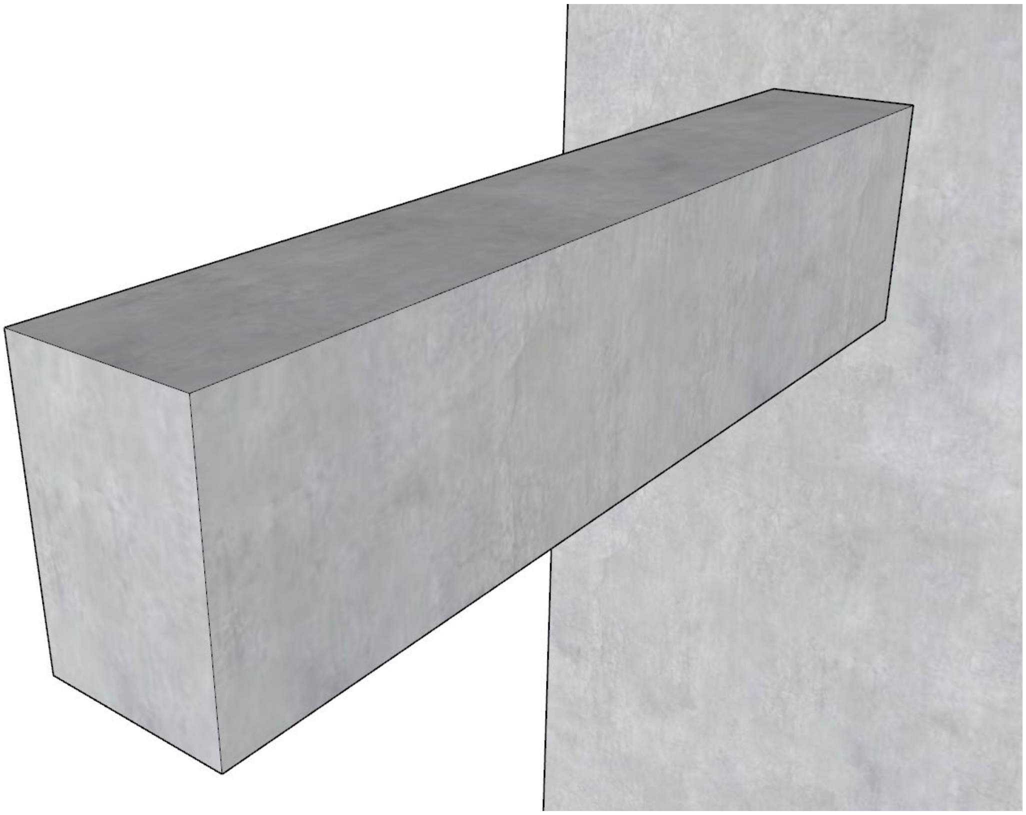
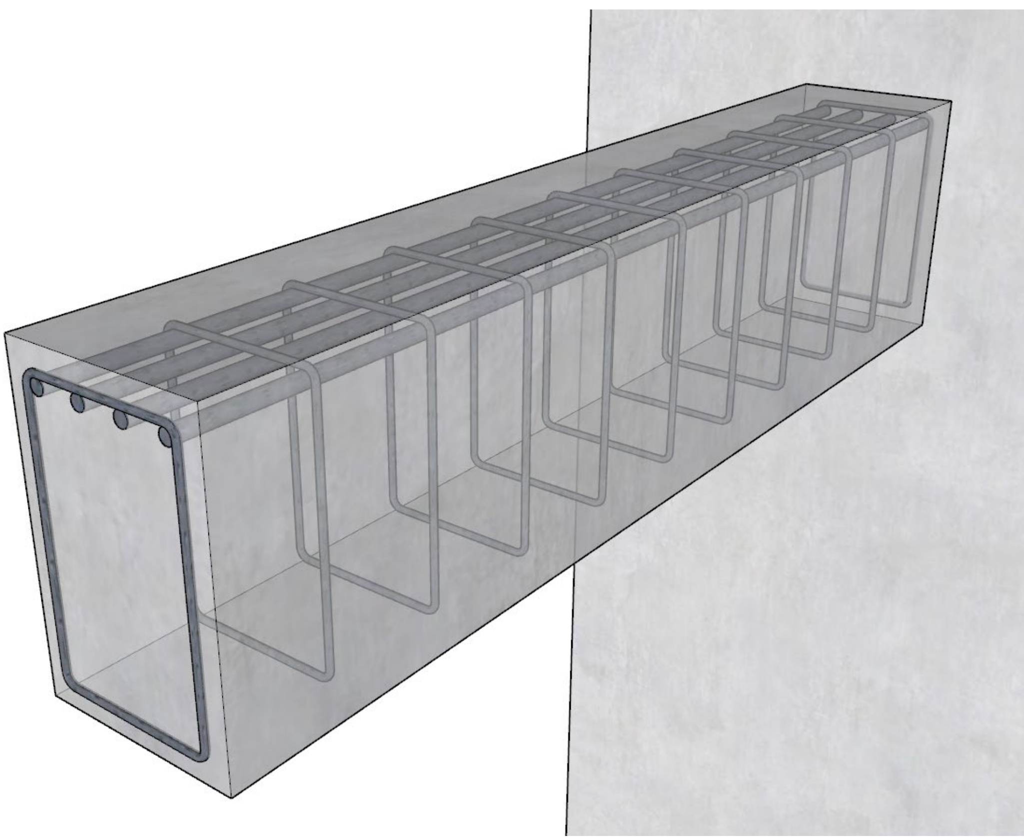
Reinforced Concrete Cantilever Beam Analysis and Design (CSA A23.3-14)





Reinforced Concrete Cantilever Beam Analysis and Design (CSA A23.3-14)

Cantilever beams consist of one span with fixed support at one end and the other end is free. There are numerous typical and practical applications of cantilever beams in buildings, bridges, industrial and special structures.

This example will demonstrate the analysis and design of the rectangular reinforced concrete cantilever beam shown below. Steps of the structural analysis, flexural design, shear design, and deflection checks will be presented. The results of hand calculations are then compared with the reference results and numerical analysis results obtained from the spBeam engineering software program by StructurePoint.

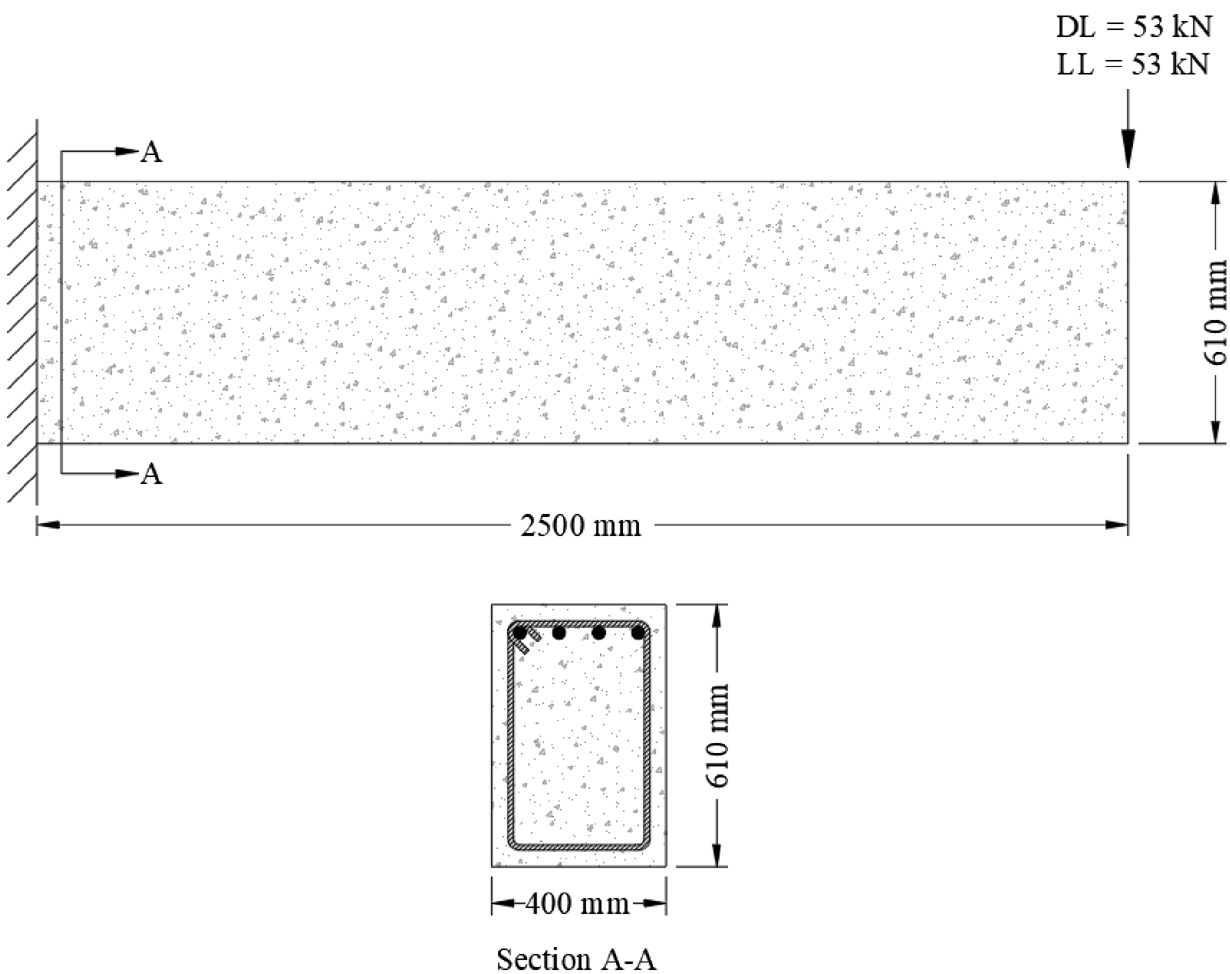


Figure 1 – Rectangular Reinforced Concrete Cantilever Beam

Contents

Code

Design of Concrete Structures (CSA A23.3-14) and Explanatory Notes on CSA Group standard A23.3-14 "Design of Concrete Structures"

References

spBeam Engineering Software Program Manual v5.00, STRUCTUREPOINT, 2015

Design Data

f_c' = 28 MPa normal weight concrete (w_c = 24 kN/m^3)

f_y = 400 MPa

Dead load, *DL* = 53 kN (self-weight is negligible) applied at the free end

Live load, *LL* = 53 kN applied at the free end

Beam span length, *L* = 2.5 m

Use No. 30M bars for longitudinal reinforcement (A_s = 700 mm^2, d_b = 29.9 mm)

Use No. 10M bars for stirrups (A_s = 100 mm^2, d_b = 11.3 mm)

Clear cover = 30 mm ***CSA A23.3-14 (Table 17)***

a_{max} = maximum aggregate size = 20 mm

Solution

1. Preliminary Member Sizing

Check the minimum beam depth requirement of ***CSA A23.3-14 (9.8.2.1)*** to waive deflection computations. Using the minimum depth for non-prestressed beams in ***Table 9.2***.

$$h_{min} = \frac{l_n}{8} = \frac{2500\ mm}{8} = 313\ mm \quad \text{(For cantilever beams)}$$ ***CSA A23.3-14 (Table 9.2)***

Therefore, since h_{min} = 313 mm < h = 610 mm the preliminary beam depth satisfies the minimum depth requirement, and the beam deflection computations are not required.

In absence of initial dimensions, the width of the rectangular section (b) may be chosen in the following range recommended by the reference:

$$\left(\frac{1}{2} \times h = 305\ \text{mm}\right) \le b = 400\ \text{mm} \le \left(\frac{2}{3} \times h = 407\ \text{mm}\right)$$ ***o.k.***

2. Load and Load combination

For the factored Load

$$w_u = 1.25 \times DL + 1.5 \times LL$$ ***CSA A23.3-14 (Annex C, Table C.1a)***

$$P_u = 1.25 \times 53 + 1.5 \times 53 = 145.75\ \text{kN}$$

Note that thee beam self-weight is neglected for comparison purposes. The effect of self-weight will be investigated later in this document.

3. Structural Analysis

Cantilever beams can be analyzed by calculating shear and moment diagrams or using Design Aid tables as shown below:

Shear and Moment Diagrams:

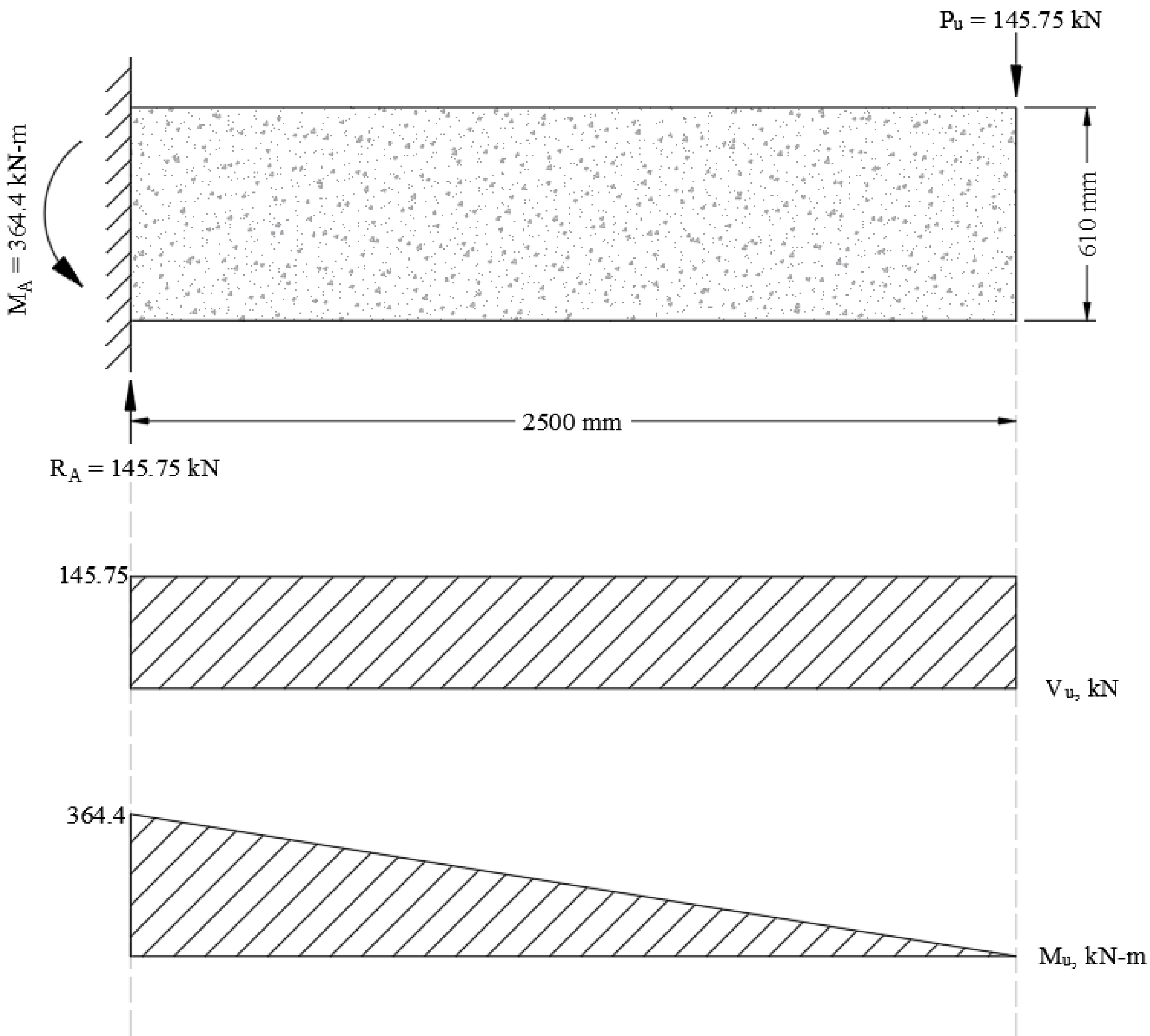


Figure 2 – Shear and Bending Moment Diagrams

Using Design Aid Tables:

$$V_u = R_A = P_u = 145.75 \text{ kN}$$

$$M_u = P_u \times L = 145.8 \times 2.5 = 364.4 \text{ kN-m}$$

CANTILEVER BEAM – CONCENTRATED LOAD AT FREE END

$R = V$ .... $= P$

M_{max} (at fixed end) .... $= P\ell$

M_x .... $= Px$

Δ_{max} (at free end) .... $= \dfrac{P\ell^3}{3EI}$

Δ_x .... $= \dfrac{P}{6EI}(2\ell^3 - 3\ell^2 x + x^3)$

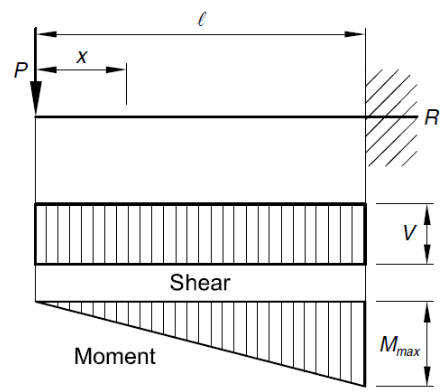


Figure 3 – Design Aid Tables (Beam Design Equations and Diagrams) – PCI Design Handbook

4. Flexural Design

4.1. Required and Provided Reinforcement

For this beam, the moment at the fixed end governs the design as shown in the previous Figure.

$$M_u = 364.4 \text{ kN-m}$$

Use 30M bars with 30 mm concrete clear cover per ***CSA A23.3-14 (Table 17)***. The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, *d*, is calculated below:

$$d = h - \left(\text{clear cover} + d_{b,stirrups} + \frac{d_{Longitudinal\ bar}}{2} \right)$$

$$d = 610 - \left(30 + 11.3 + \frac{29.9}{2} \right) = 553.75 \text{ mm}$$

In this example, *jd* is assumed equal to *0.89d.* The assumption will be verified once the area of steel is finalized.

$$jd = 0.89 \times d = 0.89 \times 553.75 = 490.6 \text{ mm}$$

$$b = 400 \text{ mm}$$

The required reinforcement at initial trial is calculated as follows:

$$A_s = \frac{M_u}{\phi_s \times f_y \times jd} = \frac{364.4 \times 10^6}{0.85 \times 400 \times 490.6} = 2184.4 \text{ mm}^2$$

$$\alpha_1 = 0.85 - 0.0015 f_c' = 0.85 - 0.0015 \times 28 = 0.81 > 0.67$$ **_CSA A23.3-14 (10.1.7)_**

$$\beta_1 = 0.97 - 0.0025 f_c' = 0.97 - 0.0025 \times 28 = 0.9 > 0.67$$ **_CSA A23.3-14 (10.1.7)_**

Recalculate '*a*' for the actual A_s = 2256.7 mm^2: $a = \frac{\phi_s \times A_s \times f_y}{\phi_c \times \alpha_1 \times f'_c \times b} = \frac{0.85 \times 2184.4 \times 400}{0.65 \times 0.81 \times 28 \times 400} = 126.3 \text{ mm}$

$$c = \frac{a}{\beta_1} = \frac{126.3}{0.9} = 140.3 \text{ mm}$$

The tension reinforcement in flexural members shall not be assumed to reach yield unless:

$$\frac{c}{d} \le \frac{700}{700 + f_y}$$ **_CSA A23.3-14 (10.5.2)_**

$$\frac{140.3}{553.75} = 0.25 \le 0.636$$

$$j = \frac{d - \frac{a}{2}}{d} = \frac{553.75 - \frac{126.3}{2}}{553.75} = 0.89$$

Therefore, the assumption that tension reinforcements will yield and *jd* equals to *0.89d* is valid.

The minimum reinforcement shall not be less than

$$A_{s,\min} = \frac{0.2 \times \sqrt{f_c'}}{f_y} \times b_t \times h = \frac{0.2\sqrt{28}}{400} \times 400 \times 610 = 645.6 \text{ mm}^2$$ **_CSA A23.3-14 (10.5.1.2)_**

Where b_t is the width of the tension zone of the section considered.

$$A_{s,req} = \max \begin{Bmatrix} A_s \\ A_{s,\min} \end{Bmatrix} = \max \begin{Bmatrix} 2184.4 \\ 645.6 \end{Bmatrix} = 2184.4 \text{ mm}^2$$

Provide 4 – 30 M bars:

$$A_{s,prov} = 4 \times 700 = 2800 \text{ mm}^2 > A_{s,req} = 2184.4 \text{ mm}^2$$

Recalculate '*a*' for the actual $A_{s,prov}$ = 2800 mm^2: $a = \frac{\phi_s \times A_{s,prov} \times f_y}{\phi_c \times \alpha_1 \times f'_c \times b} = \frac{0.85 \times 2800 \times 400}{0.65 \times 0.81 \times 28 \times 400} = 161.84 \text{ mm}$

$$M_r = \phi_s \times f_y \times A_{s,prov} \times \left(d - \frac{a}{2}\right)$$

$$M_r = 0.85 \times 400 \times 2800 \times \left(553.75 - \frac{161.84}{2}\right) = 450.13 \text{ kN-m} > M_f = 364.4 \text{ kN-m}$$

4.2. Minimum Requirements and Detailing Provisions

4.2.1. Spacing of Longitudinal Reinforcement

Check if $s_{provided}$ is greater than the minimum clear spacing (s_{min}):

$$s_{\min} = \max \begin{Bmatrix} 1.4 \times d_b \\ 1.4 \times a_{\max} \\ 30 \text{ mm} \end{Bmatrix}$$ ***CSA A23.3-14 (Annex A 6.6.5.2)***

Where a_{max} is the maximum aggregate size and is given for this example (a_{max} = 20 mm).

$$s_{\min} = \max \begin{Bmatrix} 1.4 \times 29.9 \\ 1.4 \times 20 \\ 30 \end{Bmatrix} \max \begin{Bmatrix} 42 \\ 28 \\ 30 \end{Bmatrix} = 42 \text{ mm}$$

$$s_{provided} = \frac{\left(b - 2 \times \text{clear cover} - 2 \times d_{stirrup} - n \times d_{bar}\right)}{n-1}$$

$$s_{provided} = \frac{\left(400 - 2 \times 30 - 2 \times 11.3 - 4 \times 29.9\right)}{4-1} = 66 \text{ mm} > s_{\min} = 42 \text{ mm}$$ ***o.k.***

4.2.2. Skin Reinforcement

$h = 610 \text{ mm} < 750 \text{ mm}$ skin reinforcement is not required ***CSA A23.3-14 (10.6.2)***

4.2.3. Flexural Cracking Control

Check the requirement for distribution of flexural reinforcement to control flexural cracking:

$$z = f_s (d_c A)^{1/3}$$ ***CSA A23.3-14 (10.6.1)***

Use $f_s = 0.6 f_y = 240 \text{ MPa}$ ***CAC Concrete Design Handbook – 4th Edition (2.3.2)***

$$A = \frac{2 \times X \times b}{n} = \frac{2 \times 56.25 \times 400}{4} = 11250 \text{ mm}^2$$

$$X = d_c = 30 + 11.3 + \frac{29.9}{2} = 56.25 \text{ mm}$$

$$z = 240 \times \left(56.25 \times 11250\right)^{1/3} = 20605 \text{ N/mm} < 30{,}000 \text{ N/mm}$$ ***o.k.***

5. Shear Design

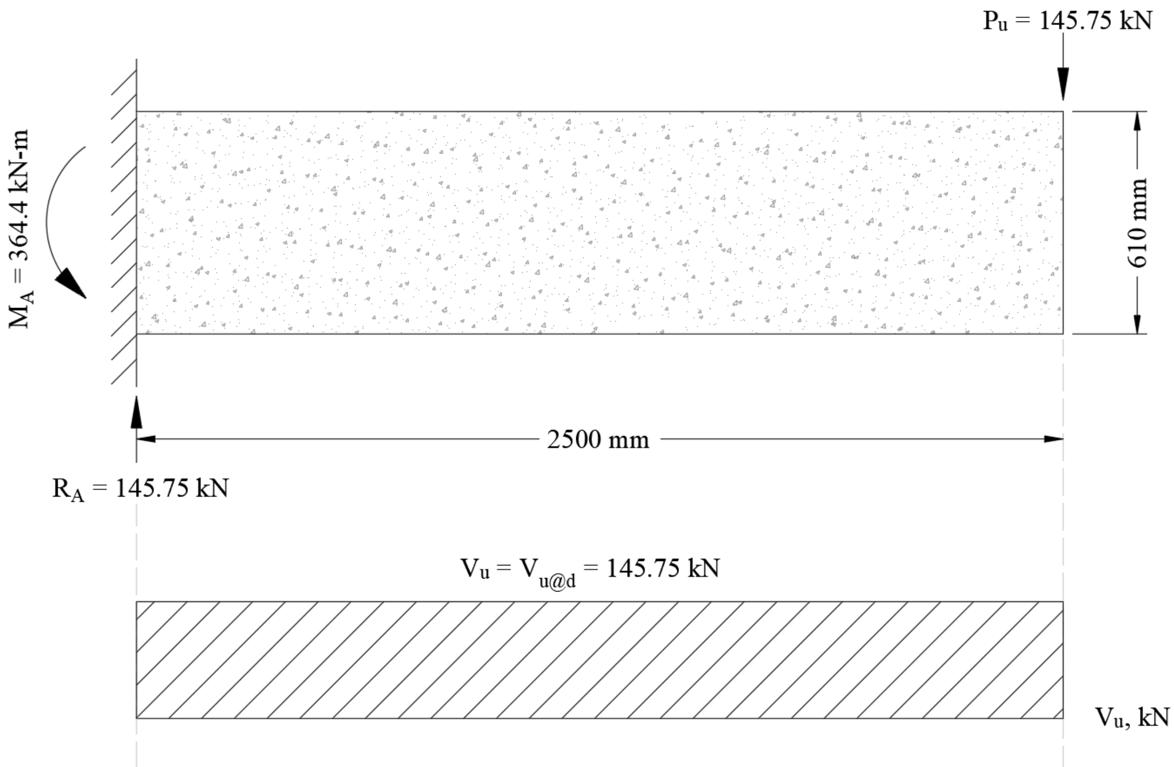


Figure 4 – Shear Diagram for Cantilever Beam

The design shear at a distance, d_v, away from the face of support,

$$d_v = \max\begin{Bmatrix} 0.9 \times d \\ 0.72 \times h \end{Bmatrix} = \max\begin{Bmatrix} 0.9 \times 553.75 \\ 0.72 \times 610 \end{Bmatrix} = \max\begin{Bmatrix} 498.4 \\ 439.2 \end{Bmatrix} = 498.4\text{ mm}$$ **_CSA A23.3-14 (3.2)_**

$$V_{f@dv} = 145.8\text{ kN}$$

The factored shear resistance shall be determined by

$$V_r = V_c + V_s + V_p = V_c$$ **_CSA A23.3-14 (Eq. 11.4)_**

However, V_r shall not exceed

$$V_{r,\max} = 0.25 \times \phi_c \times f_c^{'} \times b_w \times d_v + V_p$$ **_CSA A23.3-14 (Eq. 11.5)_**

$$V_{r,\max} = \frac{0.25 \times 0.65 \times 28 \times 400 \times 498.4}{1000} = 907.04\text{ kN}$$

Shear strength provided by concrete

$$V_c = \phi_c \times \lambda \times \beta \times \sqrt{f_c^{'}} \times b_w \times d_v$$ **_CSA A23.3-14 (Eq. 11.6)_**

$$\beta = 0.18$$ **_CSA A23.3-14 (11.3.6.3)_**

$$V_c = \frac{0.65 \times 1 \times 0.18 \times \sqrt{28} \times 400 \times 498.4}{1000} = 123.42\text{ kN} < V_{f@dv} = 145.75\text{ kN}$$ → Stirrups are required

Try 10M, two-leg stirrups (A_v = 200 mm^2).

The nominal shear strength required to be provided by shear reinforcement is

$$V_s = V_{f@dv} - V_c = 145.75 - 123.42 = 22.33 \text{ kN}$$

$$\left(\frac{A_v}{s}\right)_{req} = \frac{V_{f@dv} - V_c}{\phi \times f_{yt} \times d_v \times \cot\theta} = \frac{22.33 \times 1000}{0.85 \times 400 \times 498.4 \times \cot 35^\circ} = 0.092 \ \frac{\text{mm}^2}{\text{mm}}$$ ***CSA A23.3-14 (11.3.5.1)***

Where $\theta = 35^\circ$ ***CSA A23.3-14 (11.3.6.2)***

$$\left(\frac{A_v}{s}\right)_{\min} = \frac{0.06 \times \sqrt{f_c^{'}} \times b_w}{f_{yt}}$$ ***CSA A23.3-14 (11.2.8.2)***

$$\left(\frac{A_v}{s}\right)_{\min} = \frac{0.06 \times \sqrt{28} \times 400}{400} = 0.317 \ \frac{\text{mm}^2}{\text{mm}} > \left(\frac{A_v}{s}\right)_{req}$$

$$\therefore \left(\frac{A_v}{s}\right)_{req} = 0.317 \frac{\text{mm}^2}{\text{mm}}$$

$$s_{req} = \frac{A_v}{\left(\frac{A_v}{s}\right)_{req}} = \frac{200}{0.317} = 630 \text{ mm}$$

Check whether the required spacing based on the shear demand meets the spacing limits for shear reinforcement per ***CSA A23.3-14 (11.3.8)***.

$$0.125 \times \lambda \times \phi_c \times f_c^{'} \times b_w \times d_v > V_{f@dv}$$ ***CSA A23.3-14 (11.3.8.3)***

$$0.125 \times 1 \times 0.65 \times 28 \times 400 \times 498.4 = 453.52 \text{ kN} > V_{f@dv} = 145.75 \text{ kN}$$

Therefore, maximum stirrup spacing shall be the smallest of $0.7d_v$ and 600 mm. ***CSA A23.3-14 (11.3.8.1)***

$$s_{\max} = \min\begin{Bmatrix} 0.7 \times d_v \\ 600 \text{ mm} \end{Bmatrix} = \min\begin{Bmatrix} 0.7 \times 498.4 \\ 600 \end{Bmatrix} = \min\begin{Bmatrix} 349 \\ 600 \end{Bmatrix} = 349 \text{ mm} < s_{req}$$

$\therefore$ use $s_{provided} = 335 \text{ mm} < s_{max} = 349 \text{ mm}$

Use 10M @ 335 mm stirrups (it is more practical to round the provided spacing to 50 mm, the provided spacing is kept as 335 mm for comparison reasons with spBeam results).

$$V_r = \frac{\phi_s \times A_v \times f_y \times d_v \times \cot\theta}{s} + V_c$$ ***CSA A23.3-14 (11.3.3 and 11.5.1)***

$$V_r = \frac{0.85 \times 200 \times 400 \times 498.4 \times \cot 35^\circ}{335 \times 1000} + 123.42 = 144.48 + 123.42 = 267.89 \text{ kN} > V_{f@dv} = 145.75 \text{ kN}$$ ***o.k.***

Compute where V_f is equal to V_c, and the stirrups can be stopped ***CSA A23.3-14 (11.2.8.1)***

$$x = \frac{V_f - V_c}{V_f} \times \frac{l}{2} = \frac{145.8 - 123.42}{145.8} \times \frac{2500}{2} = 191.52 \text{ mm}$$

Use 15 – 10M @ 264 mm o.c., Place 1st stirrup 76.3 mm from the face of the support.

6. Deflection Control (Serviceability Requirements)

Since the preliminary beam depth met minimum depth requirement, the deflection calculations are not required. However, the calculations of immediate and time-dependent deflections are covered in detail in this section for illustration and comparison with spBeam model results for cantilever beam.

6.1. Immediate (Instantaneous) Deflections

Elastic analysis for three service load levels (D, $D + L_{sustained}$, $D+L_{Full}$) is used to obtain immediate deflections of the cantilever beam in this example. However, other procedures may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests.

The effective moment of inertia (I_e) is used to account for the cracking effect on the flexural stiffness of the beam. I_e for uncracked section ($M_{cr} > M_a$) is equal to I_g. When the section is cracked ($M_{cr} < M_a$), then the following equation should be used:

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \leq I_g$$

CSA A23.3-14 (Eq 9.1)

Where:

M_a = Maximum moment in member due to service loads at stage deflection is calculated.

The effective moment of inertia procedure described in the Code is considered sufficiently accurate to estimate deflections. The effective moment of inertia, I_e, was developed to provide a transition between the upper and lower bounds of I_g and I_{cr} as a function of the ratio M_{cr}/M_a.

Unless deflections are determined by a more comprehensive analysis, immediate deflection shall be computed using elastic deflection equations using the effective moment of inertia in Eq. 9.1 in CSA A23.3-14.

CSA A23.3-14 (9.8.2.3)

The values of the maximum moments for the three service load levels are calculated from structural analysis as shown previously (sustained live load = 0).

$$M_{DL} = M_{DL+LL\_sustained} = P_{DL} \times L = 53 \times 2.5 = 132.5 \text{ kN-m}$$

$$M_{DL+LL} = \left(P_{DL} + P_{LL}\right) \times L = \left(53 + 53\right) \times 2.5 = 265 \text{ kN-m}$$

M_{cr} = cracking moment.

$$M_{cr} = \frac{f_r I_g}{Y_t} = \frac{\left(\frac{3.17}{2}\right)\times\left(7.566\times10^9\right)}{305}\times10^{-6} = 39.38 \text{ kN-m}$$ ***CSA A23.3-14 (Eq. 9.2)***

f_r should be taken as half of Eq. 8.3 in CSA A23.3-14 ***CSA A23.3-14 (9.8.2.3)***

f_r = Modulus of rapture of concrete.

$$f_r = 0.6\times\lambda\times\sqrt{f_c'} = 0.6\times1.0\times\sqrt{28} = 3.17 \text{ MPa}$$ ***CSA A23.3-14 (Eq.8.3)***

I_g = Moment of inertia of the gross uncracked concrete section

$$I_g = \frac{b\times h^3}{12} = \frac{400\times610^3}{12} = 7.566\times10^9 \text{ mm}^4$$

$$y_t = \frac{h}{2} = \frac{610}{2} = 305 \text{ mm}$$

I_{cr} = moment of inertia of the cracked section transformed to concrete.

CAC Concrete Design Handbook 4th Edition (5.2.3)

The critical section at midspan is reinforced with 4 – 30M bars.

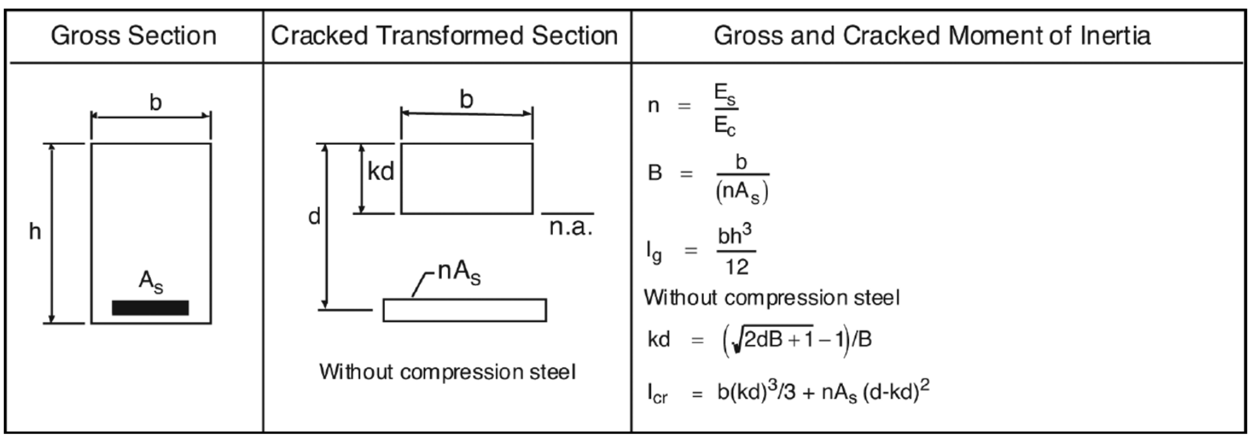


Figure 5 – Gross and Cracked Moment of Inertia of Rectangular Section (PCA Notes Table 10-2)

E_c = Modulus of elasticity of concrete.

$$E_c = \left(3300\times\sqrt{f_c'} + 6900\right)\left(\frac{\gamma_c}{2300}\right)^{1.5}$$ ***CSAA23.3-14(8.6.2.2)***

$$E_c = \left(3300\times\sqrt{28} + 6900\right)\left(\frac{2400}{2300}\right)^{1.5} = 25968 \text{ MPa}$$

$$n = \frac{E_s}{E_c} = \frac{210000}{25968} = 8.09$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$B=\frac{b}{n\ A_s}=\frac{400}{8.09\times(4\times700)}=0.018\ \text{mm}^{-1}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$kd=\frac{\sqrt{2dB+1}-1}{B}=\frac{\sqrt{2\times553.75\times0.018+1}-1}{0.018}=200.1\ \text{mm}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr}=\frac{b(kd)^3}{3}+nA_s(d-kd)^2$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr}=\frac{400\times200.1^3}{3}+8.09\times(4\times700)\times(553.75-200.1)^2=3.9002\times10^9\ \text{mm}^4$$

For dead load service load level:

$$I_{ec}=I_{cr}+(I_g-I_{cr})\left(\frac{M_{cr}}{M_a}\right)^3\ \text{, since } M_{cr}=39.38\ \text{kN.m} < M_a=132.5\ \text{kN.m}$$ ***CSA A23.3-14 (Eq. 9.1)***

$$I_e=3.9002\times10^9+(7.566\times10^9-3.9002\times10^9)\left(\frac{39.38}{132.5}\right)^3=3.9965\times10^9\ \text{mm}^4$$

The following Table provides a summary of the required parameters and calculated values needed for deflection calculation.

Table 1 – Effective Moment of Inertia Calculations (at midspan)								
I_g, mm^4 ($\times10^9$)	I_{cr}, mm^4 ($\times10^9$)	M_a, kN.m			M_{cr}, kN.m	I_e, mm^4 ($\times10^9$)		
		D	D + LL_{Sus}	D + L_{full}		D	D + LL_{Sus}	D + L_{full}
7.566	3.9002	132.5	132.5	265	39.38	3.9965	3.9965	3.9123

After obtaining the effective moment of inertia, the maximum span deflection for the cantilever beam (at the free end) can be obtained from any available procedures or design aids (see Figure 3).

$$\Delta_{max}=\frac{1}{3}\times\frac{P\times L^3}{E_c\times I_e}\ \text{(at the free end)}$$

$$\Delta_{DL}=\frac{1}{3}\times\frac{53\times2500^3}{25968\times(3.9965\times10^9)}=2.66\ \text{mm}$$

$$\Delta_{Total}=\frac{1}{3}\times\frac{(53+53)\times2500^3}{25968\times(3.9123\times10^9)}=5.43\ \text{mm}$$

$$\Delta_{LL}=\Delta_{Total}-\Delta_{DL}=5.43-2.66=2.77\ \text{mm} < \frac{L}{360}=\frac{2500}{360}=6.94\ \text{mm}$$ ***(o.k.)*** ***CSAA23.38-14 (Table 9.3)***

6.2. Time-Dependent (Long-Term) Deflections (Δ_{lt})

The additional time-dependent (long-term) deflection resulting from creep and shrinkage (Δ_{cs}) are estimated as follows.

$$\Delta_{cs} = \lambda_\Delta \times (\Delta_{sust})_{Inst}$$ ***CSA A23.3-04 (N9.8.2.5)***

The total time-dependent (long-term) deflection is calculated as:

$$(\Delta_{total})_{lt} = (\Delta_{sust})_{Inst} \times (1+\lambda_\Delta) + [(\Delta_{total})_{Inst} - (\Delta_{sust})_{Inst}]$$ ***CSA A23.3-04 (N9.8.2.5)***

Where:

$(\Delta_{sust})_{Inst}$ = Immediate (instantaneous) deflection due to sustained load, mm.

$$\xi_s = \left[1 + \frac{s}{1+50\rho'}\right]$$ ***CSA23.3-14 (Eq .9.5)***

$(\Delta_{total})_{lt}$ = Time-dependent (long-term) total delfection, mm

$(\Delta_{total})_{Inst}$ = Total immediate (instantaneous) deflection, mm

For the exterior span

s = 2, consider the sustained load duration to be 60 months or more. ***CSA A23.3-14 (9.8.2.5)***

$\rho' = 0$, conservatively.

$$\lambda_\Delta = \frac{s}{1+50\times\rho'} = \frac{2}{1+50\times 0} = 2$$ ***CSA A23.3-04 (N9.8.2.5)***

$$\Delta_{cs} = \lambda_\Delta \times (\Delta_{sust})_{Inst} = 2\times 2.66 = 5.32 \text{ mm}$$

$$\Delta_{cs} + \Delta_{LL} = 5.32 + 2.77 = 8.09 \text{ mm} \approx \frac{L}{240} = \frac{2500}{240} = 10.42 \text{ mm}$$ ***CSA A23.3-14 (Table 9.3)***

$$\xi_s = \left[1 + \frac{2}{1+50\times 0}\right] = 3$$

$$(\Delta_{total})_{lt} = 2.66\times 3 + (5.43 - 2.66) = 10.75 \text{ mm}$$

7. Cantilever Beam Analysis and Design – spBeam Software

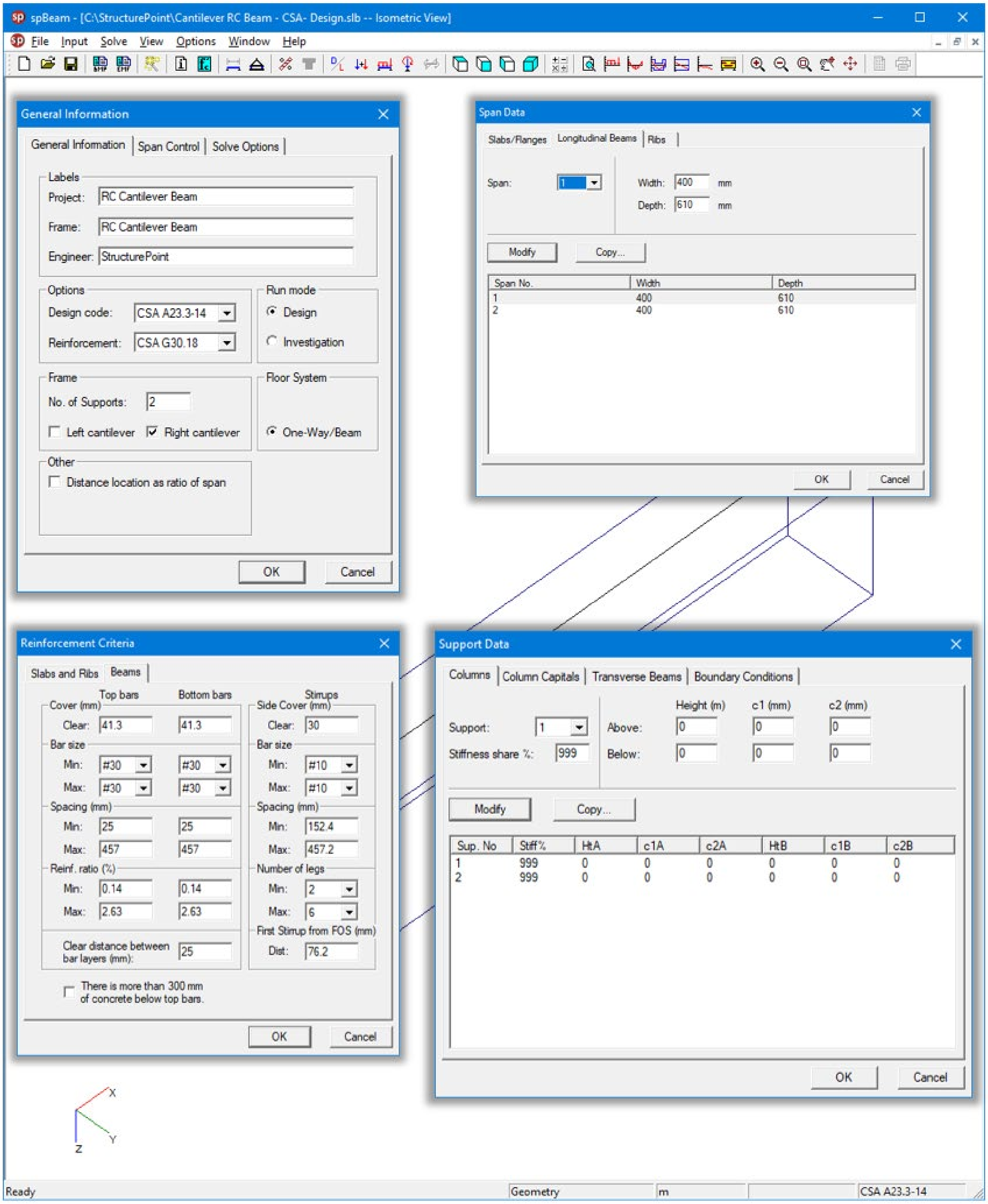
spBeam is widely used for analysis, design and investigation of beams, and one-way slab systems (including standard and wide module joist systems) per latest American (ACI 318-14) and Canadian (CSA A23.3-14) codes. spBeam can be used for new designs or investigation of existing structural members subjected to flexure, shear, and torsion loads. With capacity to integrate up to 20 spans and two cantilevers of wide variety of floor system types, spBeam is equipped to provide cost-effective, accurate, and fast solutions to engineering challenges.

spBeam provides top and bottom bar details including development lengths and material quantities, as well as live load patterning and immediate and long-term deflection results. Using the moment redistribution feature engineers can deliver safe designs with savings in materials and labor. Engaging this feature allows up to 20% reduction of negative moments over supports reducing reinforcement congestions in these areas.

Beam analysis and design requires engineering judgment in most situations to properly simulate the behavior of the targeted beam and take into account important design considerations such as: designing the beam as rectangular or T-shaped sections; using the effective flange width or the center-to-center distance between the beam and the adjacent beams. Regardless which of these options is selected, spBeam provide users with options and flexibility to:

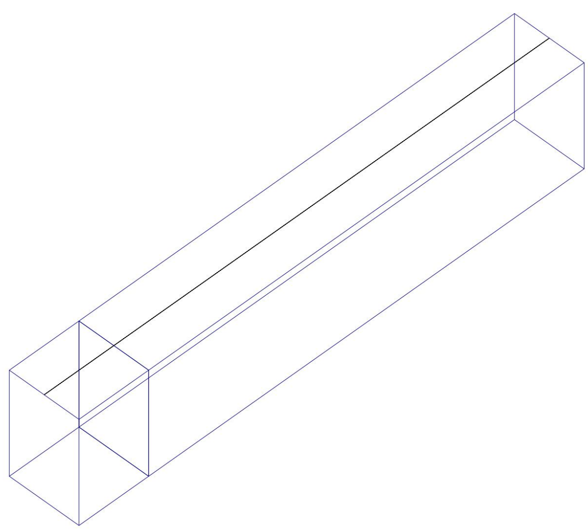
1. Design the beam as a rectangular cross-section or a T-shaped section.
2. Use the effective or full beam flange width.
3. Include the flanges effects in the deflection calculations.
4. Invoke moment redistribution to lower negative moments
5. Using gross (uncracked) or effective (cracked) moment of inertia

For illustration and comparison purposes, the following figures provide a sample of the results obtained from an spBeam model created for the cantilever beam discussed in this example.


spBeam - [C:\StructurePoint\Cantilever RC Beam - CSA- Design.slb -- Isometric View]
File
Input
Solve
View
Options
Window
Help
General Information
General Information
Span Control
Solve Options
Labels
Project:
RC Cantilever Beam
Frame:
RC Cantilever Beam
Engineer:
StructurePoint
Options
Design code:
CSA A23.3-14
Reinforcement:
CSA G30.18
Run mode
Design
Investigation
Frame
No. of Supports:
2
Left cantilever
Right cantilever
Floor System
One-Way/Beam
Other
Distance location as ratio of span
OK
Cancel
Span Data
Slabs/Flanges
Longitudinal Beams
Ribs
Span:
1
Width:
400
mm
Depth:
610
mm
Modify
Copy...
Span No.
Width
Depth
1
400
610
2
400
610
OK
Cancel
Reinforcement Criteria
Slabs and Ribs
Beams
Top bars
Bottom bars
Stirrups
Cover (mm)
Side Cover (mm)
Clear:
41.3
41.3
Clear:
30
Bar size
Min:
#30
#30
Min:
#10
Max:
#30
#30
Max:
#10
Spacing (mm)
Min:
25
25
Min:
152.4
Max:
457
457
Max:
457.2
Reinf. ratio (%)
Number of legs
Min:
0.14
0.14
Min:
2
Max:
2.63
2.63
Max:
6
Clear distance between bar layers (mm):
25
First Stirrup from FOS (mm)
Dist:
76.2
There is more than 300 mm of concrete below top bars.
OK
Cancel
Support Data
Columns
Column Capitals
Transverse Beams
Boundary Conditions
Support:
1
Stiffness share %:
999
Height (m)
c1 (mm)
c2 (mm)
Above:
0
0
0
Below:
0
0
0
Modify
Copy...
Sup. No
Stiff%
HtA
c1A
c2A
HtB
c1B
c2B
1
999
0
0
0
0
0
0
2
999
0
0
0
0
0
0
OK
Cancel
X
Y
Z
Ready
Geometry
m
CSA A23.3-14

spBeam(TM)

spBeam v5.50
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams and One-way Slab Systems



StructurePoint

Contents

1. Input Echo

1.1. General Information

File Name	C:\Struct...\Cantilever RC Beam - CSA-Design.slb
Project	RC Cantilever Beam
Frame	RC Cantilever Beam
Engineer	StructurePoint
Code	CSA A23.3-14
Reinforcement Database	CSA G30.18
Mode	Design
Number of supports =	2 + Right cantilever
Floor System	One-Way/Beam

1.2. Solve Options

Live load pattern ratio = 0%
Deflections are based on cracked section properties.
In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
Combined M-V-T reinforcement design NOT selected.
Moment redistribution NOT selected.
Effective flange width calculations NOT selected.
Rigid beam-column joint NOT selected.
Torsion analysis and design NOT selected.

1.3. Material Properties

1.3.1. Concrete: Slabs / Beams

w_c	2400	kg/m³
f'_c	28	MPa
E_c	25968	MPa
f_r	1.5875	MPa
Precast concrete	No	

1.3.2. Concrete: Columns

w_c	2400	kg/m³
f'_c	28	MPa
E_c	25968	MPa
f_r	3.1749	MPa
Precast concrete	No	

1.3.3. Reinforcing Steel

f_y	400	MPa
f_{yt}	400	MPa
E_s	210000	MPa
Epoxy coated bars	No	

1.4. Reinforcement Database

Size	Db	Ab	Wb	Size	Db	Ab	Wb
	mm	mm^2	kg/m		mm	mm^2	kg/m
#10	11	100	1	#15	16	200	2
#20	20	300	2	#25	25	500	4
#30	30	700	5	#35	36	1000	8
#45	44	1500	12	#55	56	2500	20

1.5. Span Data

1.5.1. Slabs

Span	Loc	L1	t	wL	wR	H_{min}	
		m	mm	m	m	mm	
1	Int	0.400	0	0.200	0.200	0	
2	Int	2.500	0	0.200	0.200	0	RC

1.5.2. Ribs and Longitudinal Beams

Notes:
*c - Deep beam. Additional design and bar detailing required.

Span	Ribs			Beams		Span	
	b	h	Sp	b	h	H_{min}	
	mm	mm	mm	mm	mm	mm	
1	0	0	0	400	610	25	*c
2	0	0	0	400	610	313	

1.6. Support Data

1.6.1. Columns

Support	c1a	c2a	Ha	c1b	c2b	Hb	Red %
	mm	mm	m	mm	mm	m	
1	0	0	0.000	0	0	0.000	999
2	0	0	0.000	0	0	0.000	999

1.6.2. Boundary Conditions

Support	Spring		Far End	
	K_z	K_{ry}	Above	Below
	kN/mm	kN-mm/rad		
1	0	0	Fixed	Fixed
2	0	0	Fixed	Fixed

1.7. Load Data

1.7.1. Load Cases and Combinations

Case	Dead	Live
Type	DEAD	LIVE
U1	1.250	1.500

1.7.2. Point Forces

Case/Patt	Span	Wa	La
		kN	m
Dead	2	53.00	2.500
Live	2	53.00	2.500

1.8. Reinforcement Criteria

1.8.1. Slabs and Ribs

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Bar Size		#20	#35	#20	#35
Bar spacing	mm	25	457	25	457
Reinf ratio	%	0.14	5.00	0.14	5.00
Clear Cover	mm	38		38	

There is NOT more than 300 mm of concrete below top bars.

1.8.2. Beams

	Units	Top Bars		Bottom Bars		Stirrups	
		Min.	Max.	Min.	Max.	Min.	Max.
Bar Size		#30	#30	#30	#30	#10	#10
Bar spacing	mm	25	457	25	457	152	457
Reinf ratio	%	0.14	2.63	0.14	2.63		
Clear Cover	mm	41		41			
Layer dist.	mm	25		25			
No. of legs						2	6
Side cover	mm					30	
1st Stirrup	mm					76	

There is NOT more than 300 mm of concrete below top bars.

2. Design Results

2.1. Top Reinforcement

Notes:
*3 - Design governed by minimum reinforcement.

Span	Zone	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars	
		m	kNm	m	mm²	mm²	mm²	mm		
1	Left	0.40	0.00	0.000	0	5487	0	0	---	
	Midspan	0.40	0.00	0.140	0	5487	0	0	---	
	Right	0.40	0.00	0.400	646	5487	0	94	4-#30	*3
2	Left	0.40	364.37	0.000	646	5487	2184	94	4-#30	
	Midspan	0.40	236.84	0.875	646	5487	1354	94	4-#30	
	Right	0.40	127.53	1.625	646	5487	703	94	4-#30	

2.2. Top Bar Details

	Left				Continuous		Right			
Span	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
		m		m		m		m		m
1	---		---		---		2-#30	0.40	2-#30	0.40
2	---		---		4-#30	2.50	---		---	

2.3. Top Bar Development Lengths

	Left				Continuous		Right			
Span	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen
		mm		mm		mm		mm		mm
1	---		---		---		2-#30	300.00	2-#30	300.00
2	---		---		4-#30	524.59	---		---	

2.4. Bottom Reinforcement

Span	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars
	m	kNm	m	mm²	mm²	mm²	mm	
1	0.40	0.00	0.400	0	5487	0	0	---
2	0.40	0.00	1.250	0	5487	0	0	---

2.5. Bottom Bar Details

	Long Bars			Short Bars		
Span	Bars	Start	Length	Bars	Start	Length
		m	m		m	m
1	---			---		
2	---			---		

2.6. Bottom Bar Development Lengths

	Long Bars		Short Bars	
Span	Bars	DevLen	Bars	DevLen
		mm		mm
1	---		---	
2	---		---	

2.7. Flexural Capacity

	Top							Bottom					
Span	x	$A_{s,top}$	ΦM_n-	M_u-	Comb	Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb	Pat	Status
	m	mm²	kNm	kNm				mm²	kNm	kNm			
1	0.000	2800	-450.13	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.140	2800	-450.13	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.200	2800	-450.13	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.260	2800	-450.13	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.300	2800	-450.13	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.400	2800	-450.13	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
2	0.000	2800	-450.13	-364.37	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.875	2800	-450.13	-236.84	U1	All	OK	0	0.00	0.00	U1	All	OK
	1.250	2800	-450.13	-182.19	U1	All	OK	0	0.00	0.00	U1	All	OK
	1.625	2800	-450.13	-127.53	U1	All	OK	0	0.00	0.00	U1	All	OK
	2.500	2800	-450.13	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK

2.8. Longitudinal Beam Transverse Reinforcement Demand and Capacity

2.8.1. Section Properties

Span	d_v	$(A_v/s)_{min}$	ΦV_c	$V_{r,max}$
	mm	mm²/mm	kN	kN
1	498.4	0.317	123.42	907.04
2	498.4	0.317	123.42	907.04

2.8.2. Beam Transverse Reinforcement Demand

Notes:
*8 - Minimum transverse (stirrup) reinforcement governs.

			Required				Demand	
Span	Start	End	X_u	V_u	Comb/Patt	A_v/s	A_v/s	
	m	m	m	kN		mm²/mm	mm²/mm	
1	0.076	0.324	0.200	0.00	U1/All	0.000	0.000	
2	0.076	1.166	0.498	145.75	U1/All	0.092	0.317	*8
	1.166	1.833	1.166	145.75	U1/All	0.092	0.317	*8
	1.833	2.500	1.833	145.75	U1/All	0.092	0.317	*8

2.8.3. Beam Transverse Reinforcement Details

Span	Size	Stirrups (2 legs each unless otherwise noted)
1	#10	--- None ---
2	#10	8 @ 335

2.8.4. Beam Transverse Reinforcement Capacity

Notes:
*8 - Minimum transverse (stirrup) reinforcement governs.

			Required					Provided				
Span	Start	End	X_u	V_u	Comb/Patt	A_v/s	Reqd/Min	A_v	Sp	A_v/s	ΦV_n	
	m	m	m	kN		mm²/mm		mm²	mm	mm²/mm	kN	
1	0.000	0.400	0.200	0.00	U1/All	0.000	0.00	-----	-----	-----	105.25	
2	0.000	0.076	0.498	145.75	U1/All	-----	-----	-----	-----	-----	-----	
	0.076	2.424	0.498	145.75	U1/All	0.092	0.29	200.0	335	0.596	267.73	*8
	2.424	2.500	2.424	145.75	U1/All	-----	-----	-----	-----	-----	-----	

2.9. Slab Shear Capacity

Span	b	d_v	β	V_{ratio}	ΦV_c	V_u	X_u
	mm	mm			kN	kN	m
1	--- Not checked ---						
2	--- Not checked ---						

2.10. Material TakeOff

2.10.1. Reinforcement in the Direction of Analysis

Top Bars	63.7 kg	<=>	21.98 kg/m	<=>	54.950 kg/m²
Bottom Bars	0.0 kg	<=>	0.00 kg/m	<=>	0.000 kg/m²
Stirrups	11.0 kg	<=>	3.79 kg/m	<=>	9.463 kg/m²
Total Steel	74.7 kg	<=>	25.77 kg/m	<=>	64.413 kg/m²
Concrete	0.7 m³	<=>	0.24 m³/m	<=>	0.610 m³/m²

3. Deflection Results: Summary

3.1. Section Properties

3.1.1. Frame Section Properties

Notes:
M+ve values are for positive moments (tension at bottom face).
M-ve values are for negative moments (tension at top face).

Span	Zone	M_{+ve}			M_{-ve}		
		I_g	I_{cr}	M_{cr}	I_g	I_{cr}	M_{cr}
		mm^4	mm^4	kNm	mm^4	mm^4	kNm
1	Left	7.566e+009	0	39.38	7.566e+009	0	-39.38
	Midspan	7.566e+009	0	39.38	7.566e+009	3.9002e+009	-39.38
	Right	7.566e+009	0	39.38	7.566e+009	3.9002e+009	-39.38
2	Left	7.566e+009	0	39.38	7.566e+009	3.9002e+009	-39.38
	Midspan	7.566e+009	0	39.38	7.566e+009	3.9002e+009	-39.38
	Right	7.566e+009	0	39.38	7.566e+009	3.9002e+009	-39.38

3.1.2. Frame Effective Section Properties

Span	Zone	Weight	Load Level					
			Dead		Sustained		Dead+Live	
			M_{max}	I_e	M_{max}	I_e	M_{max}	I_e
			kNm	mm^4	kNm	mm^4	kNm	mm^4
1	Left	0.150	0.00	7.566e+009	0.00	7.566e+009	0.00	7.566e+009
	Middle	0.700	0.00	7.566e+009	0.00	7.566e+009	0.00	7.566e+009
	Right	0.150	0.00	7.566e+009	0.00	7.566e+009	0.00	7.566e+009
	Span Avg	----	----	7.566e+009	----	7.566e+009	----	7.566e+009
2	Left	1.000	-132.50	3.9965e+009	-132.50	3.9965e+009	-265.00	3.9123e+009
	Span Avg	----	----	3.9965e+009	----	3.9965e+009	----	3.9123e+009

3.2. Instantaneous Deflections

3.2.1. Extreme Instantaneous Frame Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Sustained	Live		Total	
						Unsustained	Total	Sustained	Dead+Live
1	Down	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
	Up	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
2	Down	Def	mm	2.66	---	2.77	2.77	2.66	5.43
		Loc	m	2.500	---	2.500	2.500	2.500	2.500
	Up	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---

3.3. Long-term Deflections

3.3.1. Long-term Deflection Factors

Notes:
Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

Span	Zone	M_{+ve}					M_{-ve}				
		$A_{s,top}$	b	d	Rho'	Lambda	$A_{s,bot}$	b	d	Rho'	Lambda
		mm^2	mm	mm	%		mm^2	mm	mm	%	
1	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

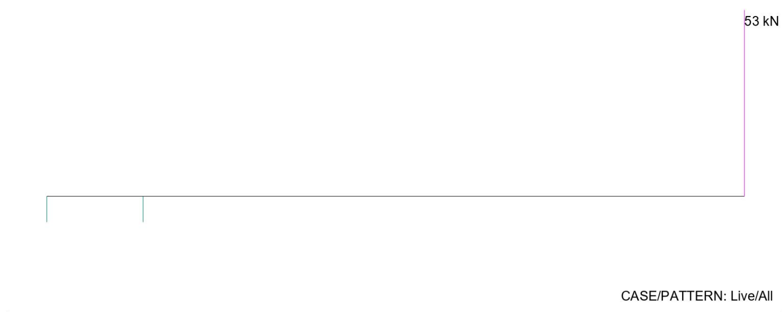
3.3.2. Extreme Long-term Frame Deflections and Corresponding Locations

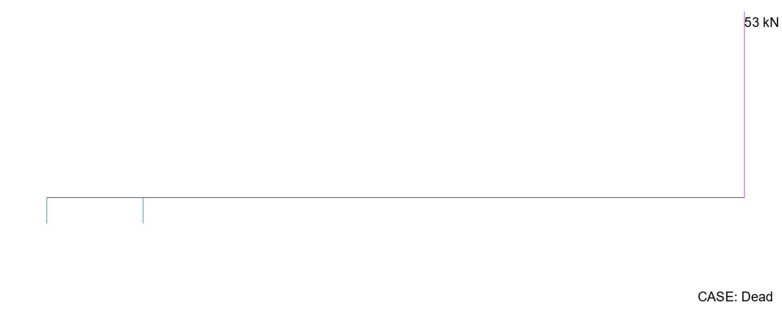
Notes:
Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
1	Down	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
	Up	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
2	Down	Def	mm	5.32	8.09	8.09	10.75
		Loc	m	2.500	2.500	2.500	2.500
	Up	Def	mm	---	---	---	---
		Loc	m	---	---	---	---

4. Diagrams

4.1. Loads





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File: C:\StructurePoint\Cantilever RC Beam - CSA- Design.slb

Project: RC Cantilever Beam

Frame: RC Cantilever Beam

Engineer: StructurePoint

Code: CSA A23.3-14

Date: 10/14/21

Time: 10:29:31

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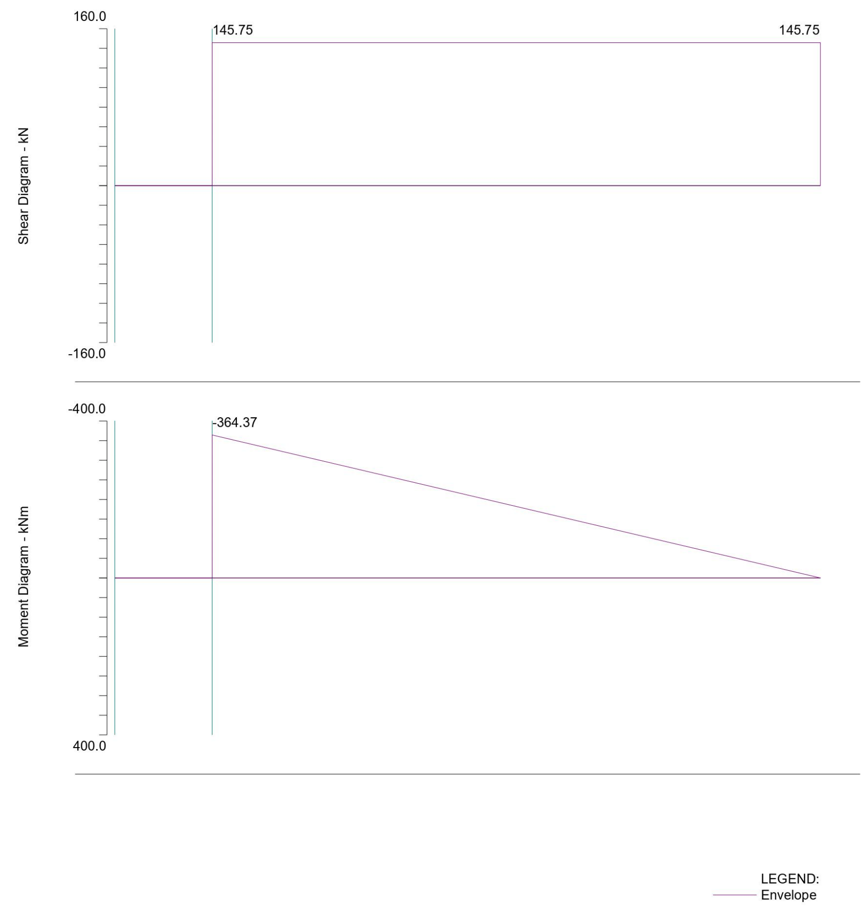
10:37

4.2. Internal Forces



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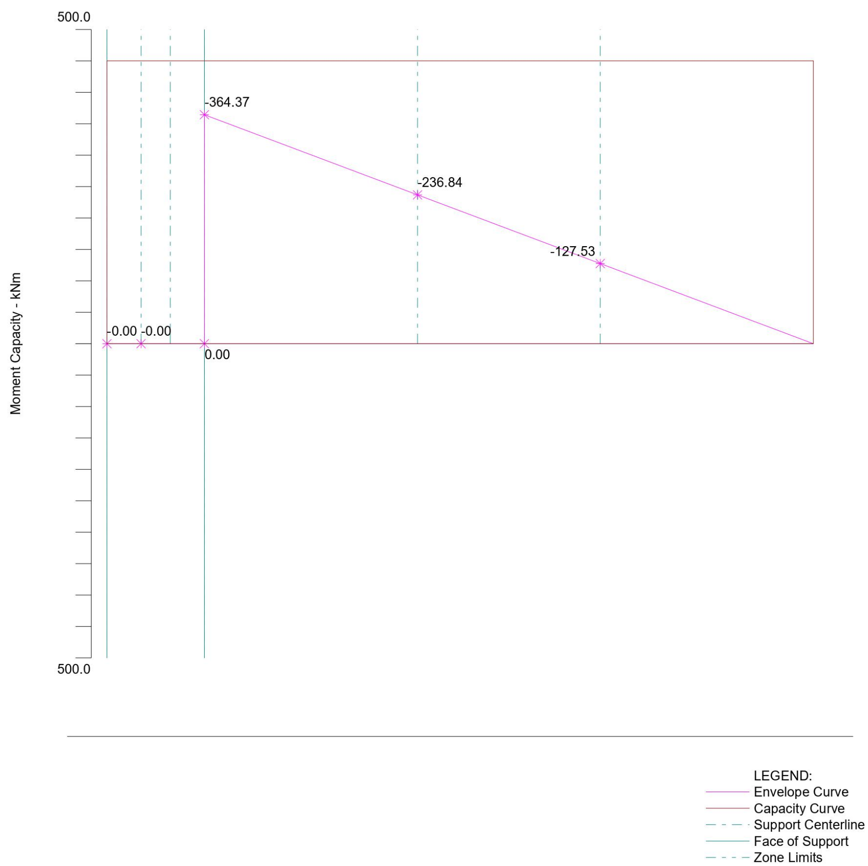
4.3. Moment Capacity



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Project: RC Cantilever Beam

Frame: RC Cantilever Beam

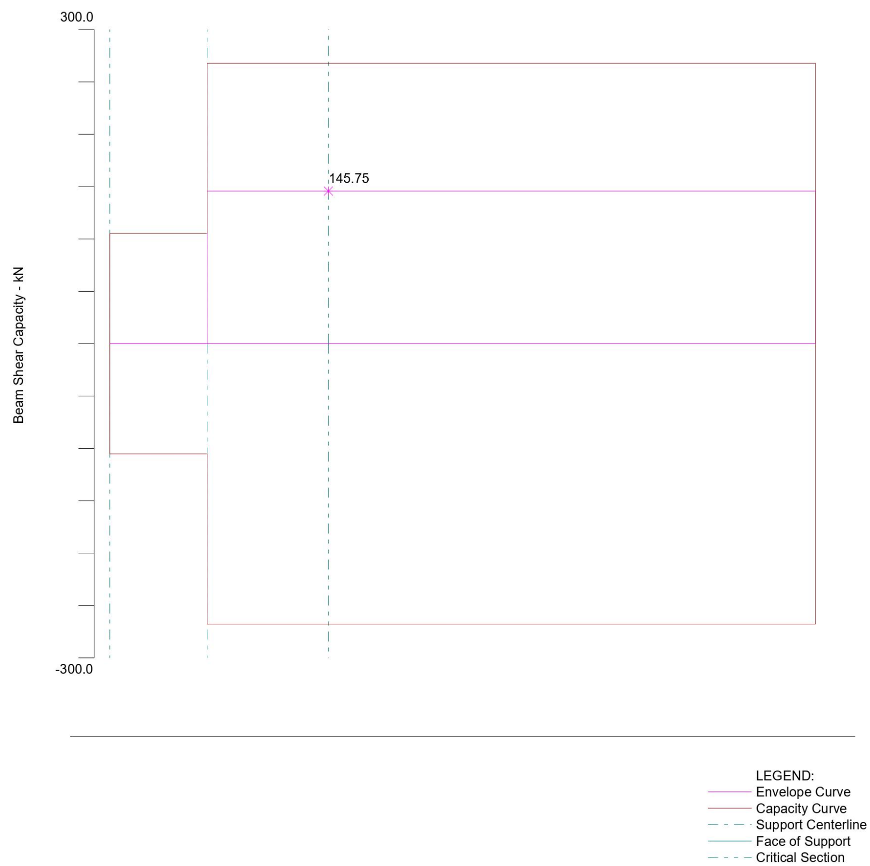
Engineer: StructurePoint

Code: CSA A23.3-14

Date: 10/14/21

Time: 10:29:31

4.4. Shear Capacity



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File: C:\StructurePoint\Cantilever RC Beam - CSA- Design.slb

Project: RC Cantilever Beam

Frame: RC Cantilever Beam

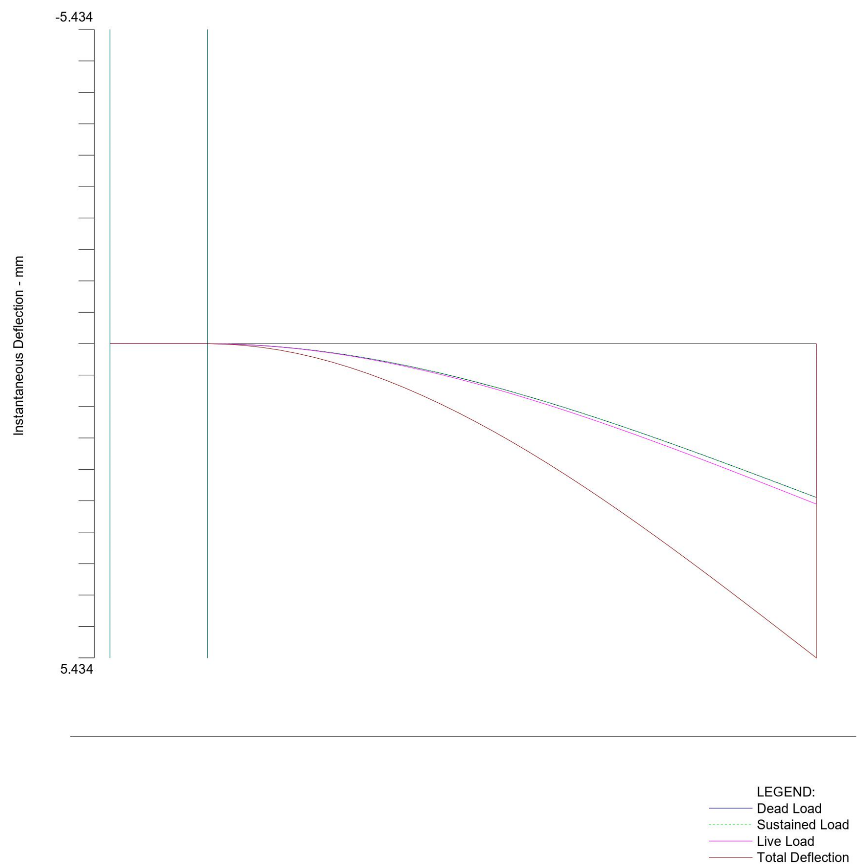
Engineer: StructurePoint

Code: CSA A23.3-14

Date: 10/14/21

Time: 10:29:31

4.5. Deflection



spBeam v5.50. Licensed to: STRUCTUREPOINT. License ID: 00000-0000000-4-20C84-20C84

File: C:\StructurePoint\Cantilever RC Beam - CSA- Design.slb

Project: RC Cantilever Beam

Frame: RC Cantilever Beam

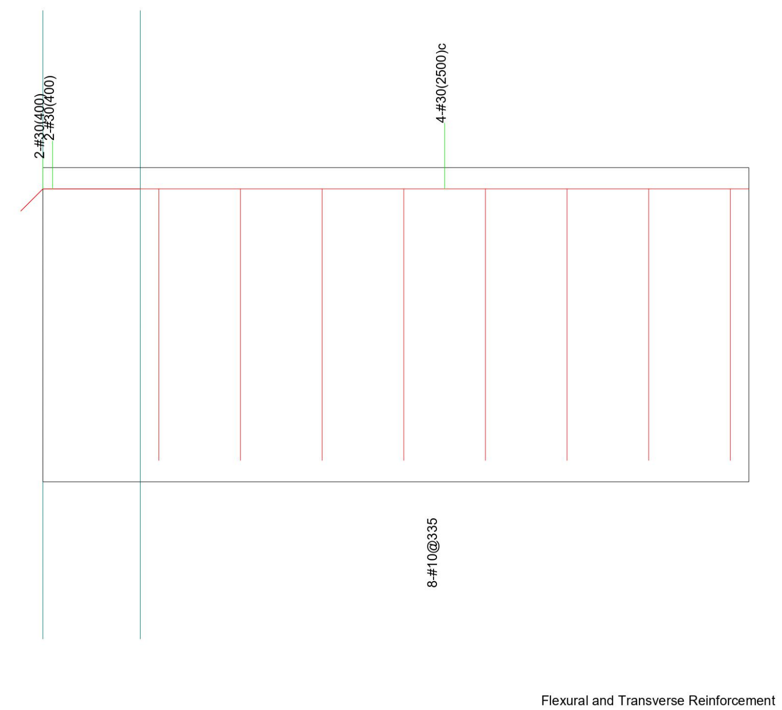
Engineer: StructurePoint

Code: CSA A23.3-14

Date: 10/14/21

Time: 10:29:31

4.6. Reinforcement



spBeam v5.50. Licensed to: STRUCTUREPOINT. License ID: 00000-0000000-4-20C84-20C84

File: C:\StructurePoint\Cantilever RC Beam - CSA- Design.slb

Project: RC Cantilever Beam

Frame: RC Cantilever Beam

Engineer: StructurePoint

Code: CSA A23.3-14

Date: 10/14/21

Time: 10:29:31

8. Analysis and Design Results Comparison and Conclusions

The following tables show the comparison between hand results and spBeam model results.

Table 2 - Comparison of Moments and Flexural Reinforcement (At Fixed End)				
Location	M_f, kN-m	Reinforcement	$A_{s,provided}$, mm^2	M_r, kN-m
Hand	364.37	4 – 30M	2800	450.13
spBeam	364.37	4 – 30M	2800	450.13

Table 3 - Comparison of Shear and lateral Reinforcement									
V_f^*, kN		$(A_v/s)_{req}^{**}$, mm^2/mm		$(A_v/s)_{min}^{**}$, mm^2/mm		Reinforcement		V_r, kN	
Hand	spBeam	Hand	spBeam	Hand	spBeam	Hand	spBeam	Hand	spBeam
145.75	145.75	0.092	0.092	0.317	0.317	10M @ 335 mm	10M @ 335 mm	267.89	267.73

* Shear values are taken at distance d_v from the faces of supports
** Minimum transverse reinforcement governs

Table 4 - Comparison of Section Properties								
Location	I_{cr}, mm^4 ($\times 10^9$)		I_e, mm^4 ($\times 10^9$)					
	Hand	spBeam	Hand			spBeam		
			DL	DL+LL_{sus}	Total	DL	DL+LL_{sus}	Total
Midspan	3.9002	3.9002	3.9965	3.9965	3.9123	3.9965	3.9965	3.9123

Table 5 - Comparison of Maximum Instantaneous Deflection (At Free End), mm		
Deflection Type	Hand	spBeam
Δ_{DL}	2.66	2.66
Δ_{LL}	2.77	2.77
Δ_{total}	5.43	5.43

Table 6 - Comparison of Maximum Long-Term Deflection (At Free End), mm		
Deflection Type	Hand	spBeam
Δ_{cs}	5.32	5.32
$\Delta_{cs} + \Delta_{LL}$	8.09	8.09
$(\Delta_{total})_{lt}$	10.75	10.75

The results of all the hand calculations used illustrated above are in agreement with the automated exact results obtained from the spBeam program.