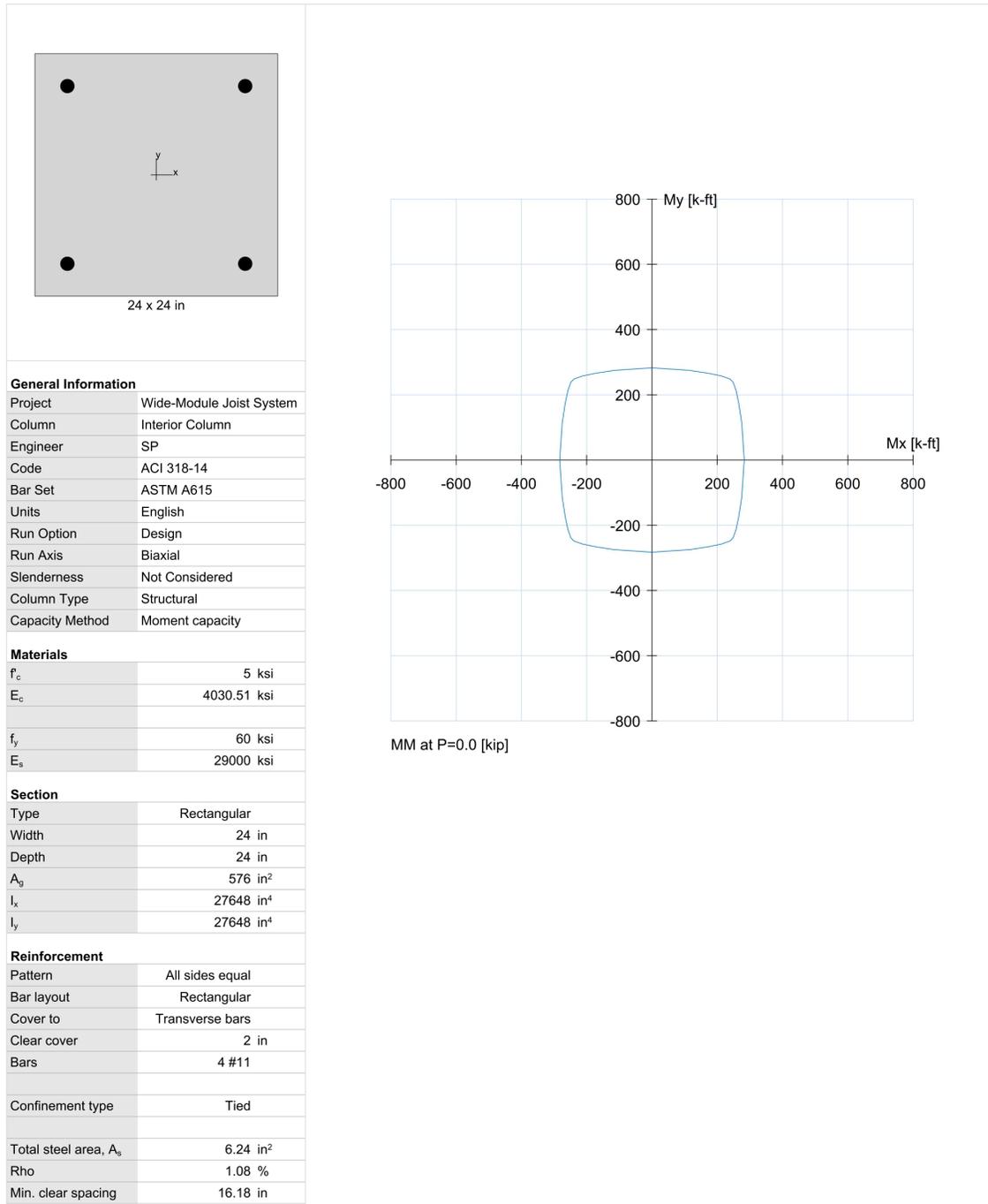


Figure 47 – [spColumn](#) Model – Biaxial Moment Interaction

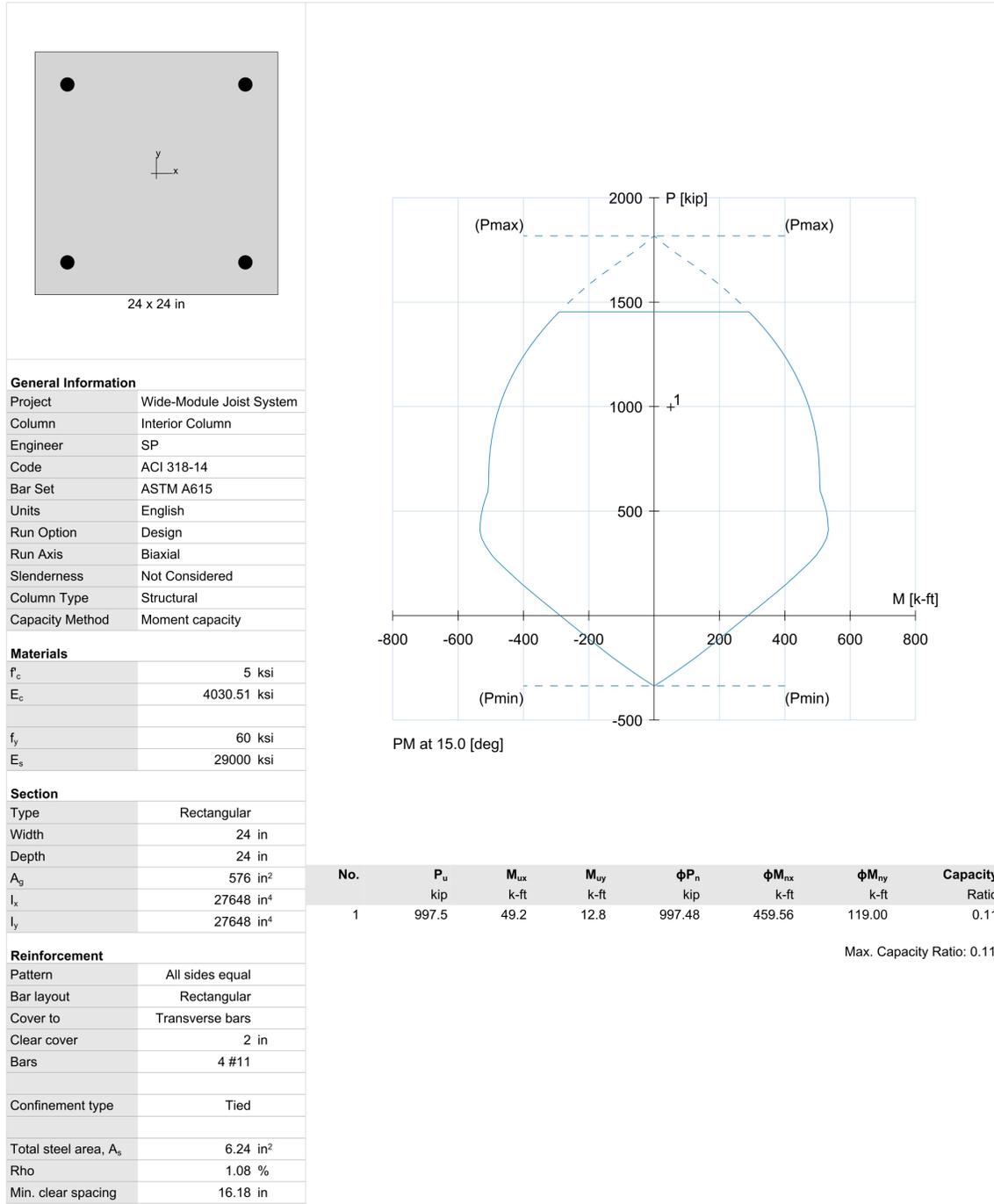


Figure 48 – spColumn Model – Axial Moment Interaction

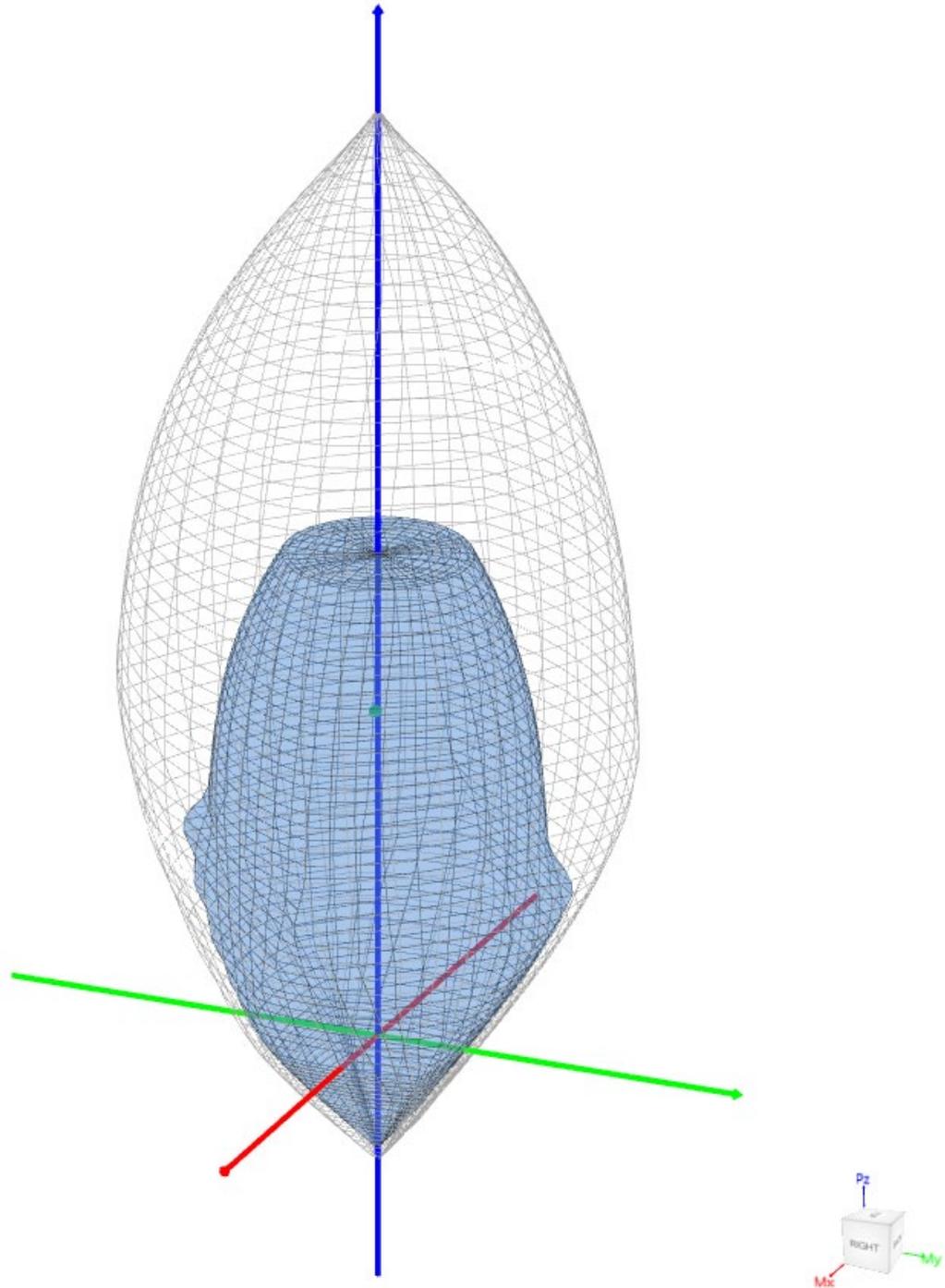


Figure 49 – [spColumn](#) Model – Nominal and Factored Failure Surfaces

Edge (Exterior) Column:

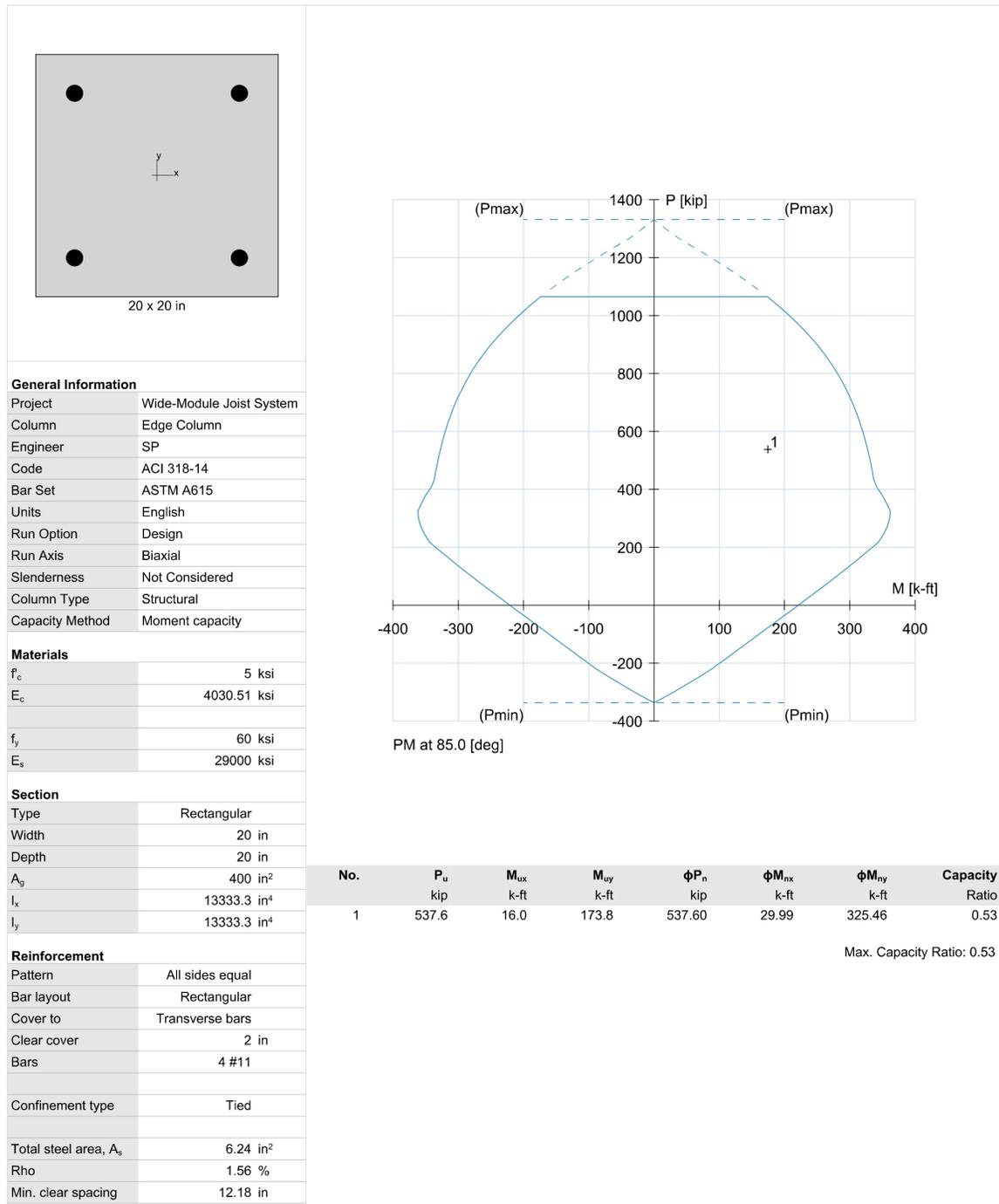


Figure 50 – spColumn Model – Axial Moment Interaction

Corner Column:

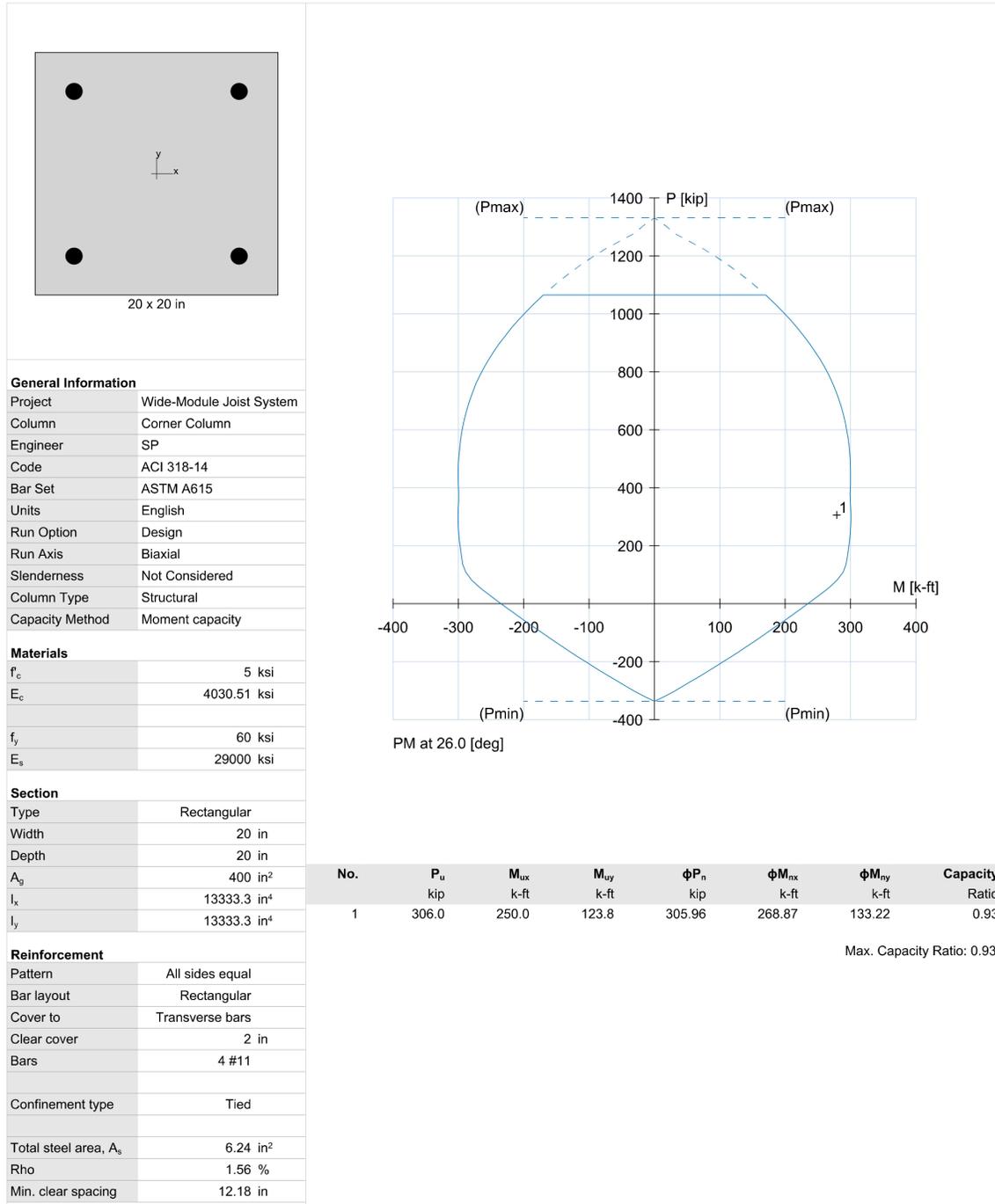


Figure 51 – spColumn Model –Axial Moment Interaction